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imprs-is

ICLR 2023

Trading Information between Latents in Hierarchical Variational Autoencoders

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Presentation on 11th International Conference on Learning Representations (ICLR), 2023

SCAN ME

Controlling Information in β-VAEs

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$$\underbrace{\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}\left[\log p_\theta(\mathbf{x}|\mathbf{z})\right]}_{-\text{ distortion } D} - \beta \underbrace{D_{\mathrm{KL}}\left(q_\phi(\mathbf{z}|\mathbf{x}) \,\|\, p(\mathbf{z})\right)}_{\text{rate } R}$$

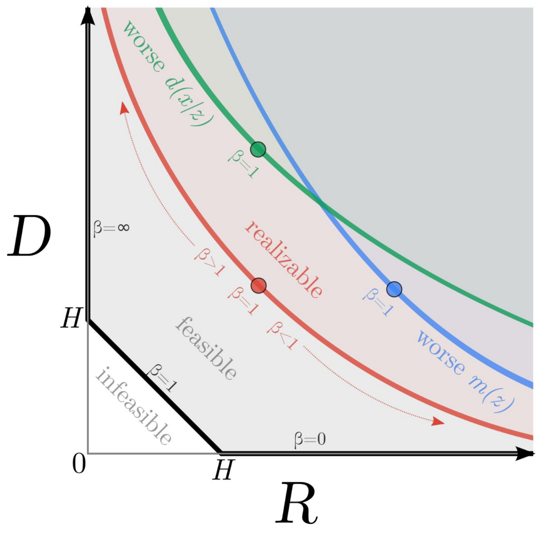
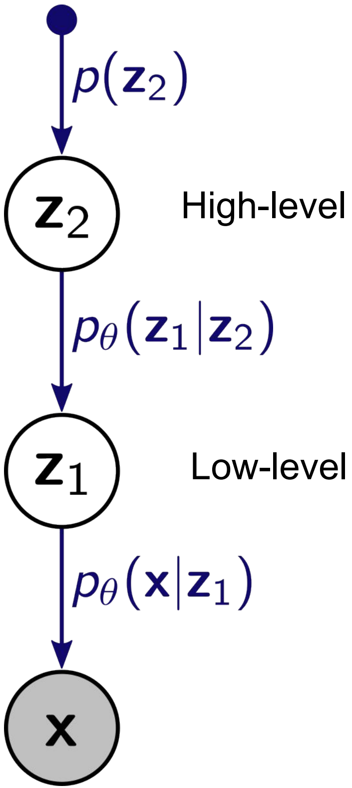


Figure taken from Alemi et al., *Fixing a Broken ELBO*, ICML 2018.



Defining Layer-Wise Bit Rates

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For one architecture, total bit rate separates into:

$$R = R(\boldsymbol{z}_L) + R(\boldsymbol{z}_{L-1}|\boldsymbol{z}_L) + R(\boldsymbol{z}_{L-2}\,|\,\boldsymbol{z}_{L-1}, \boldsymbol{z}_L) + \,\ldots\, + R(\boldsymbol{z}_1|\boldsymbol{z}_{\geq 2})$$

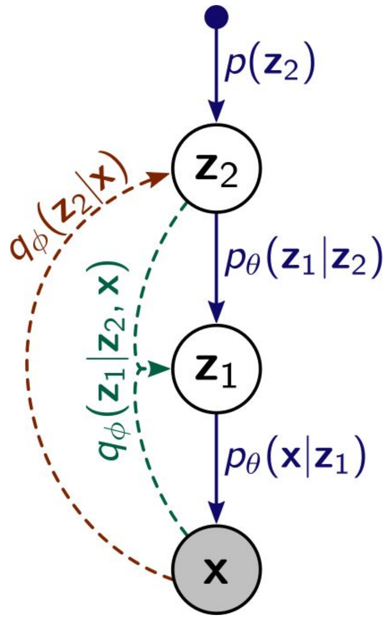
where:

$$R(\boldsymbol{z}_\ell|\boldsymbol{z}_{\geq \ell+1}) = \mathbb{E}_{q(\boldsymbol{z}_{\geq \ell+1}|\boldsymbol{x})}\left[D_{\mathrm{KL}}\left[q_\phi(\boldsymbol{z}_\ell\,|\,\boldsymbol{z}_{\geq \ell+1}, \boldsymbol{x}) \,\|\, p_\theta(\boldsymbol{z}_\ell\,|\,\boldsymbol{z}_{\geq \ell+1})\right]\right]$$

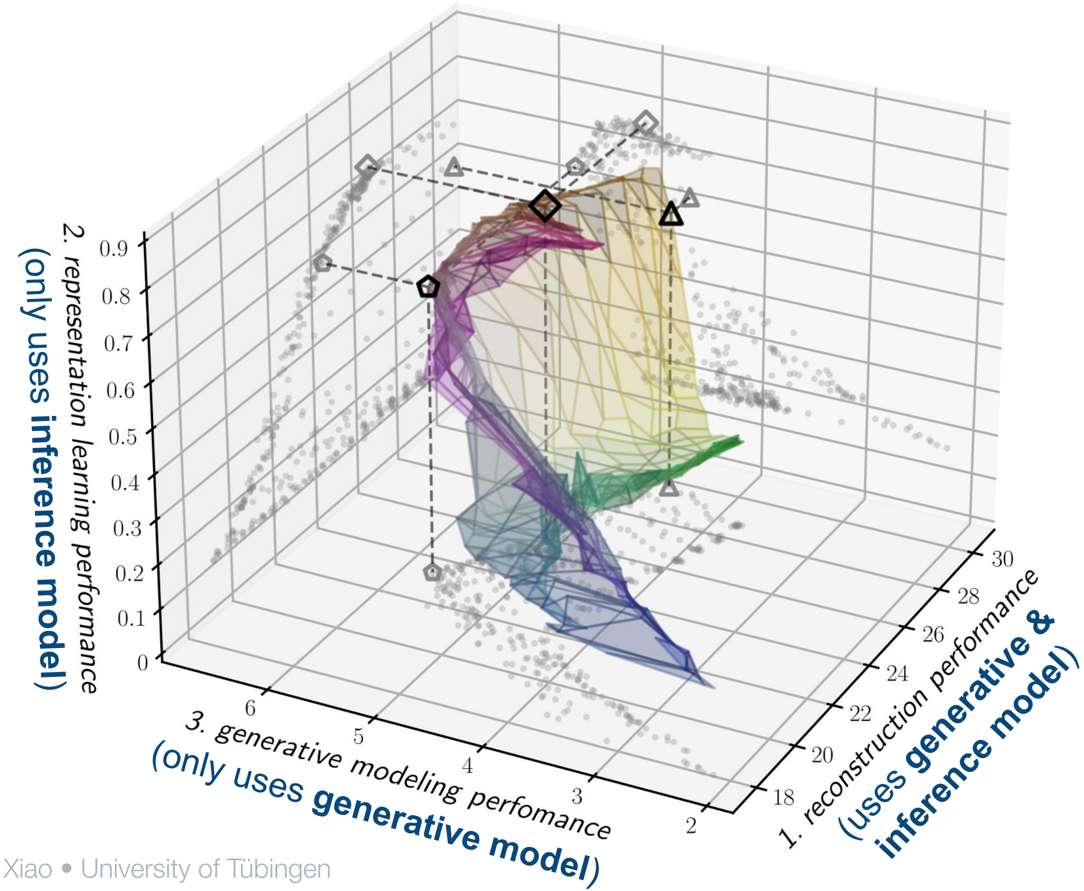
⇒ Proposed training objective:

$$\mathbb{E}_{\boldsymbol{x}\sim\mathbb{X}_{\text{train}}}\left[D + \beta_L R(\boldsymbol{z}_L) + \beta_{L-1} R(\boldsymbol{z}_{L-1}|\boldsymbol{z}_L) + \ldots + \beta_1 R(\boldsymbol{z}_1|\boldsymbol{z}_{\geq 2})\right]$$

L independent Lagrange multipliers



There is no "One VAE Fits All"



diverse application domains

⇓

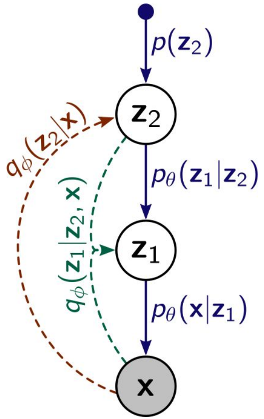
need fine-grained control
(no one-size-fits-all hierarchical VAE)

Application Type 1: Reconstruction

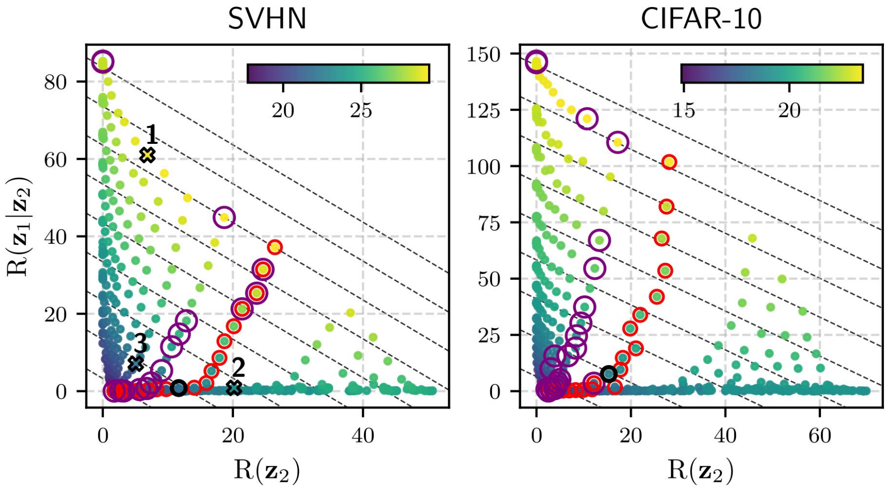
Theory:

$$\mathbb{E}_{\mathbf{x}\in p_{\text{data}}}[D] \geq H[p_{\text{data}}] - E_{\mathbf{x}\in p_{\text{data}}}\left[R(\mathbf{z}_L) + R(\mathbf{z}_{L-1}|\mathbf{z}_L) + \ldots + R(\mathbf{z}_1|\mathbf{z}_{\geq 2})\right]$$

distortion



Experiment:



Application Type 2: Rep. Learning

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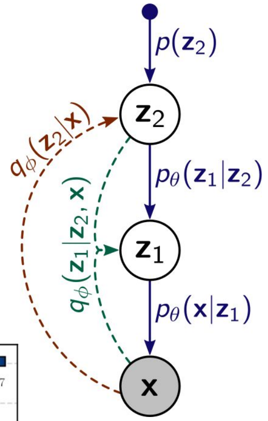
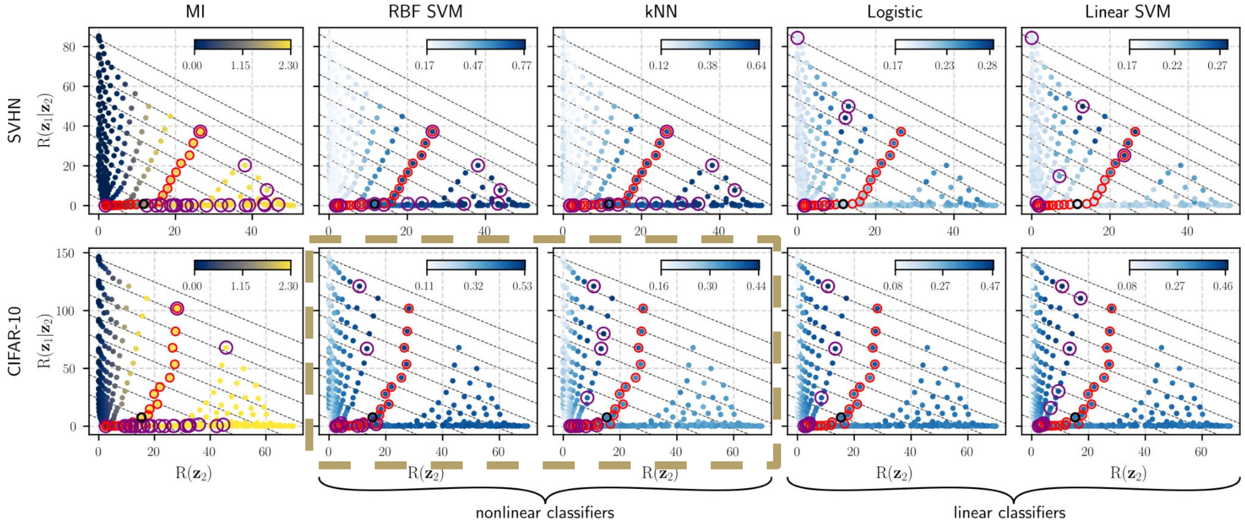


Theory: consider classifier operating on $\mathbf{z}_2$

$$\Rightarrow \text{accuracy} \leq f^{-1}\big(I_q(\text{label}; \mathbf{z}_2)\big) \leq f^{-1}\big(\mathbb{E}_{p_\text{data}}[R(\mathbf{z}_2)]\big)$$

$$f(\alpha) = H[p_\text{data}(y)] + \alpha \log \alpha + (1-\alpha) \log \frac{1-\alpha}{M-1}$$ [analogous to Meyen, 2016]

Experiment:



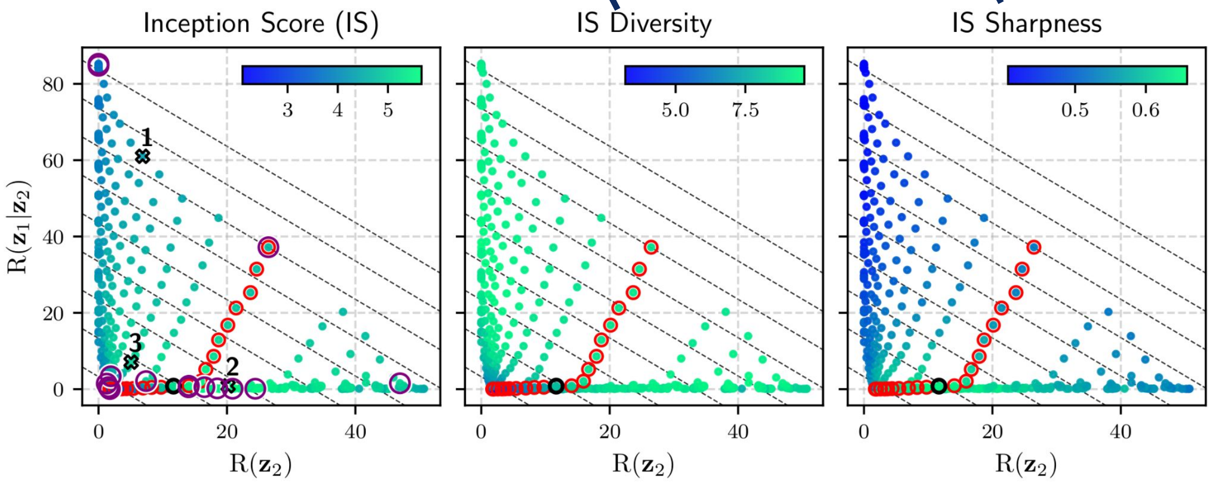
Application Type 3: Generation

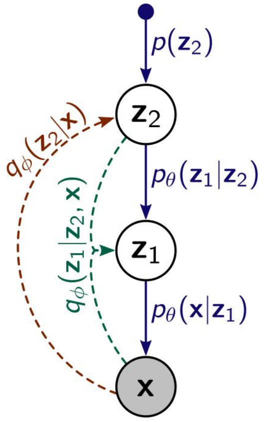
Theory: expect best generative performance when all $\beta = 1$

Experiment:

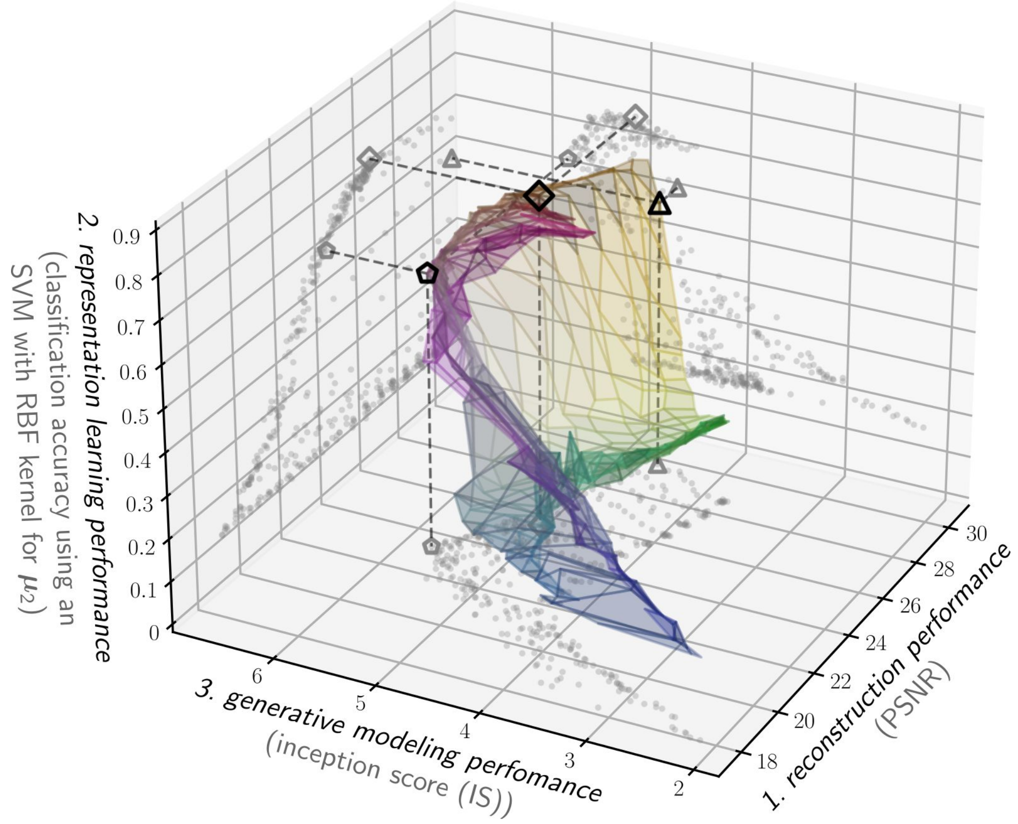
$$\mathrm{IS} = \exp\left\{\mathbb{E}_{p_\theta(\boldsymbol{x})}\left[D_{\mathrm{KL}}[p_{\mathrm{cls.}}(y|\boldsymbol{x}) \,\|\, p_{\mathrm{cls.}}(y)]\right]\right\}$$
$$= e^{H[p_{\mathrm{cls.}}(y)]} \times e^{-\mathbb{E}_{p_\theta(\boldsymbol{x})}[H[p_{\mathrm{cls.}}(y|\boldsymbol{x})]]}$$

[Salimans et al., 2016]





Summary



diverse application domains

⇓

need fine-grained control

⇓

control layer-wise rates