

# *Distributed Differential Privacy* in **Multi-Armed Bandits**

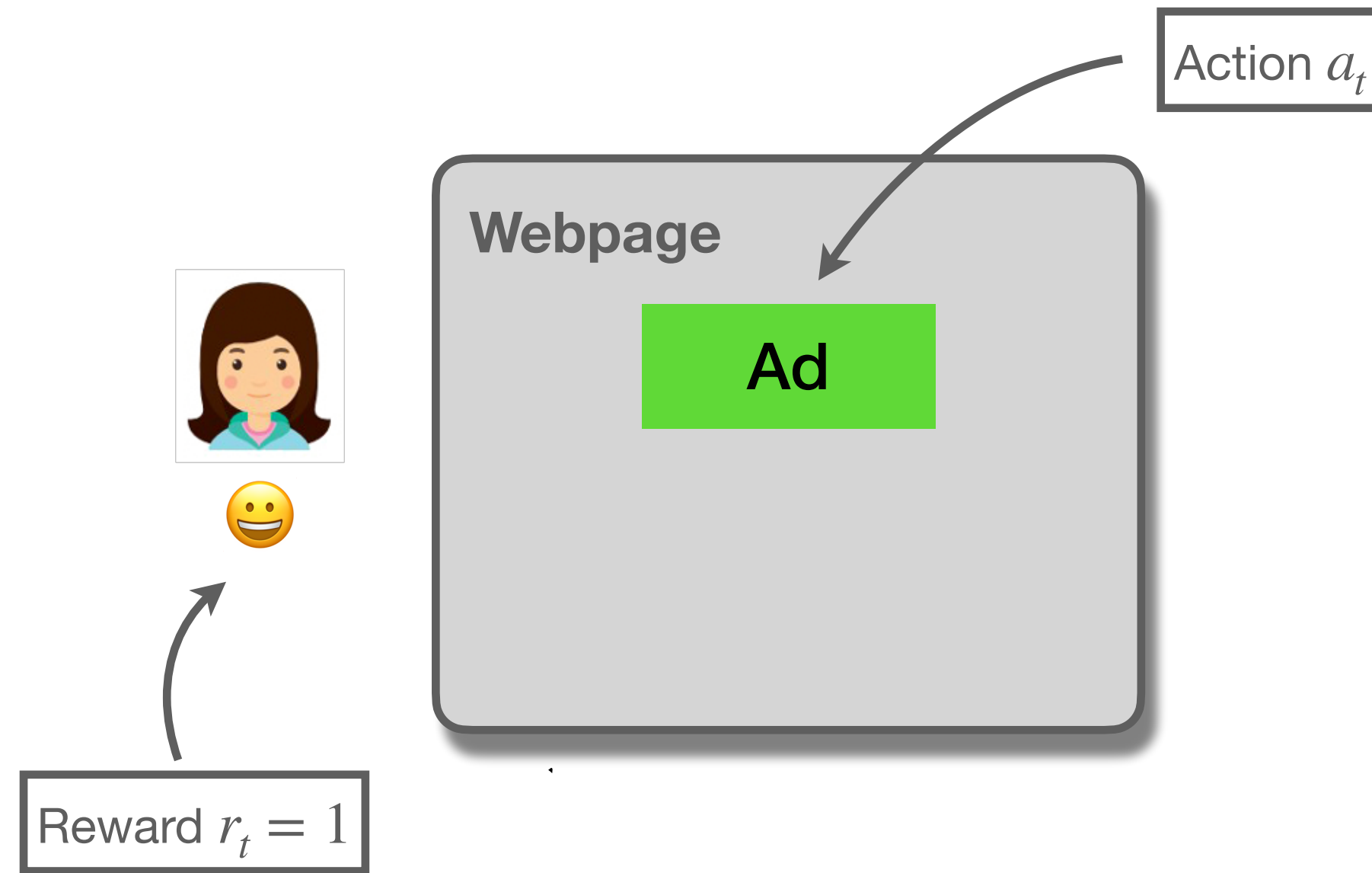
**Xingyu Zhou\***, Sayak Ray Chowdhury\*

Wayne State University, Microsoft Research, India

\* Equal Contributions

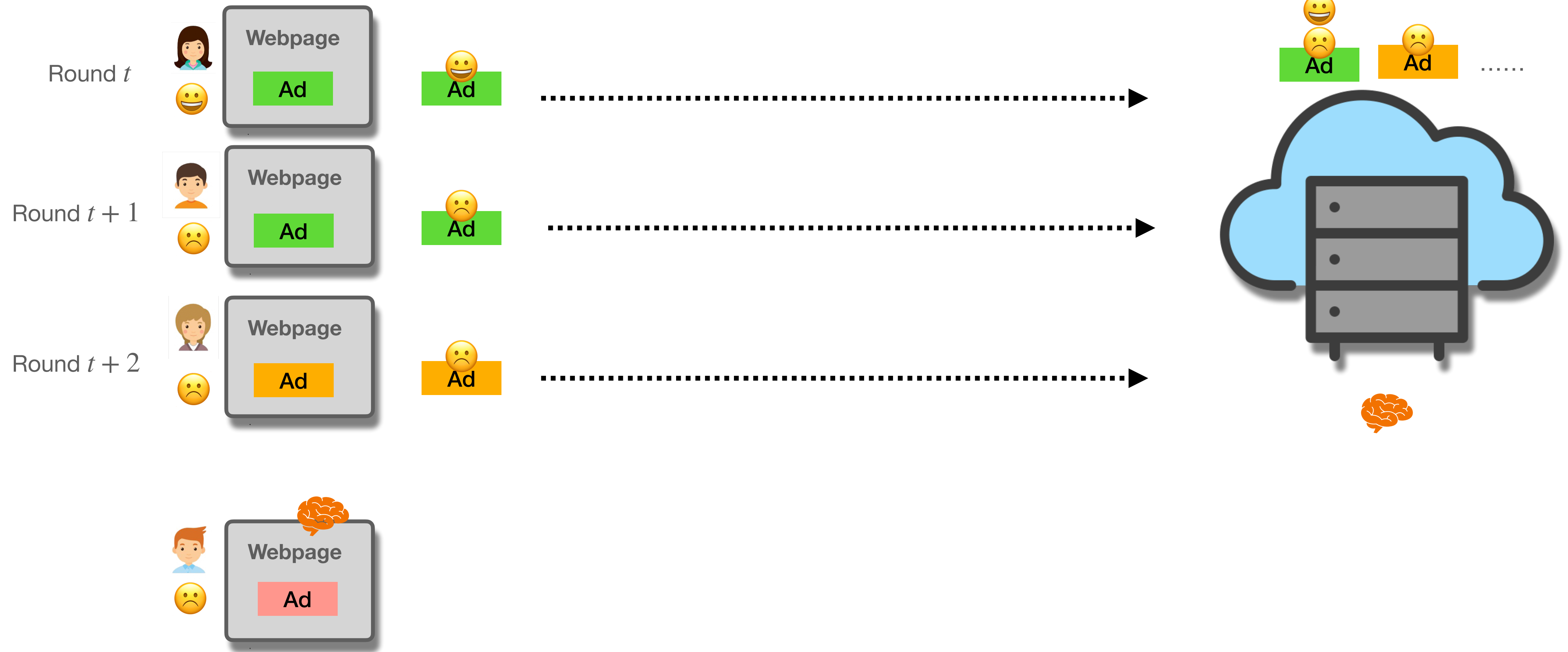
# Multi-Armed Bandits

## Ads recommendation example



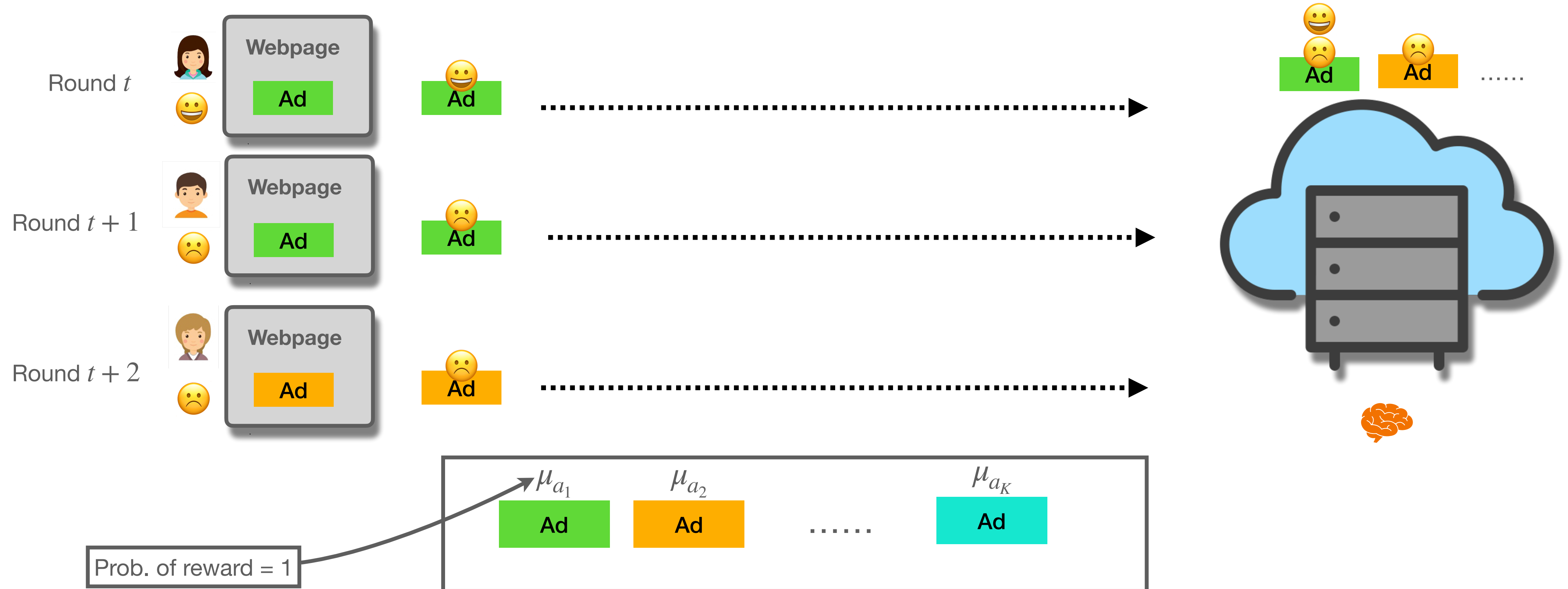
# Multi-Armed Bandits

## Ads recommendation example



# Multi-Armed Bandits

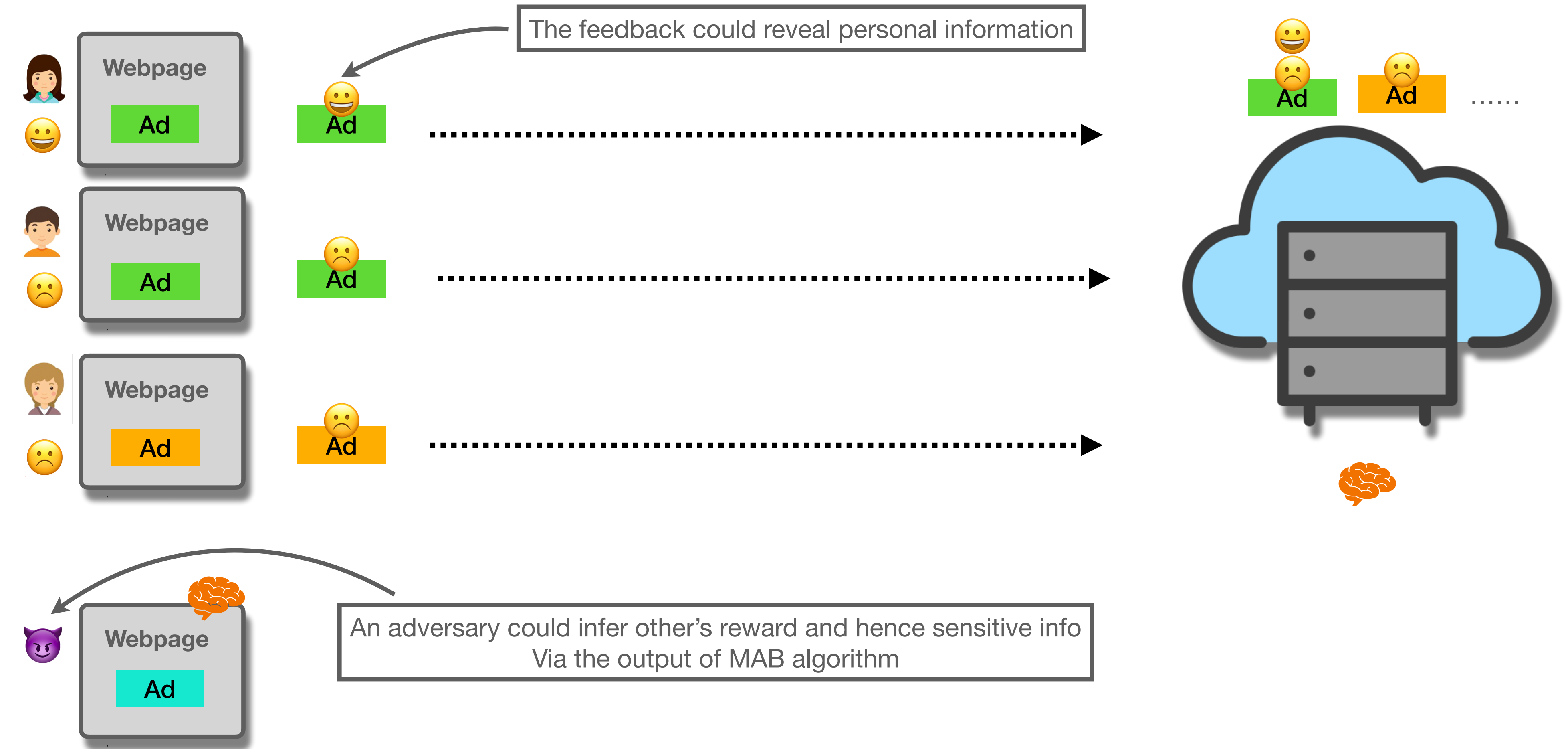
## Regret minimization



$$\mathbb{E}[\text{Reg}(T)] = T \cdot \mu_{a^*} - \mathbb{E} \left[ \sum_{t=1}^T \mu_{a_t} \right]$$

# Privacy Concern

## Reward is sensitive



# Differential Privacy

## Central model

### Differential Privacy

For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$

$$\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$$

# Differential Privacy

## Central model

### Differential Privacy

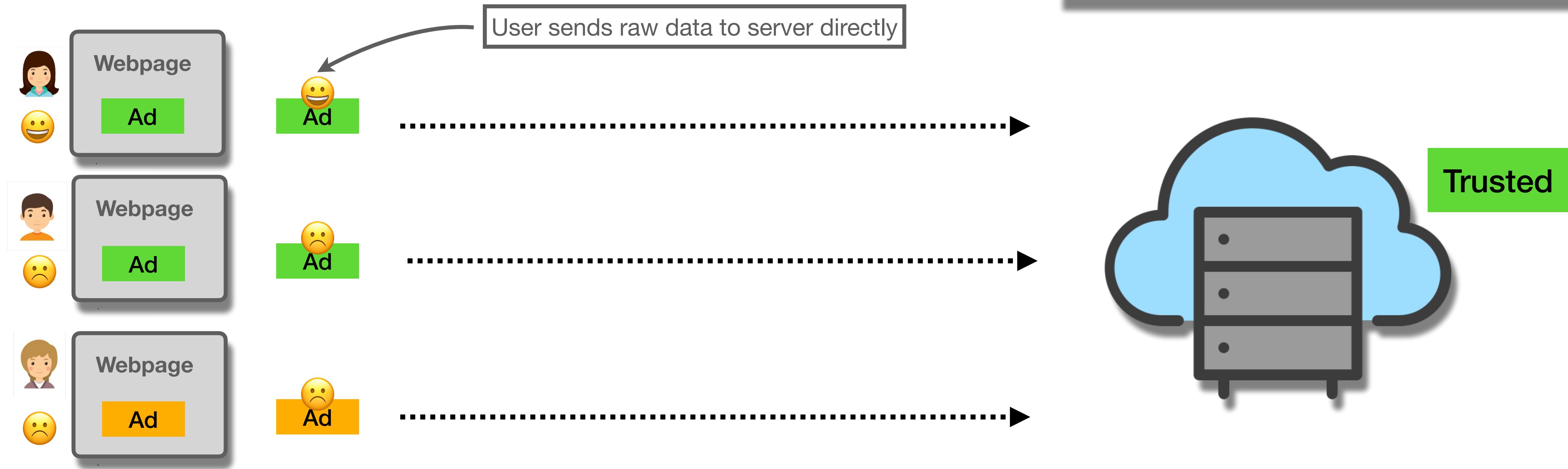
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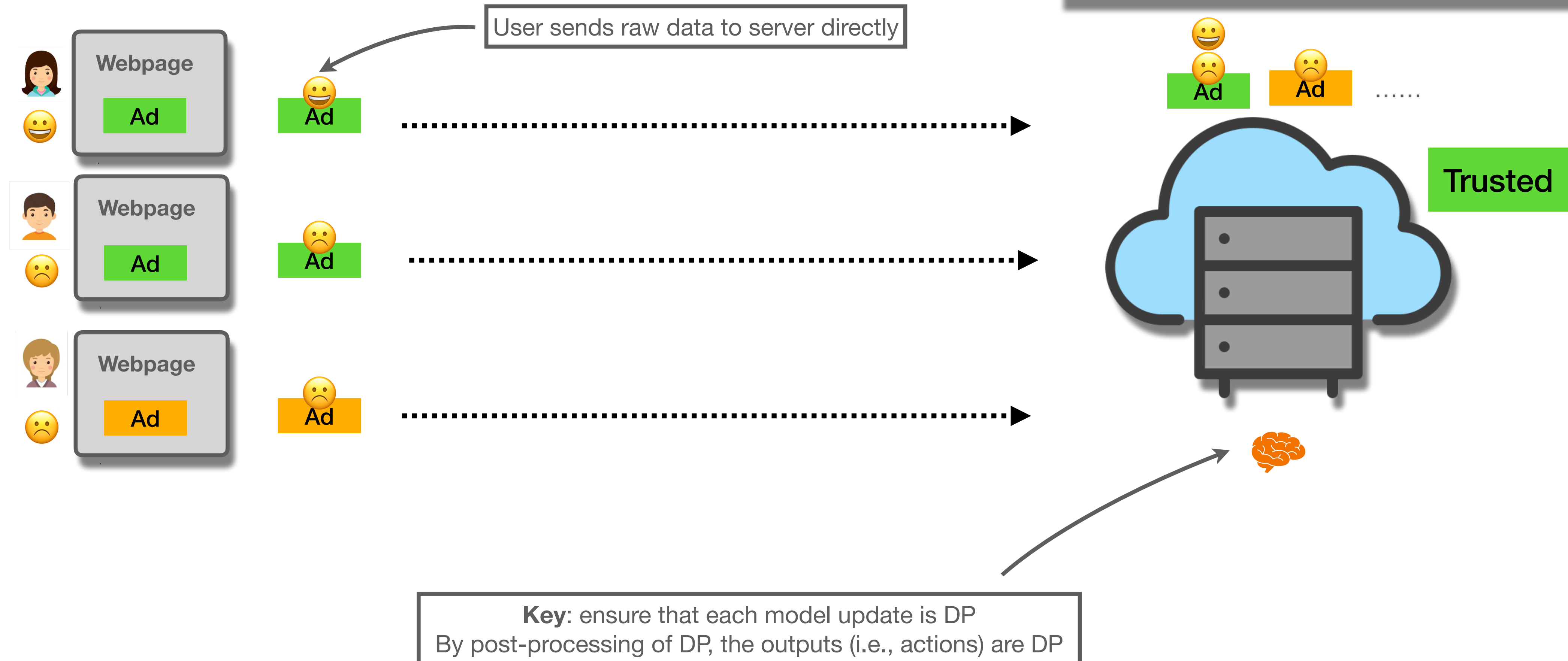
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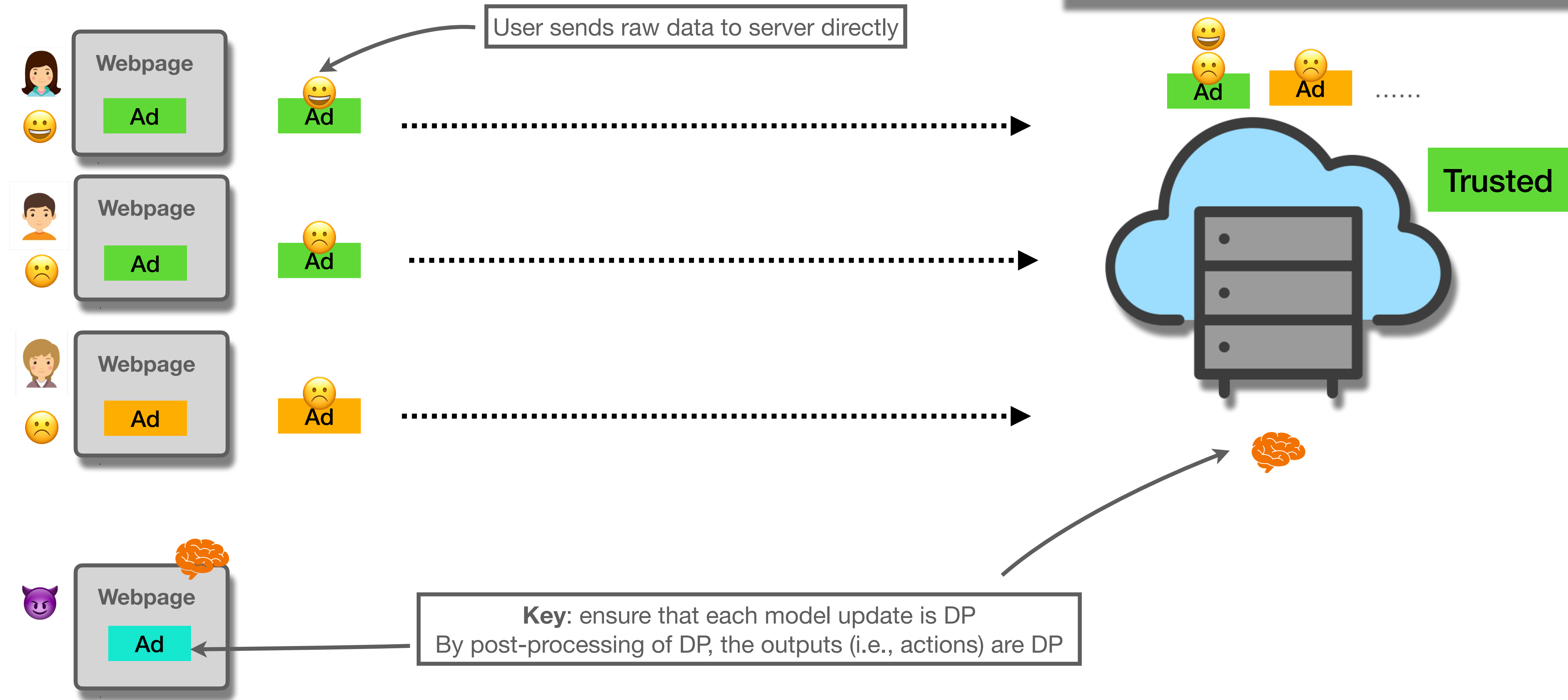
# Differential Privacy

## Central model

### Differential Privacy

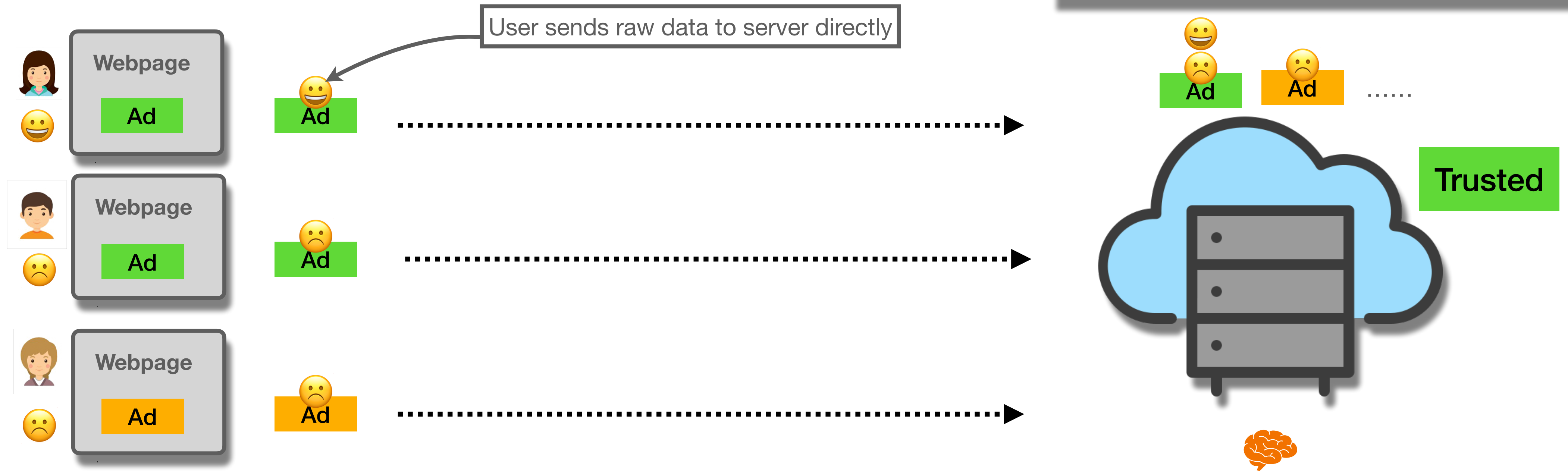
For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$

$$\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$$



# Optimal Regret in MAB

## Central model



**Differential Privacy  $(\epsilon, \delta)$**   
 For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$   
 $\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$

[Sajed & Sheffet'19]  
**Algorithm:** successive arm elimination + batching + Laplace noise  
**Privacy :** pure DP , i.e.,  $(\epsilon, 0)$ -DP  
**Regret:** optimal non-private regret +  $\Theta\left(\frac{K \log T}{\epsilon}\right)$

Only an additive privacy cost  
 And it is optimal!

# Differential Privacy

## Local model

### Differential Privacy

For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$

$$\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$$



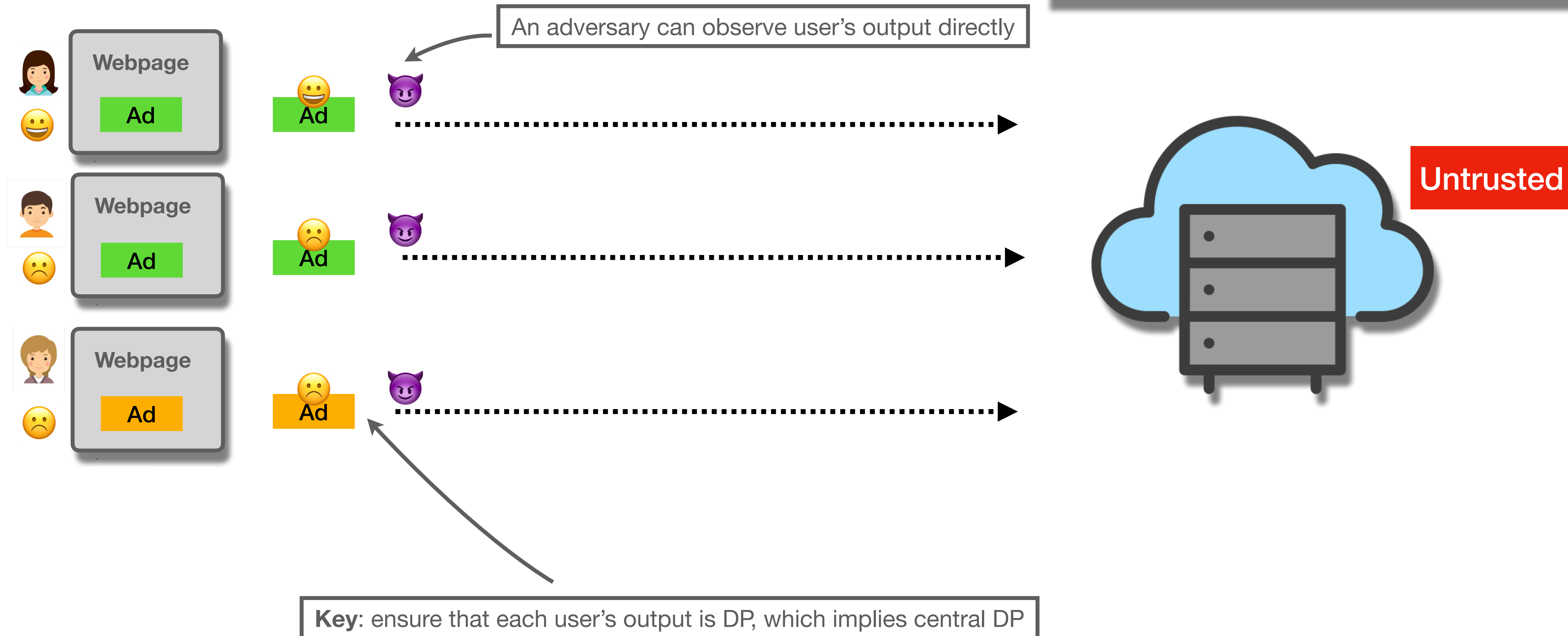
# Differential Privacy

## Local model

### Differential Privacy

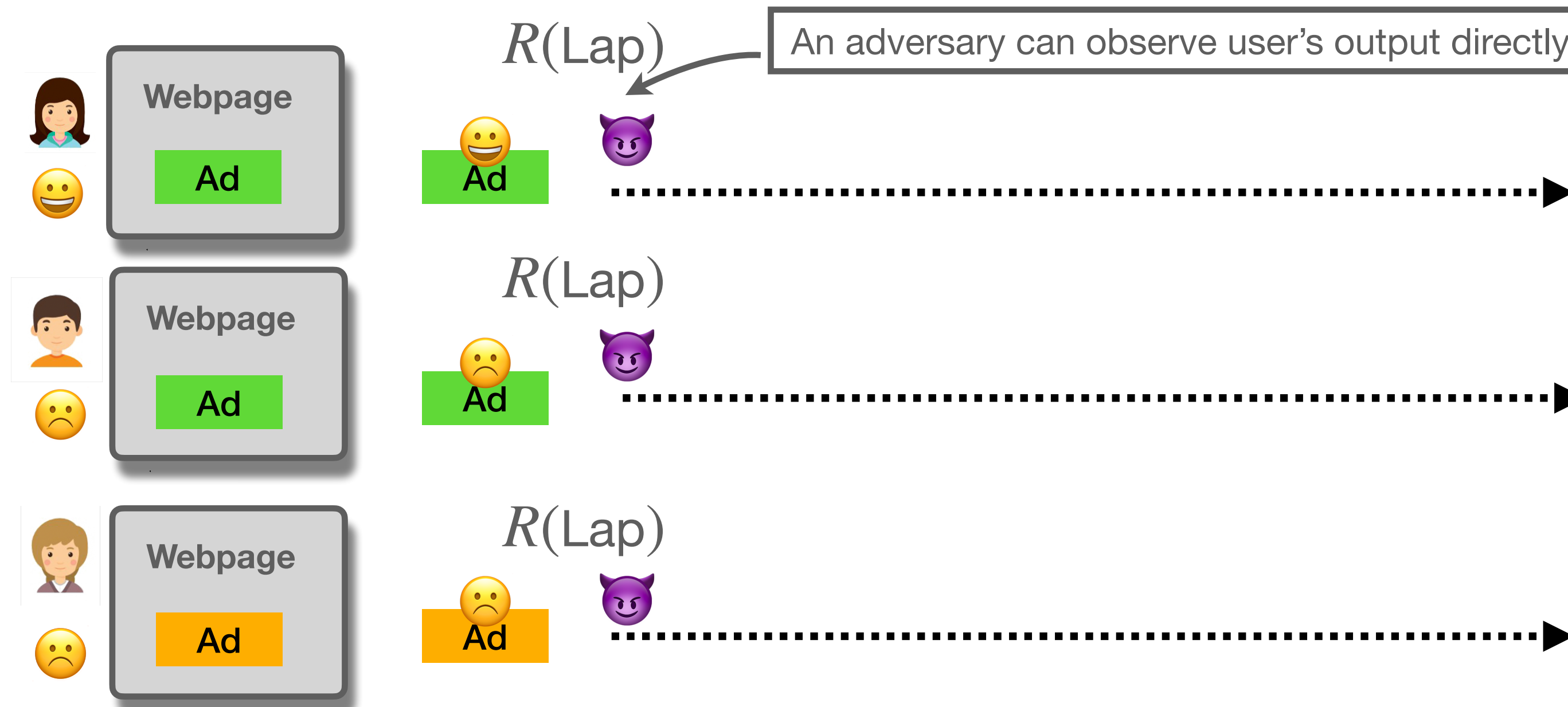
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# Optimal Regret in MAB

## Local model



**Differential Privacy**

For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$

$$\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$$


[Ren, Zhou, Liu, Shroff'20]

**Algorithm:** Laplace noise at local side + UCB

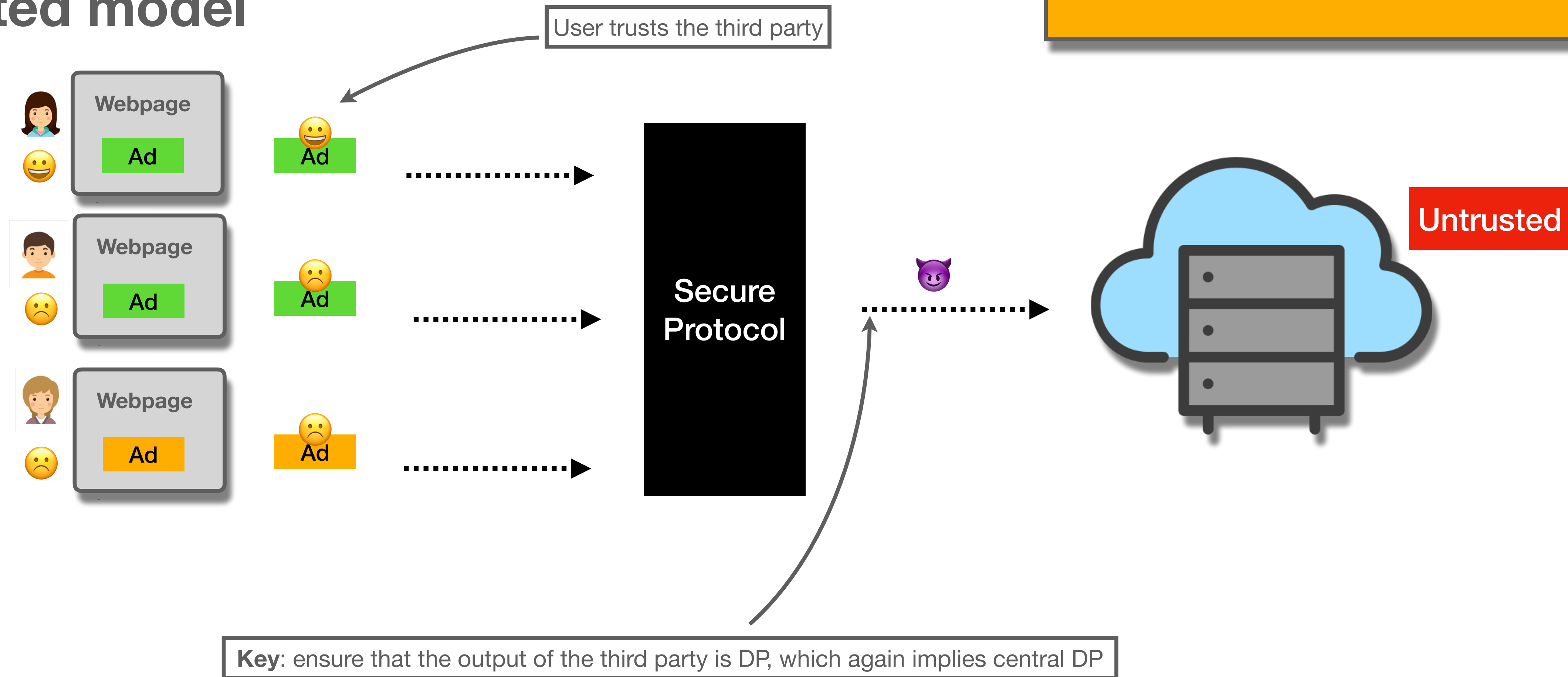
**Privacy :** pure DP , i.e.,  $(\epsilon, 0)$ -DP

**Regret:**  $\approx \frac{1}{\epsilon^2}$  optimal non-private regret

Due to a large amount of noise  
Privacy cost is now **multiplicative**

# Differential Privacy

## Distributed model



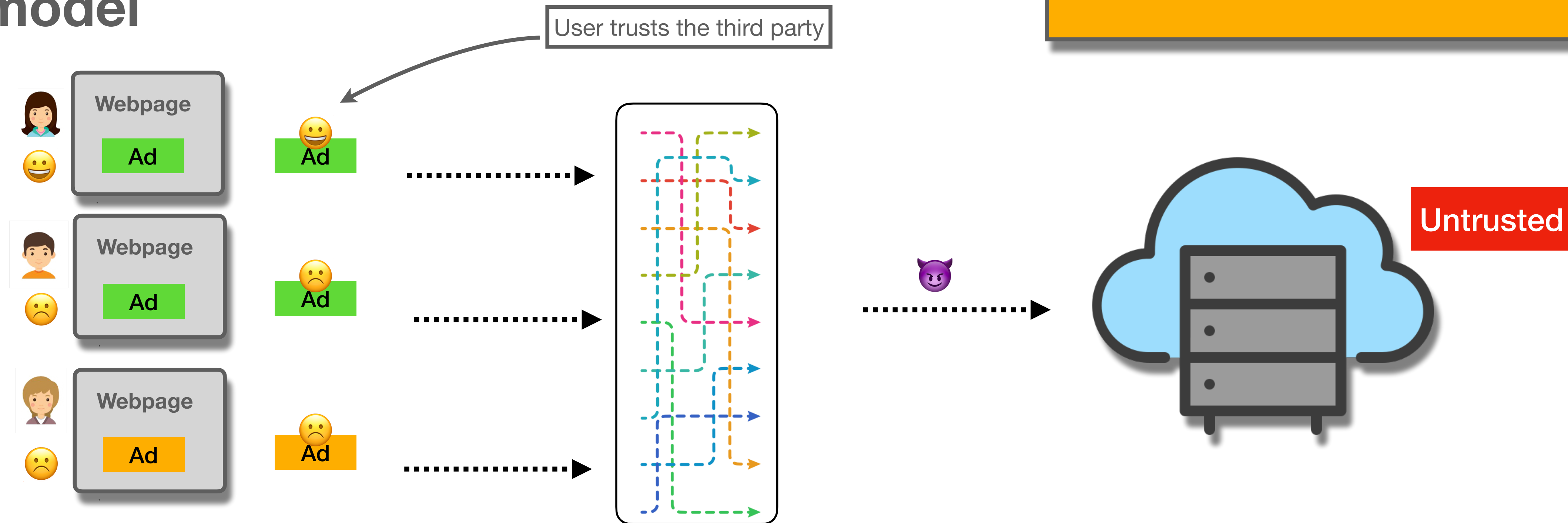
**Differential Privacy**

For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$

$$\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$$

# Distributed DP in MAB

## Shuffle model



### Differential Privacy

For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$

$$\mathbb{P}(M(D) \in E) \leq e^\epsilon \mathbb{P}(M(D') \in E) + \delta$$

[Tenenbaum, Kaplan, Mansou, Stemme'21]

**Algorithm:** successive arm elimination + batching + shuffle protocol

**Privacy:** approximate DP, i.e.,  $(\epsilon, \delta)$ -DP

**Regret:** optimal non-private regret +  $O\left(\frac{K \log T \sqrt{\log(1/\delta)}}{\epsilon}\right)$

Shuffler

[Limitations]

**Privacy:** only approximate DP rather than pure DP

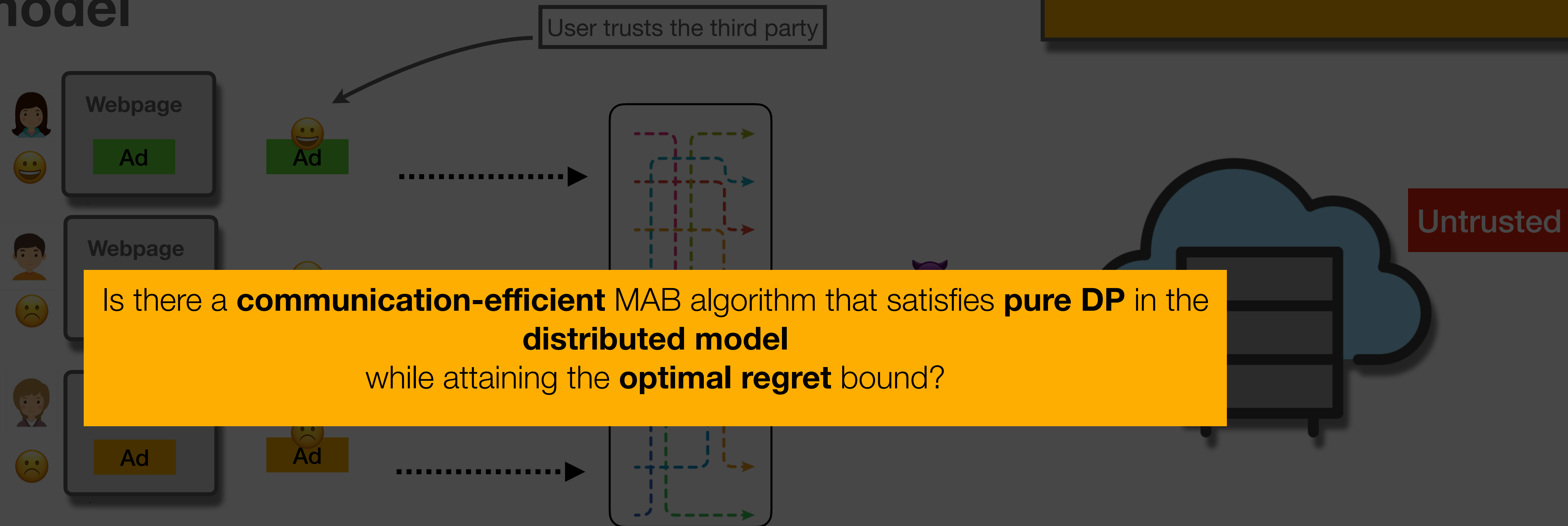
**Regret:** not optimal, additional log factors

**Communication:** current scheme only works for binary reward  
\* adapt to other scheme incurs extensive communication



# Distributed DP in MAB

## Shuffle model



### Differential Privacy

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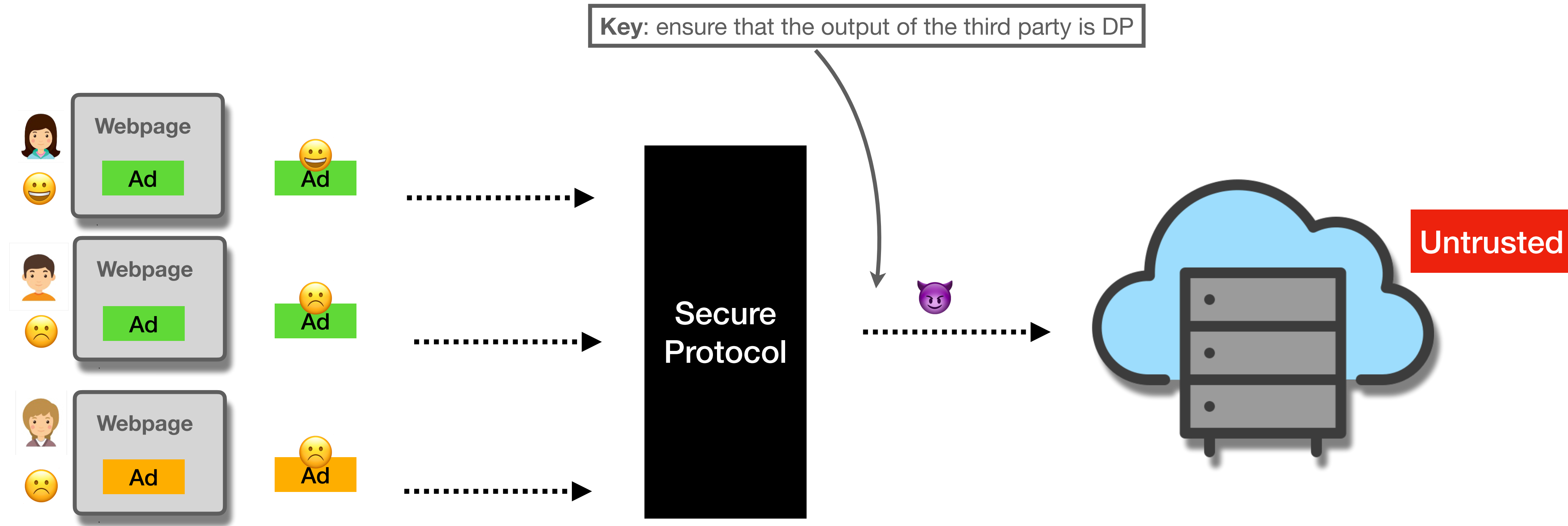
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# Contribution

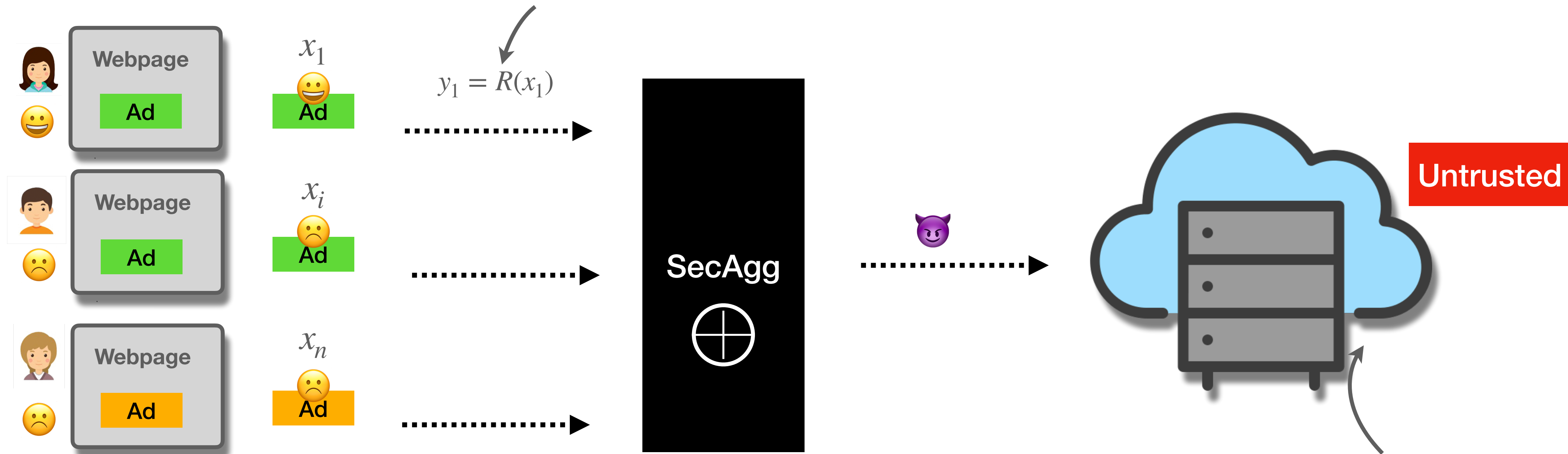
# Main Results



1. The first algorithm to achieve optimal regret with pure DP in distributed model
2. The first algorithm to achieve RDP using only discrete privacy noise
3. A unified algorithmic framework for achieving optimal regret under central, local, distributed model
4. Extensive simulations and experiments to validate our theoretical results

# Our Algorithm

Each local randomizer adds privacy noise on reward  
 Due to SecAgg: (i) only **discrete** privacy noise (ii) **modular** clipping



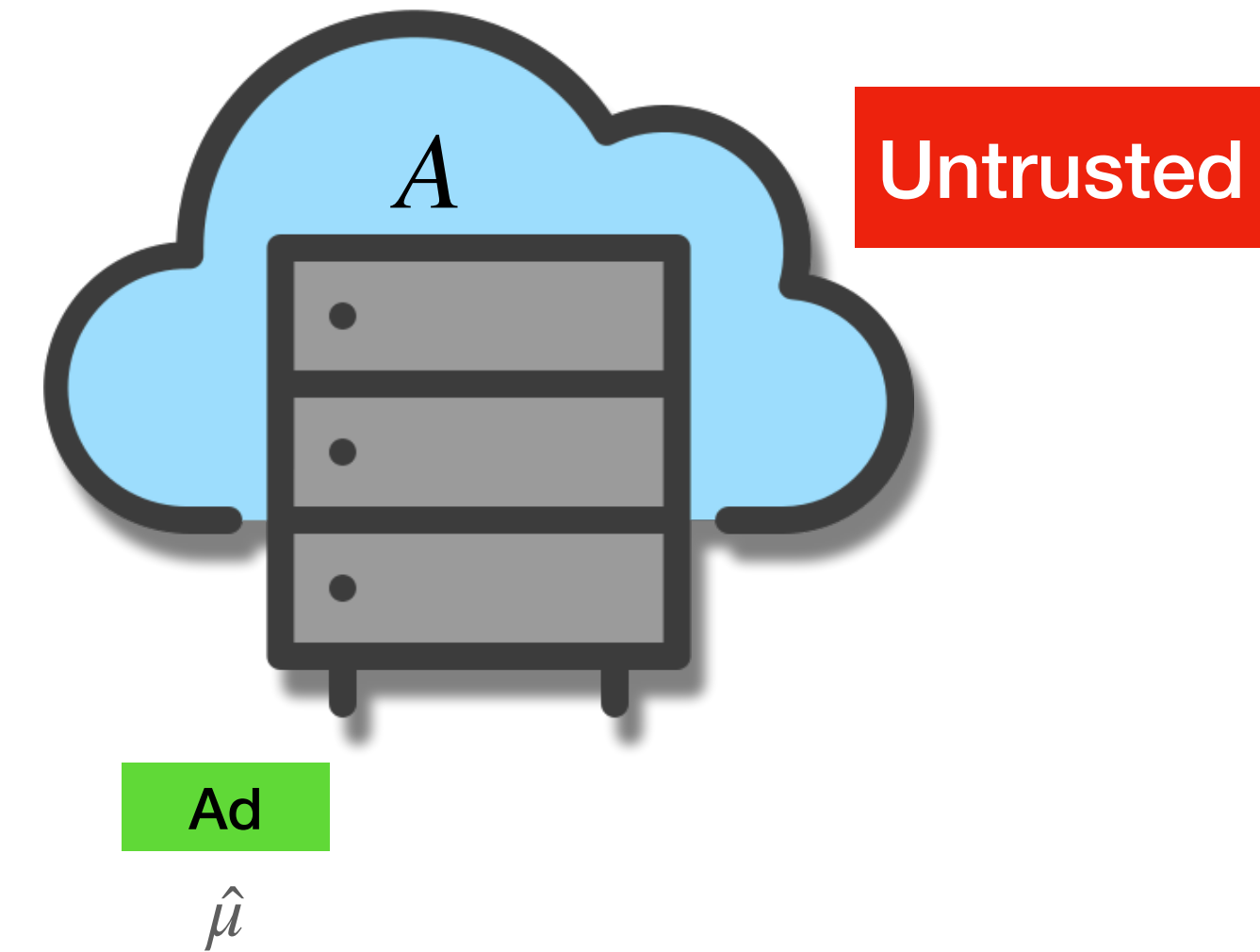
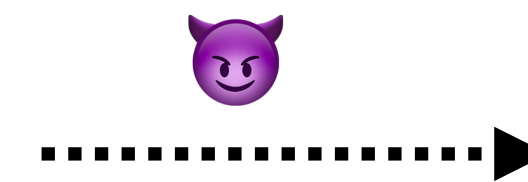
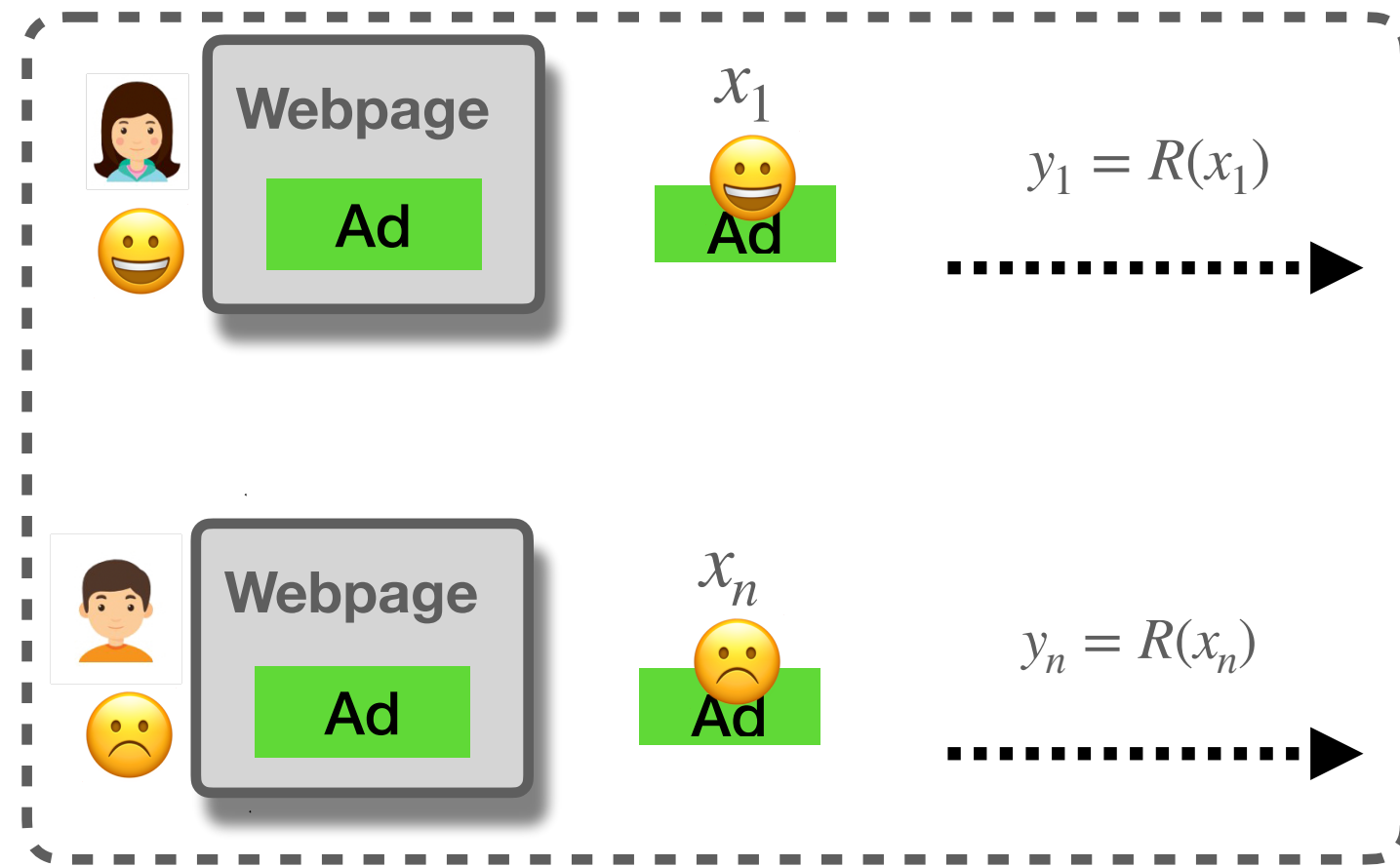
Given output from SecAgg,  $\hat{y}$ , try to calculate  $z = A(\hat{y}) \approx \sum_{i=1}^n x_i$

Given  $y_1, y_2, \dots, y_n$ , output  $\hat{y} = \left( \sum_{i=1}^n y_i \right) \text{ mod } m$   
 This alone does not provide formal privacy

# Our Algorithm

 = Current active actions Ad Ad

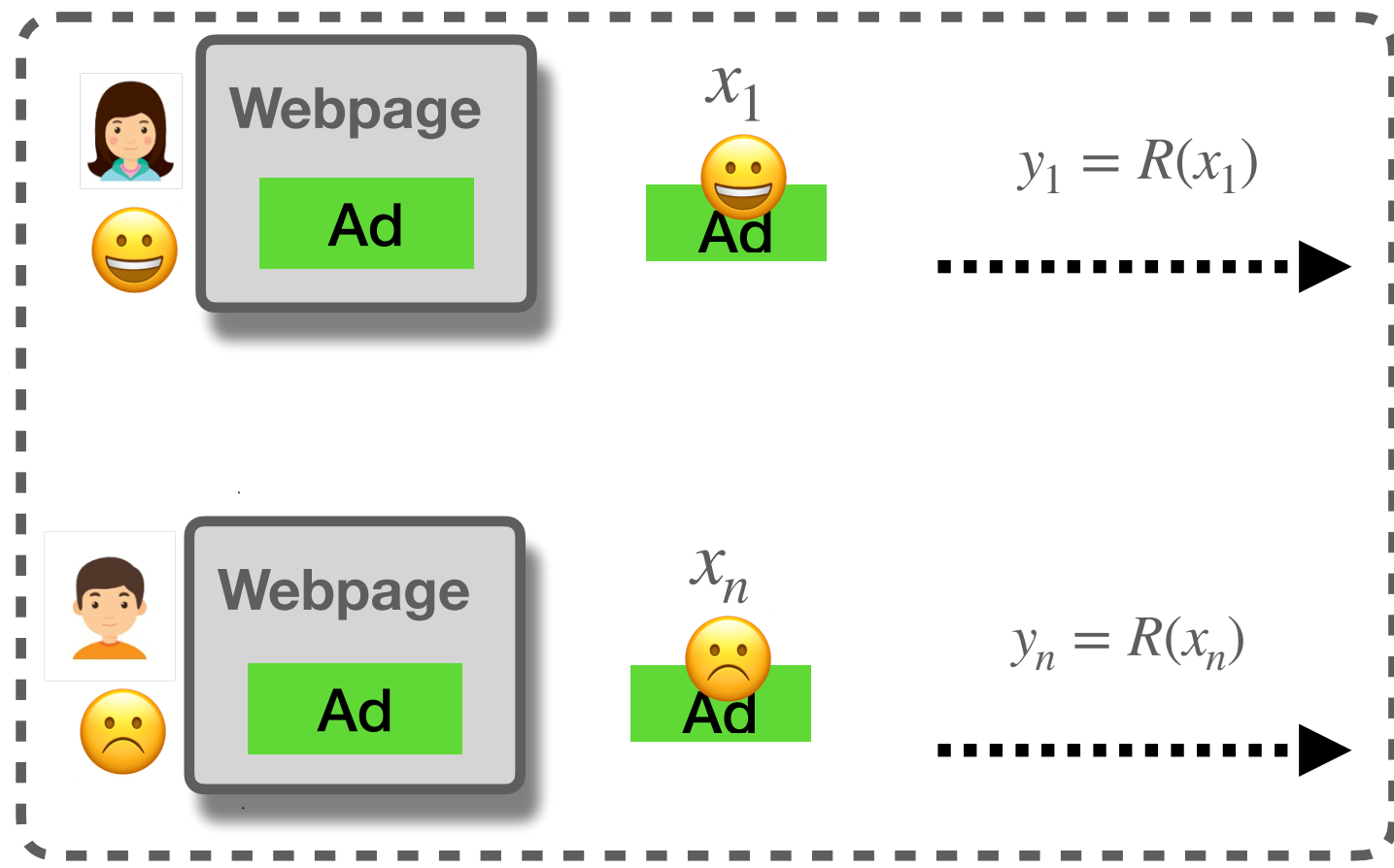
Batch size:  $l(b) = 2^b$



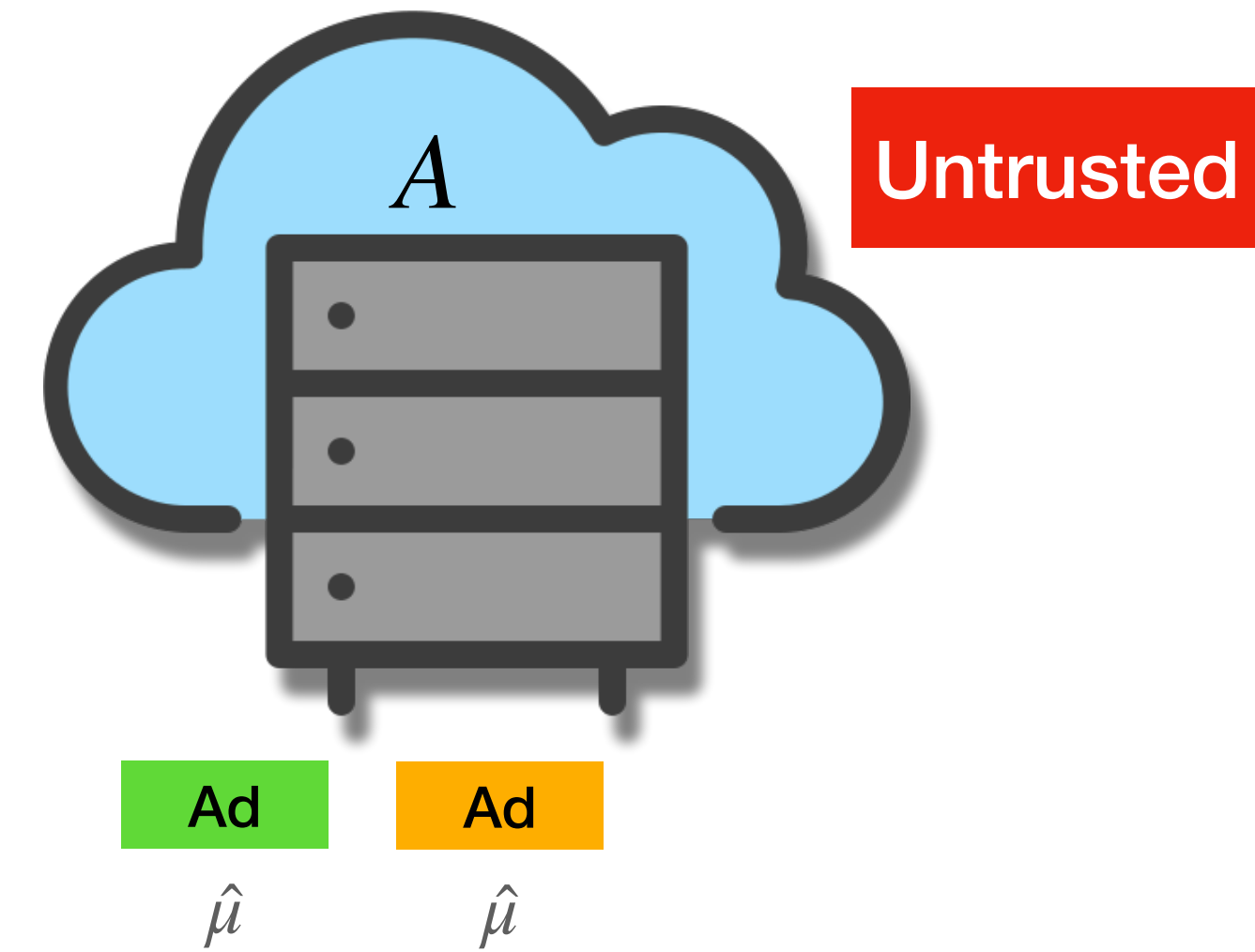
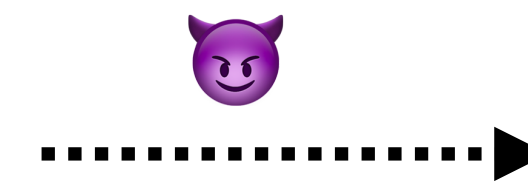
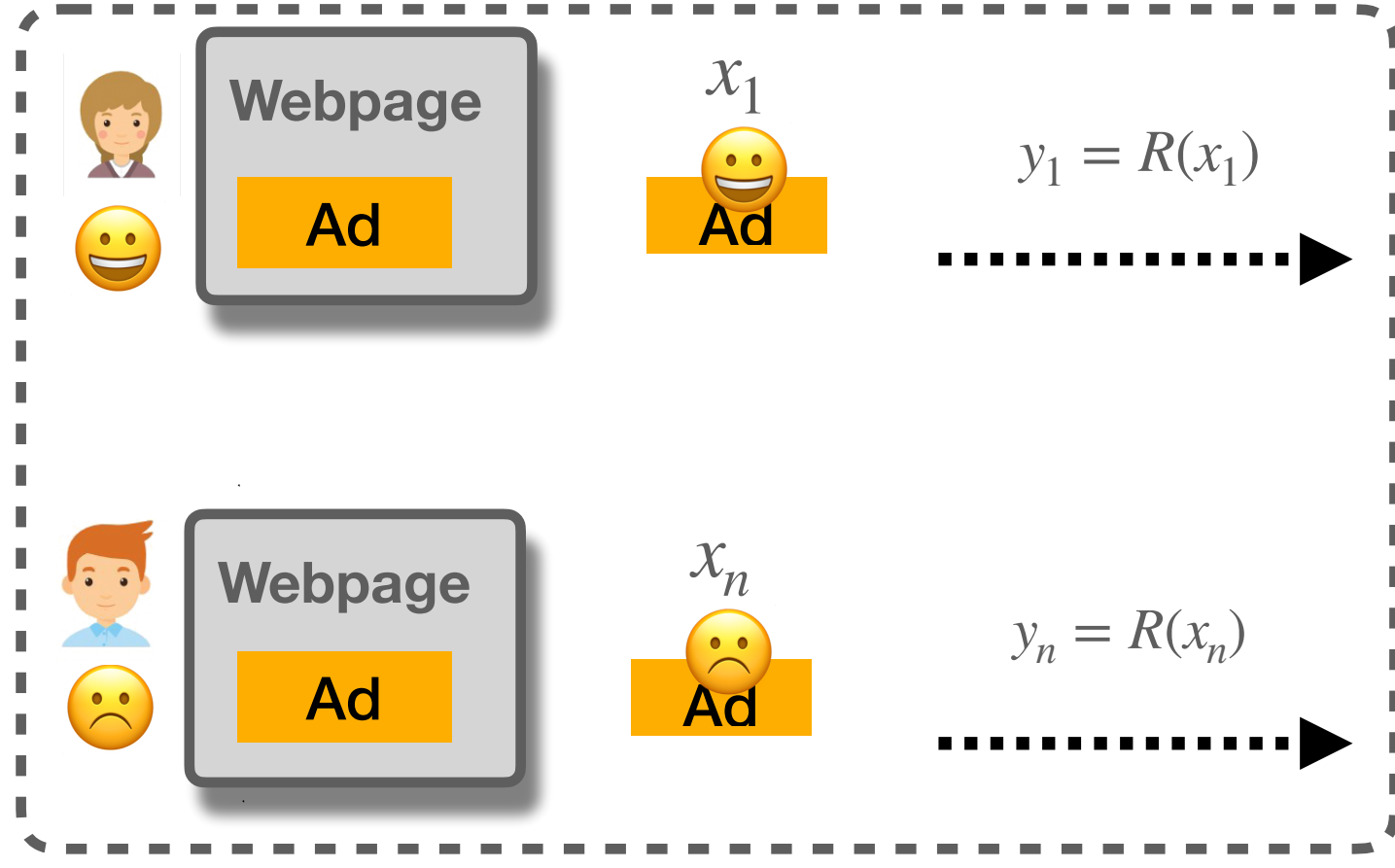
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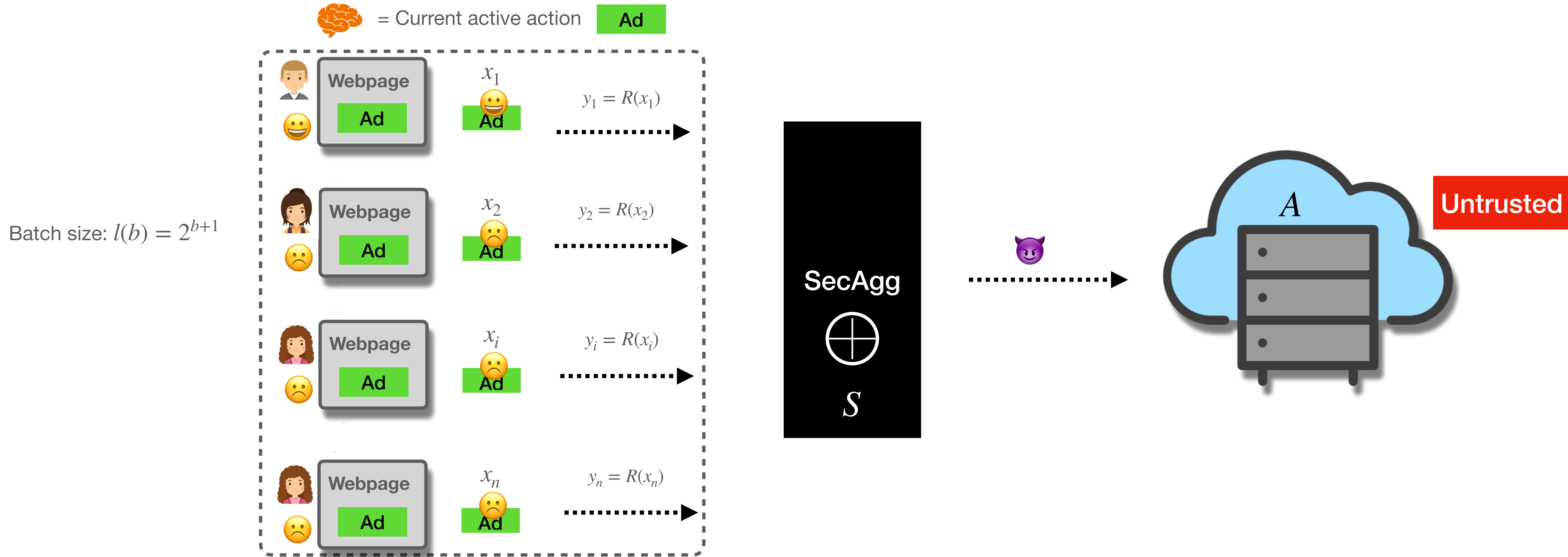


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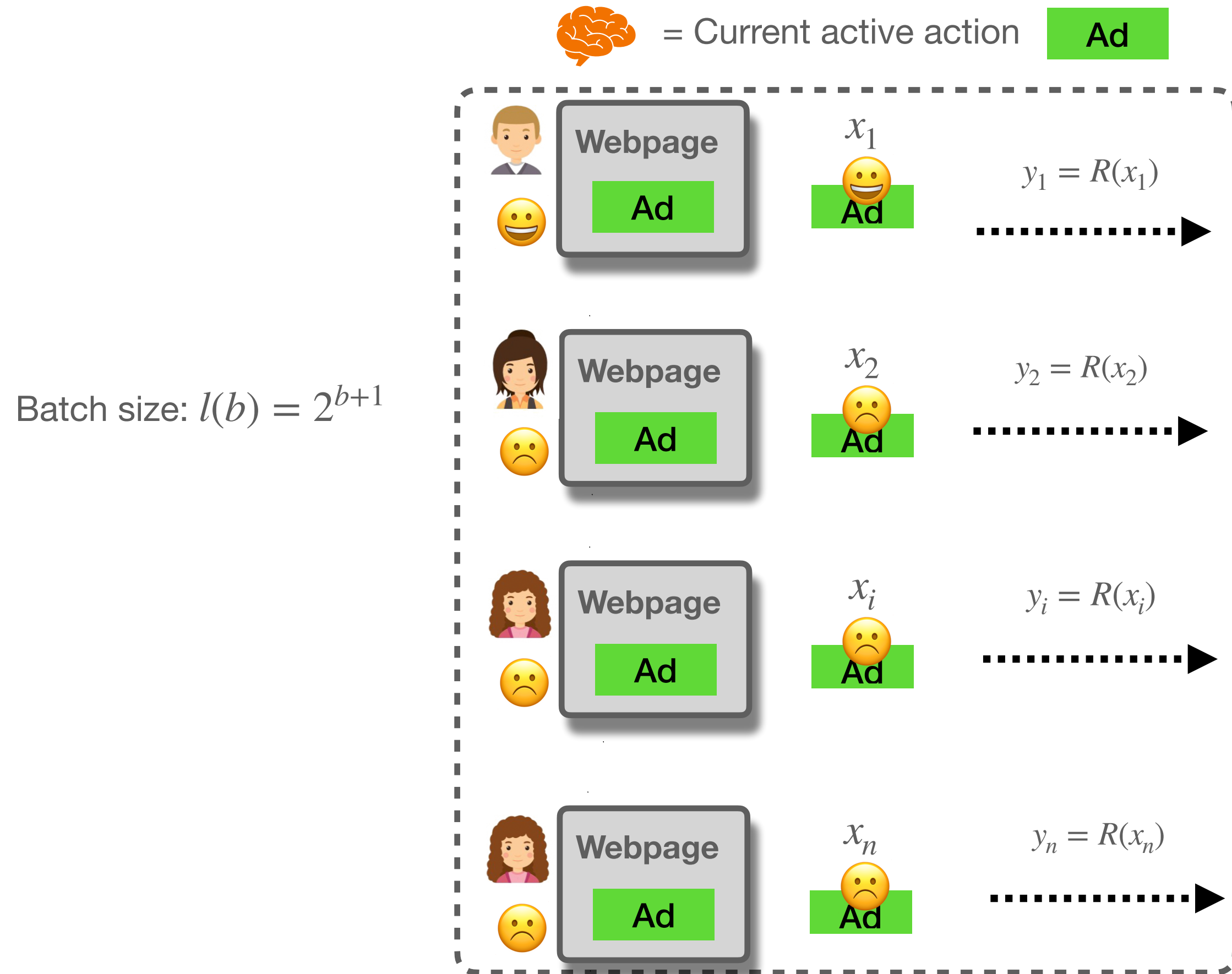


 = after action elimination Ad

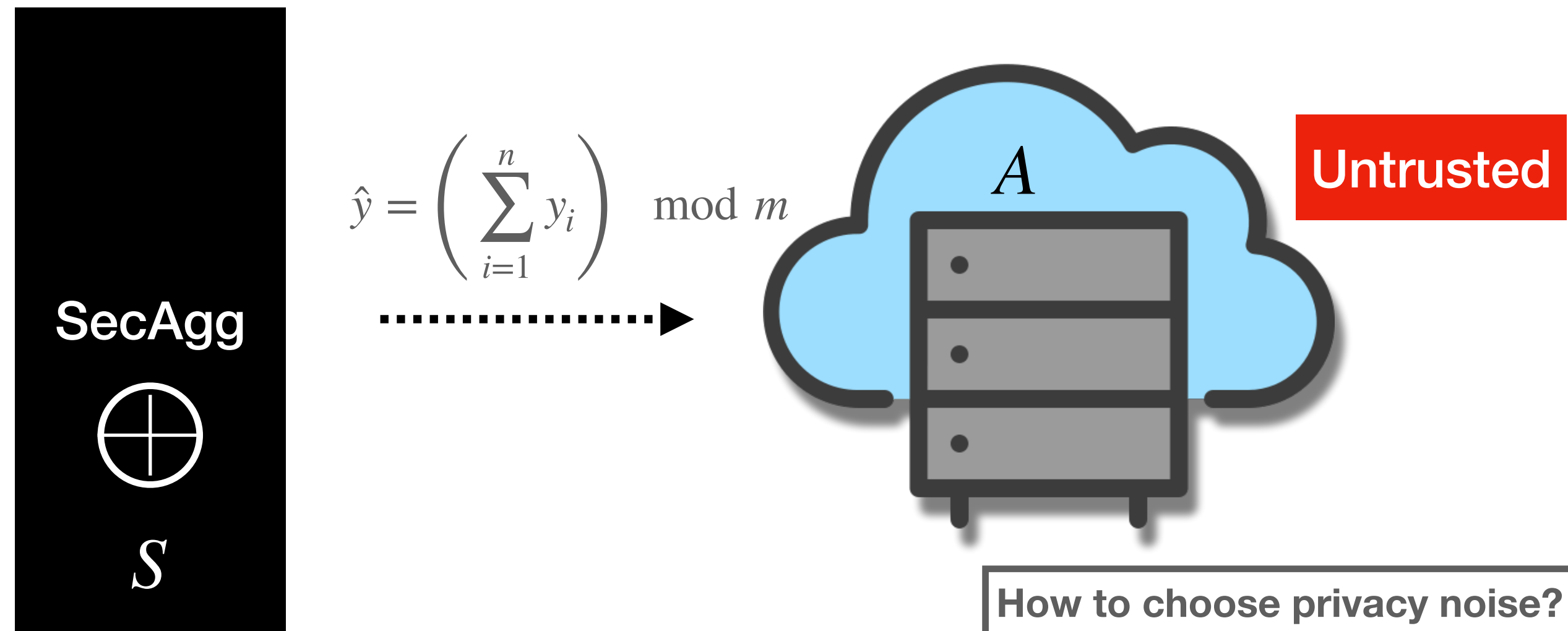
# Our Algorithm



# Our Algorithm



**Analyzer  $A$**   
 [Balle et al' 20, Cheu & Yan' 21]  
 Input  $\hat{y}$ , Output  $z$   
 If  $\hat{y} > ng + \tau$ :  $z = (\hat{y} - m)/g$ ; else  $z = \hat{y}/g$



**Local Randomizer  $R$**   
 [Balle et al' 20, Cheu & Yan' 21]  
 Input  $x_i \in [0,1]$ , Output  $y_i$

$x_i \xrightarrow{[x_i g] + \text{Ber}(x_i g - [x_i g])} \hat{x}_i \xrightarrow{(\hat{x}_i + \eta_i) \bmod m} y_i$



# Achieving Pure DP

## Simulate discrete Laplace using Polya noise

### Theorem 1 (Pure-DP via SecAgg)

Fix  $\epsilon > 0$  and  $T$ . For each batch  $b$ , the noise  $\eta_i = \gamma_i^+ - \gamma_i^-$ , where  $\gamma_i^+, \gamma_i^- \sim^{i.i.d} \text{Polya}(1/n, e^{-\epsilon/g})$ .

There exist proper choices of  $g, m, \tau$  such that

**Privacy:** pure DP in the distributed model

**Regret:** optimal non-private regret +  $\Theta\left(\frac{K \log T}{\epsilon}\right)$

**Communication:** bits scales logarithmically with the batch size

### Local Randomizer $R$

[Balle et al' 20, Cheu & Yan' 21]

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#### Remark on privacy

- First result on pure DP in distribute model for MABs
- Also achieve pure DP using advanced shuffle protocol

#### Remark on regret

- Match the optimal regret under central model
- Only use discrete privacy noise, w/o finite precision approx

#### Remark on communication

- Only communicate bits
- Previous works scale polynomially

# Achieving RDP

## Skellam noise

$(\alpha, \epsilon(\alpha))$ -RDP

For any two neighboring datasets  $D$  and  $D'$ , and any outcome  $E$ ,  $D_\alpha(M(D), M(D')) \leq \epsilon(\alpha)$

### Theorem 2 (RDP via SecAgg)

Fix  $\epsilon > 0$  and  $T$ . For each batch  $b$ , let the noise be  $\eta_i \sim SK\left(0, \frac{g^2}{n\epsilon^2}\right)$ .

There exist proper choices of  $g, m, \tau$  such that

**Privacy:**  $\approx (\alpha, \frac{\alpha\epsilon^2}{2})$ -RDP

**Regret:**  $\approx$  optimal non-private regret +  $\Theta\left(\frac{K\sqrt{\log T}}{\epsilon}\right)$

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# Achieving RDP

## Skellam noise

### Proposition (Tail bound of Skellam noise)

Skellam random variable has a sub-exponential tail

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#### Remark 1 (Conversion to approximate DP)

- After conversion, it is  $\sqrt{\log T}$  tighter than SOTA

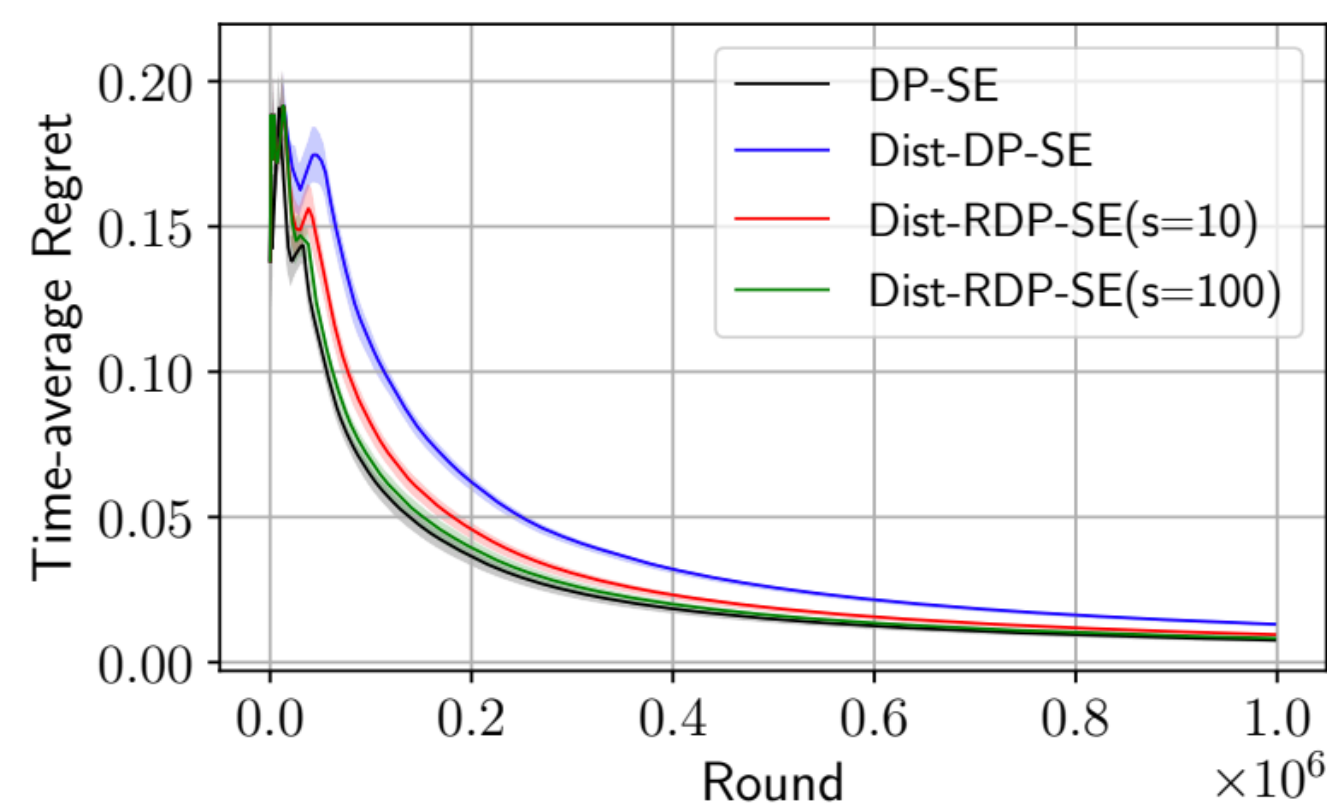
#### Remark 2 (Tight privacy accounting)

- Useful for analyzing returning users

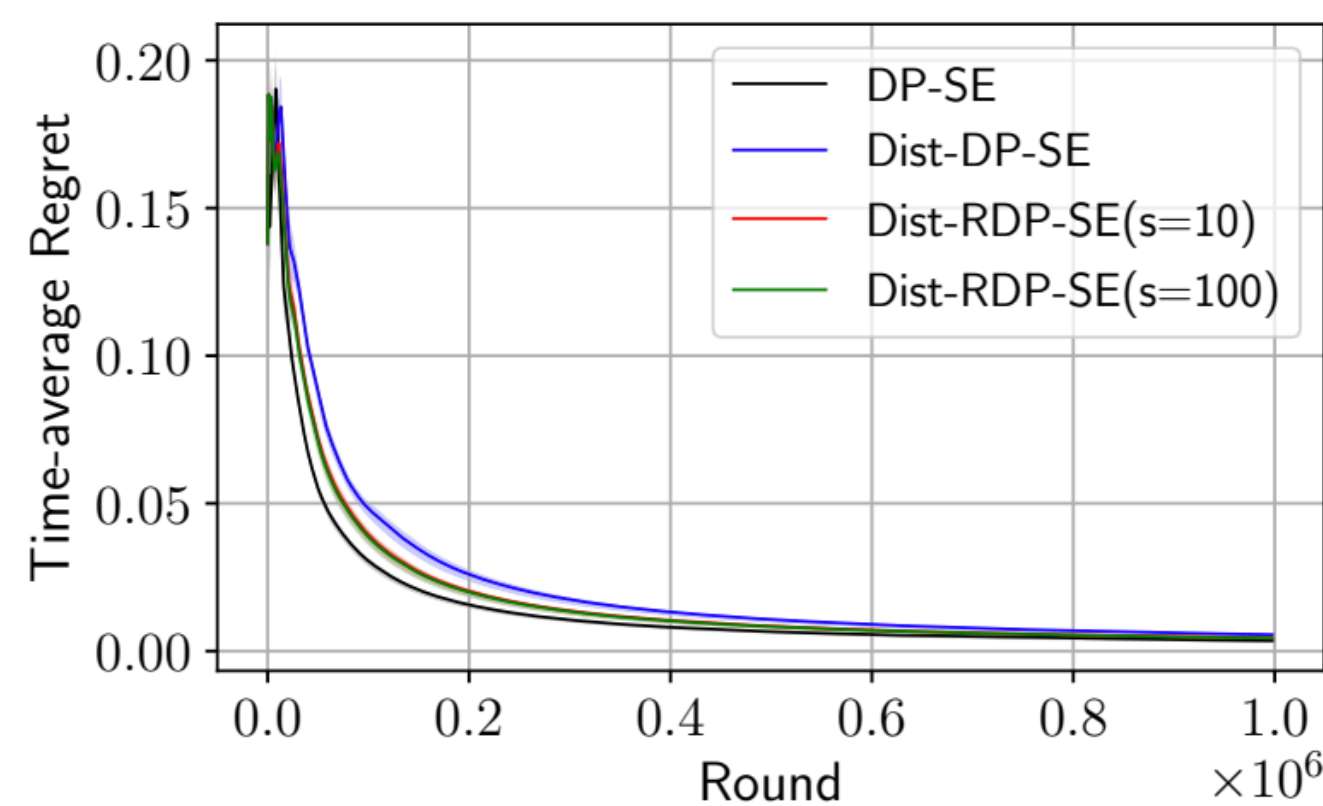
#### Remark 3 (RDP in other models)

- Our algorithm can be adapted to central and local
- Hence, the first result of RDP guarantees

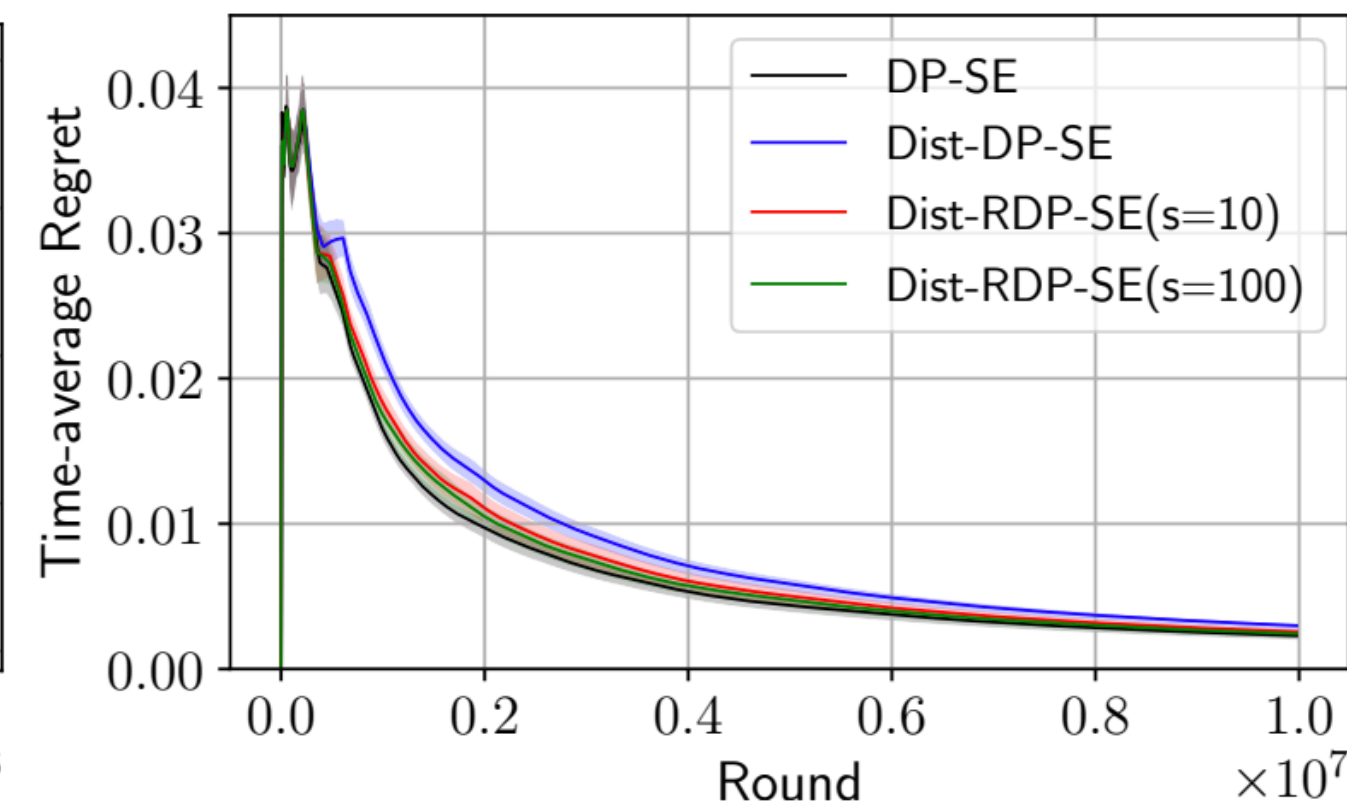
# Simulations



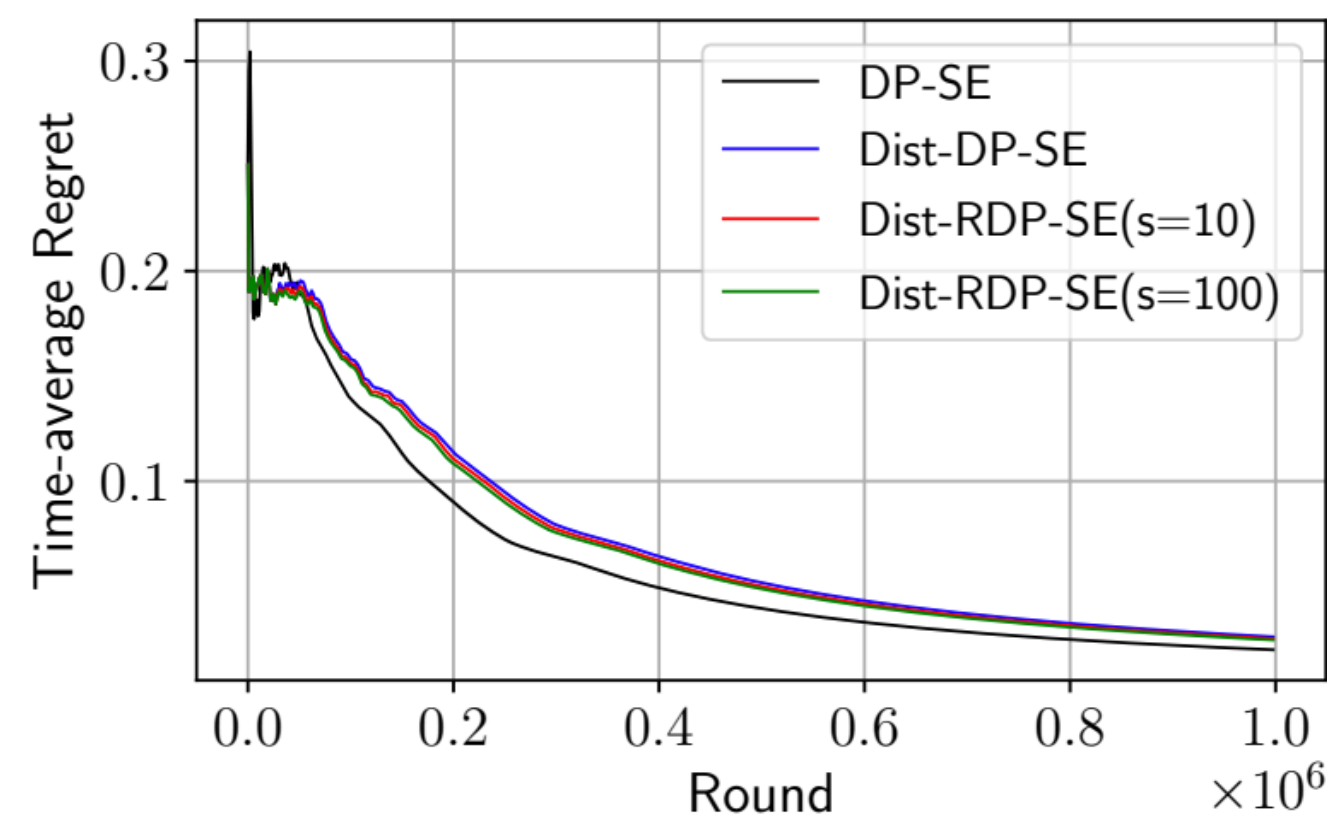
(a)  $\epsilon = 0.1, K = 10$



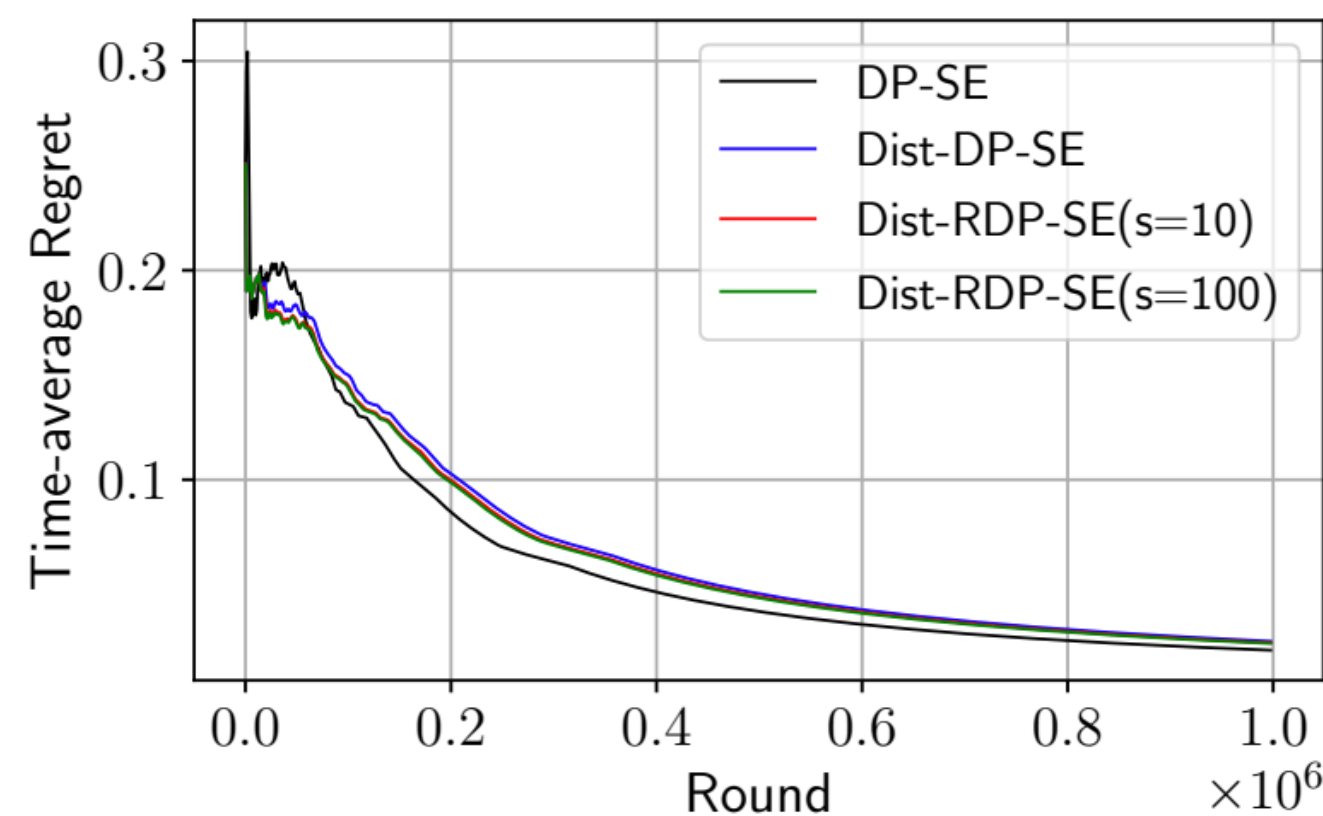
(b)  $\epsilon = 0.5, K = 10$



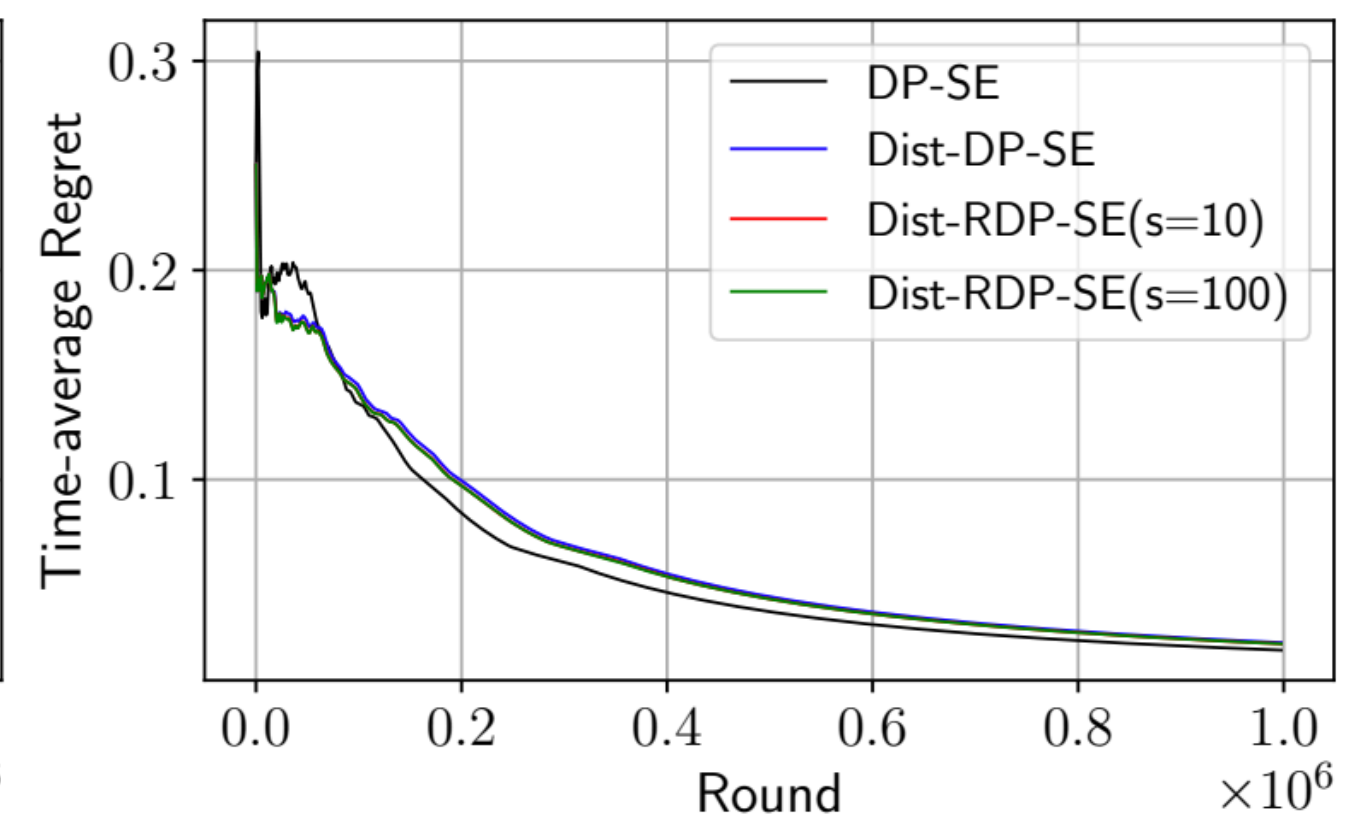
(c)  $\epsilon = 0.1, K = 10$



(d)  $\epsilon = 1, K = 50$



(e)  $\epsilon = 5, K = 50$



(f)  $\epsilon = 10, K = 50$

**Thank you!**