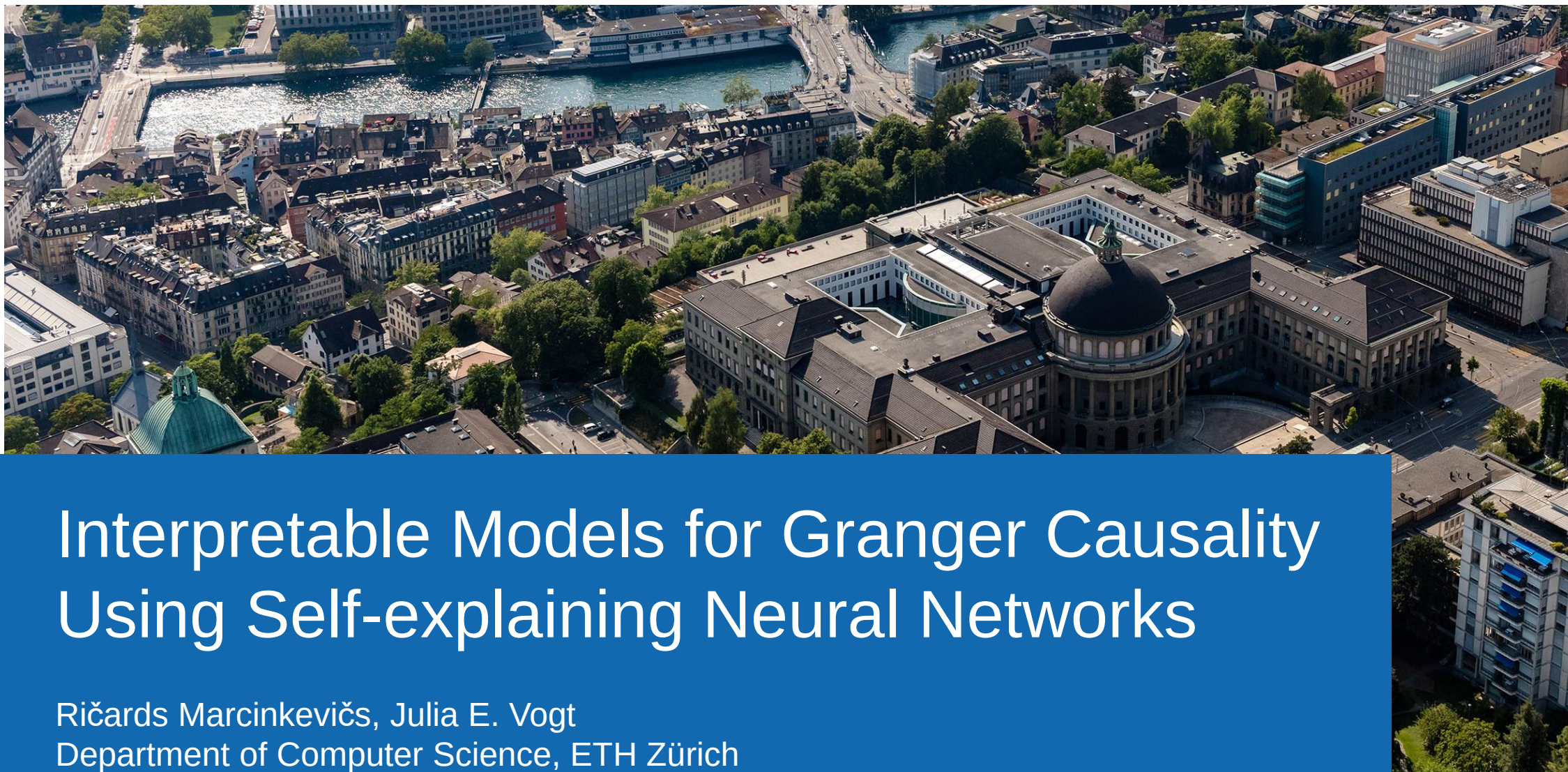
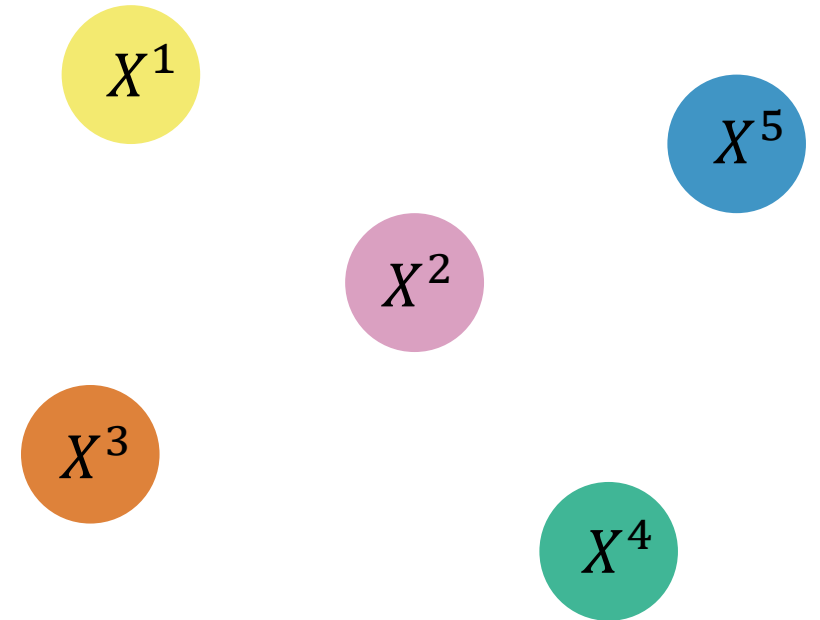
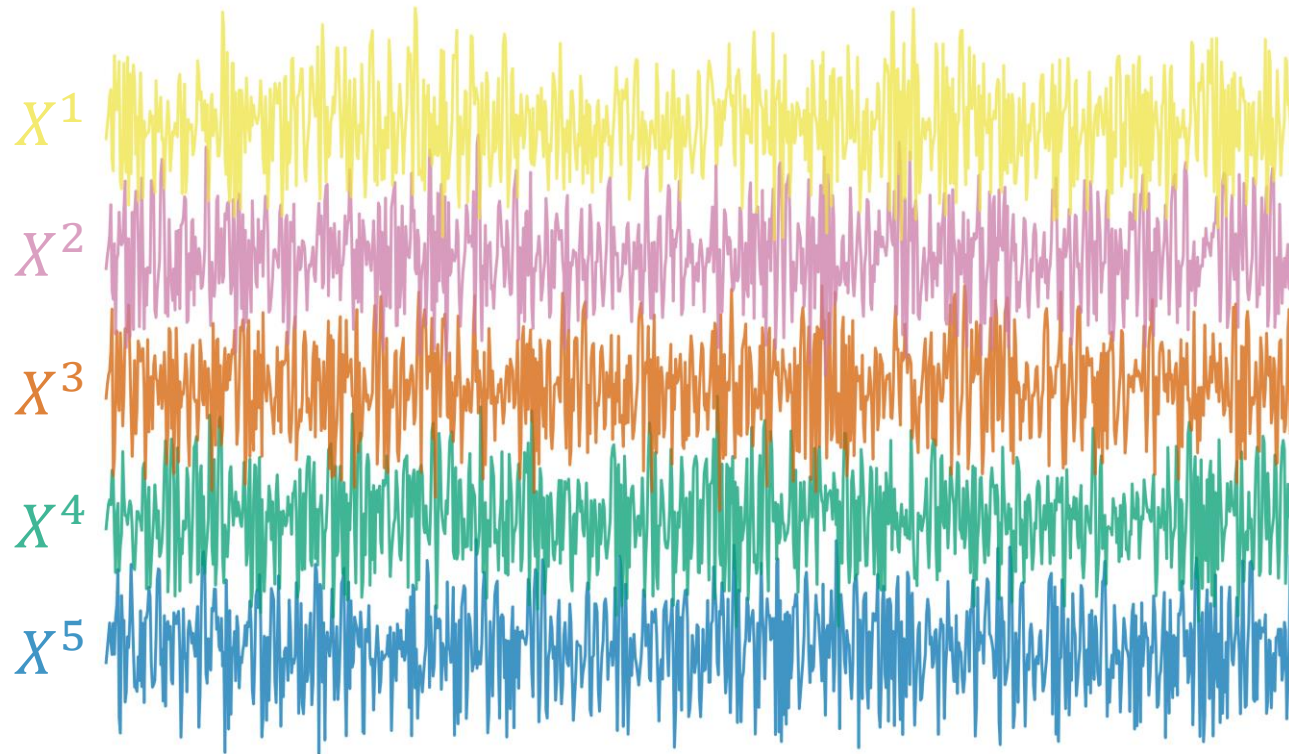




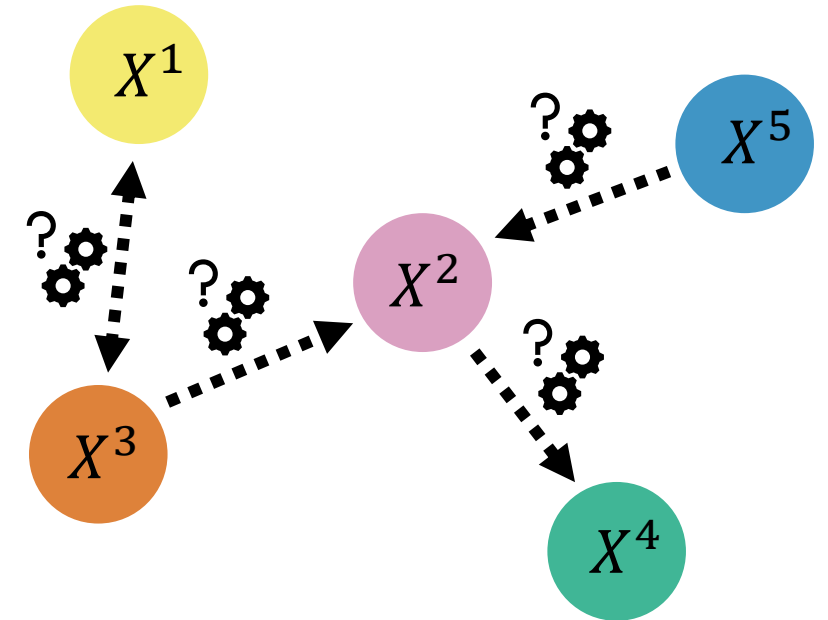
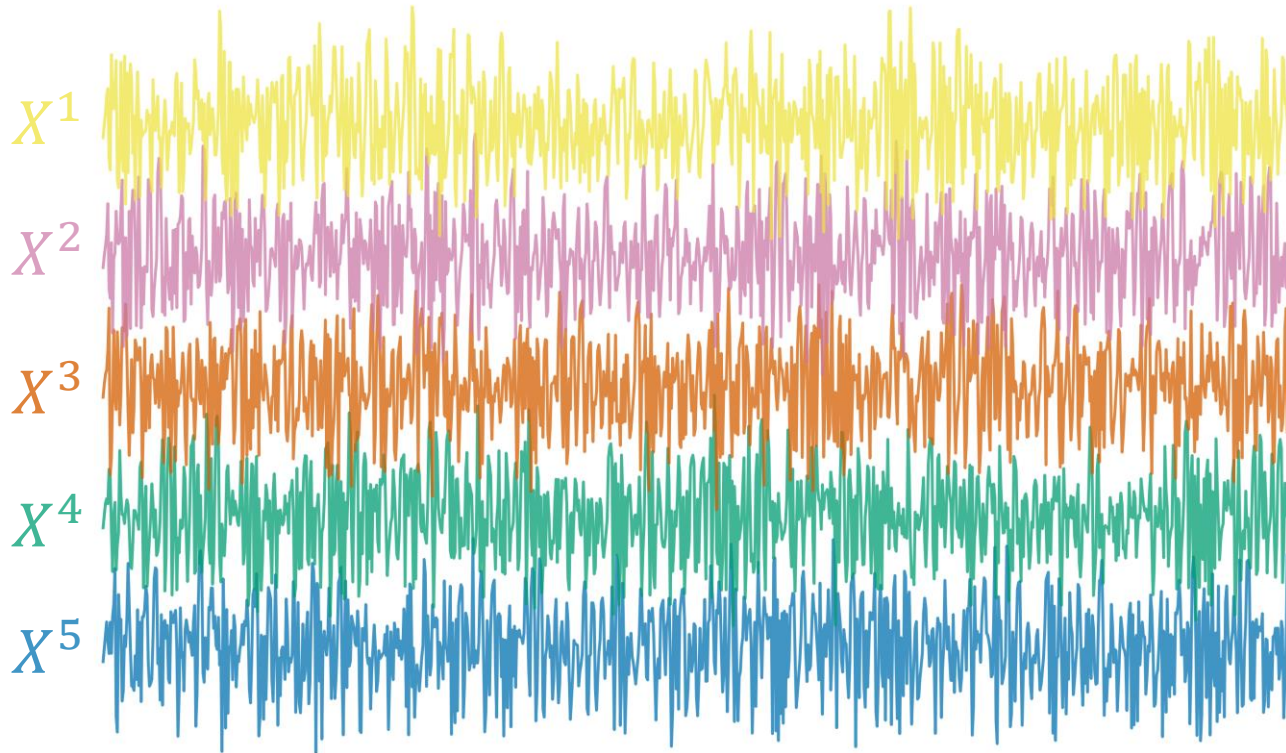
Interpretable Models for Granger Causality Using Self-explaining Neural Networks

Ričards Marcinkevičs, Julia E. Vogt
Department of Computer Science, ETH Zürich

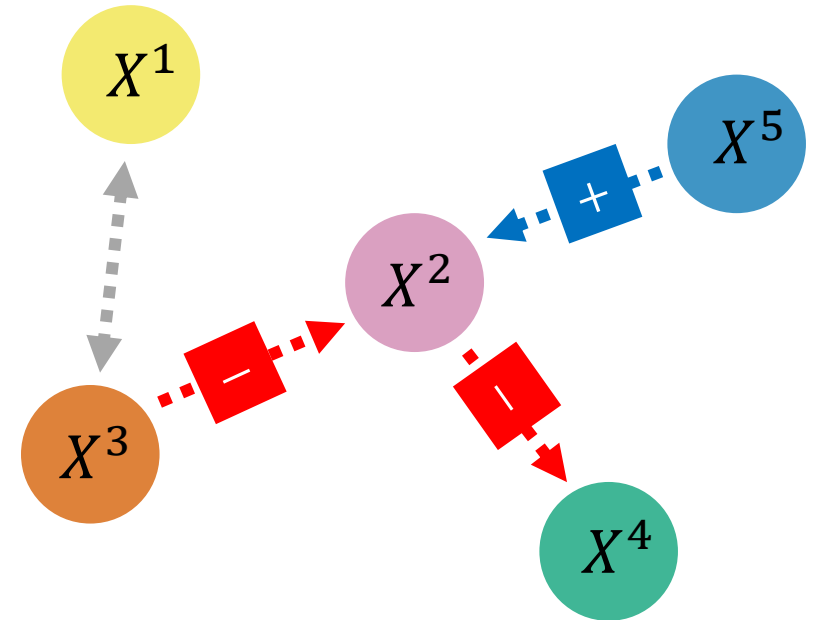
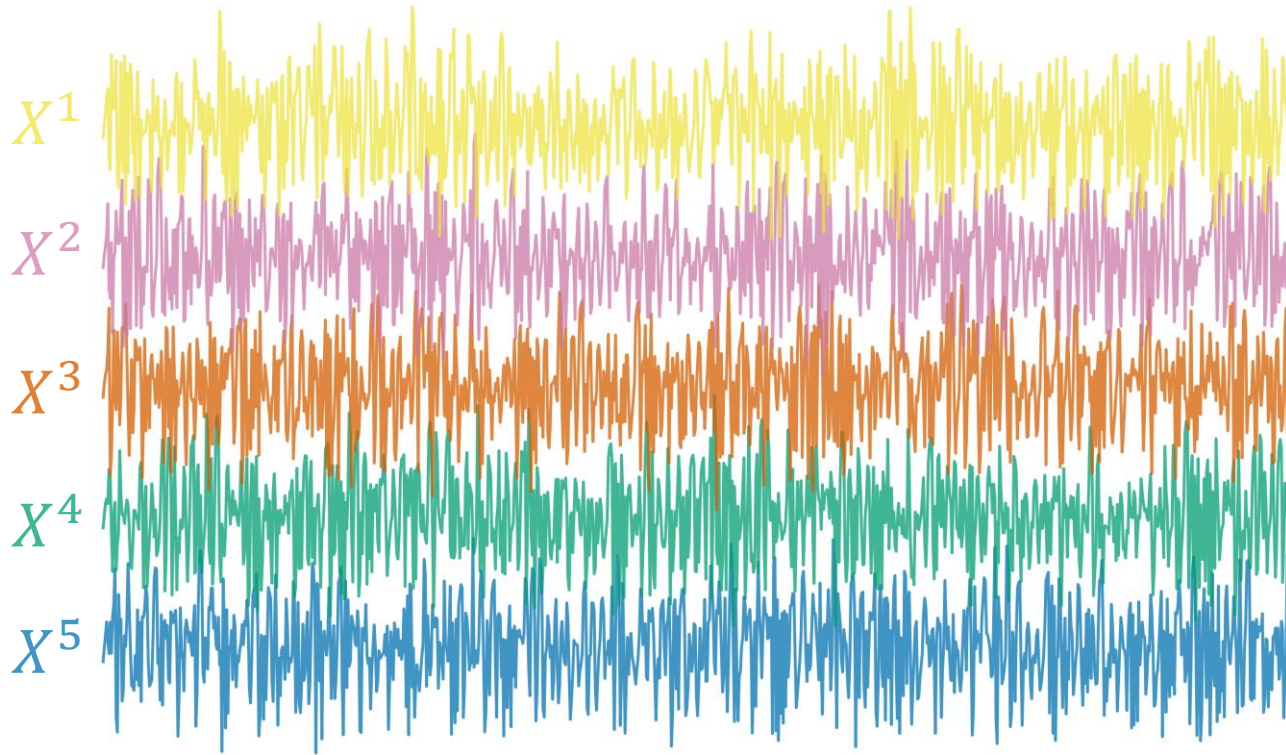
Time Series Analysis



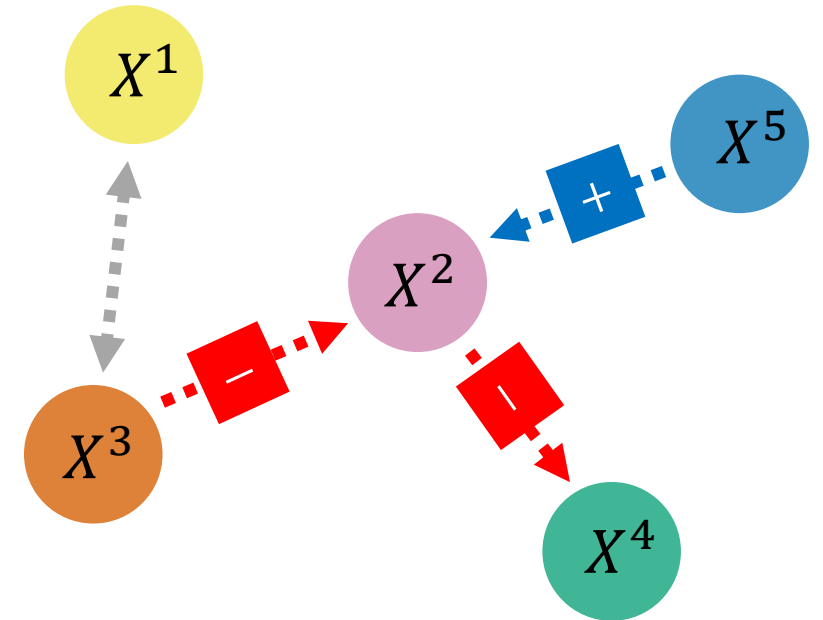
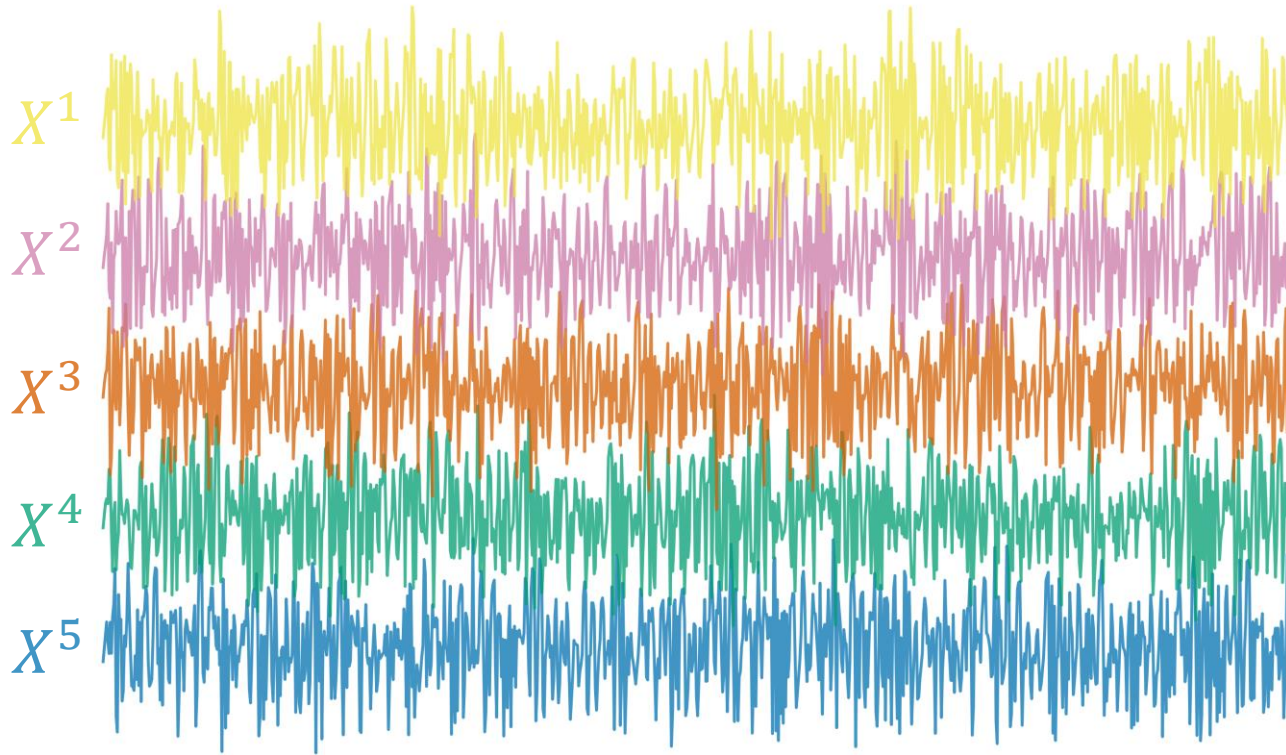
Time Series Analysis



Time Series Analysis



Time Series Analysis



We propose an **interpretable model** for inferring **nonlinear relationships** in **multivariate time series...**

X Granger-causes Y iff

$$\mathbb{P}[Y_{t+1} \in \mathcal{A} \mid \mathcal{I}(t)] \neq \mathbb{P}[Y_{t+1} \in \mathcal{A} \mid \mathcal{I}_{-X}(t)].$$

X Granger-causes Y iff

$$\mathbb{P}[Y_{t+1} \in \mathcal{A} \mid \underbrace{\mathcal{I}(t)}_{\text{past including } X_{<t+1}}] \neq \mathbb{P}[Y_{t+1} \in \mathcal{A} \mid \underbrace{\mathcal{I}_{-X}(t)}_{\text{past without } X_{<t+1}}].$$

past including $X_{<t+1}$

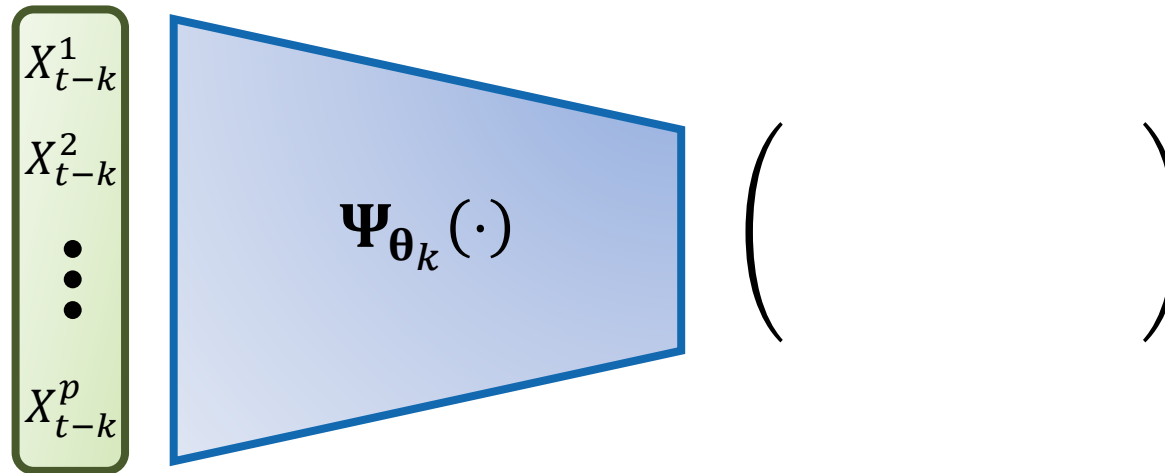
past without $X_{<t+1}$

VAR + Self-explaining Neural Networks =
Generalised Vector Autoregression (GVAR)

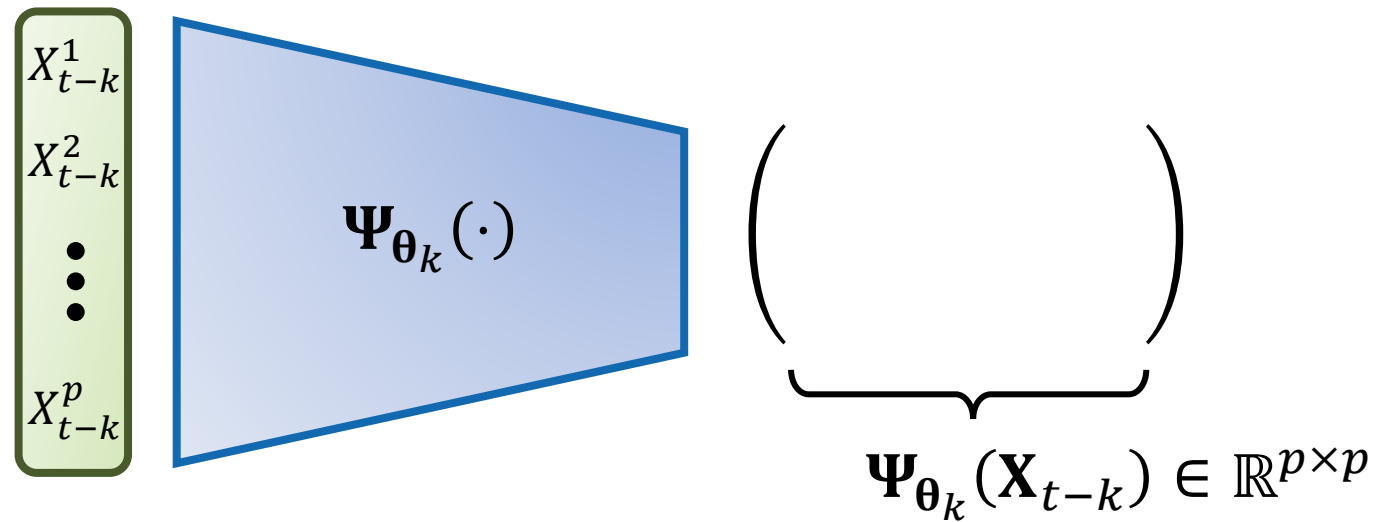
Generalised Vector Autoregression

$$\begin{pmatrix} X_{t-k}^1 \\ X_{t-k}^2 \\ \vdots \\ X_{t-k}^p \end{pmatrix}$$

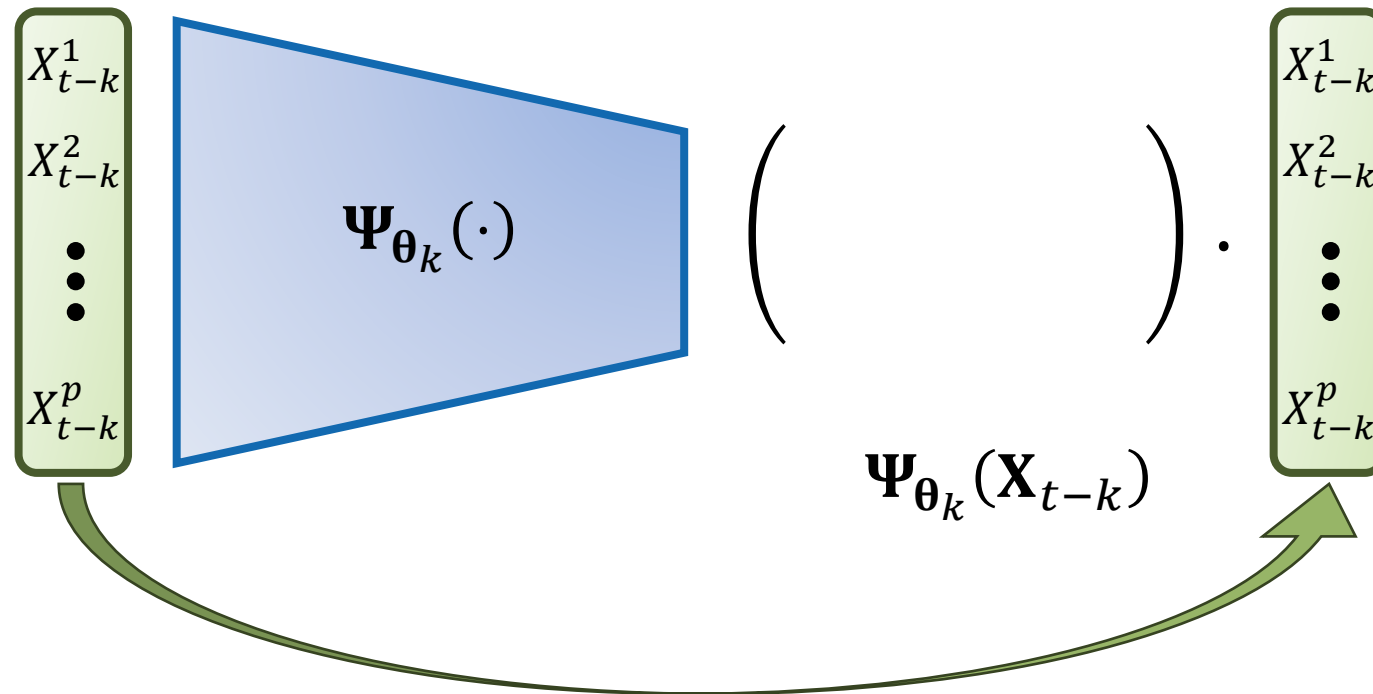
Generalised Vector Autoregression



Generalised Vector Autoregression



Generalised Vector Autoregression



Generalised Vector Autoregression

$$\sum_{k=1}^K \begin{pmatrix} X_{t-k}^1 \\ X_{t-k}^2 \\ \vdots \\ X_{t-k}^p \end{pmatrix} \Psi_{\theta_k}(\cdot) = \widehat{\mathbf{X}}_t$$

Generalised Vector Autoregression

$$\sum_{k=1}^K \begin{pmatrix} X_{t-k}^1 \\ X_{t-k}^2 \\ \vdots \\ X_{t-k}^p \end{pmatrix} \begin{array}{c} \text{Generalised coefficients} \\ \left(\begin{array}{c} \Psi_{\theta_k}(\cdot) \end{array} \right) \\ \Psi_{\theta_k}(\mathbf{X}_{t-k}) \end{array} \cdot \begin{pmatrix} X_{t-k}^1 \\ X_{t-k}^2 \\ \vdots \\ X_{t-k}^p \end{pmatrix} = \hat{\mathbf{X}}_t$$

$$\sum_{t=K+1}^T \ell(\mathbf{x}_t, \hat{\mathbf{x}}_t) + \lambda \sum_{t=K+1}^T R(\Psi_t) + \gamma \sum_{t=K+1}^{T-1} \|\Psi_{t+1} - \Psi_t\|_2^2$$

$$\underbrace{\sum_{t=K+1}^T \ell(\mathbf{x}_t, \hat{\mathbf{x}}_t)}_{\text{forecasting loss}} + \lambda \sum_{t=K+1}^T R(\Psi_t) + \gamma \sum_{t=K+1}^{T-1} \|\Psi_{t+1} - \Psi_t\|_2^2$$

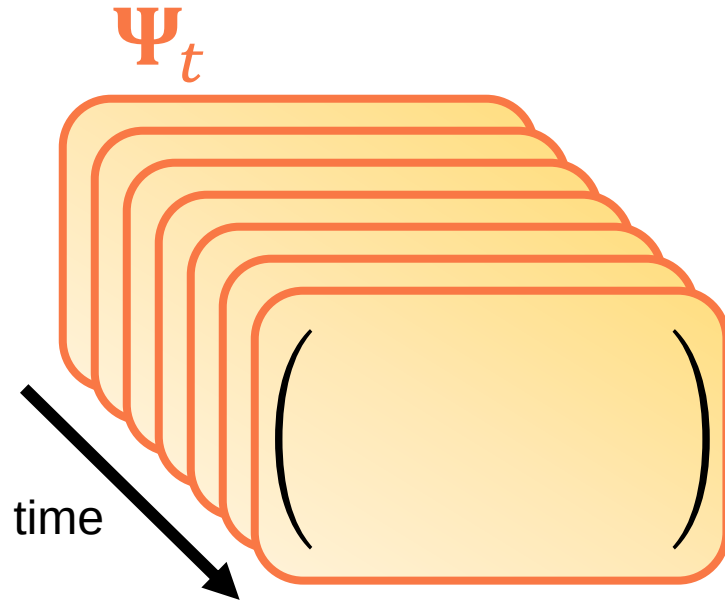
Loss Function

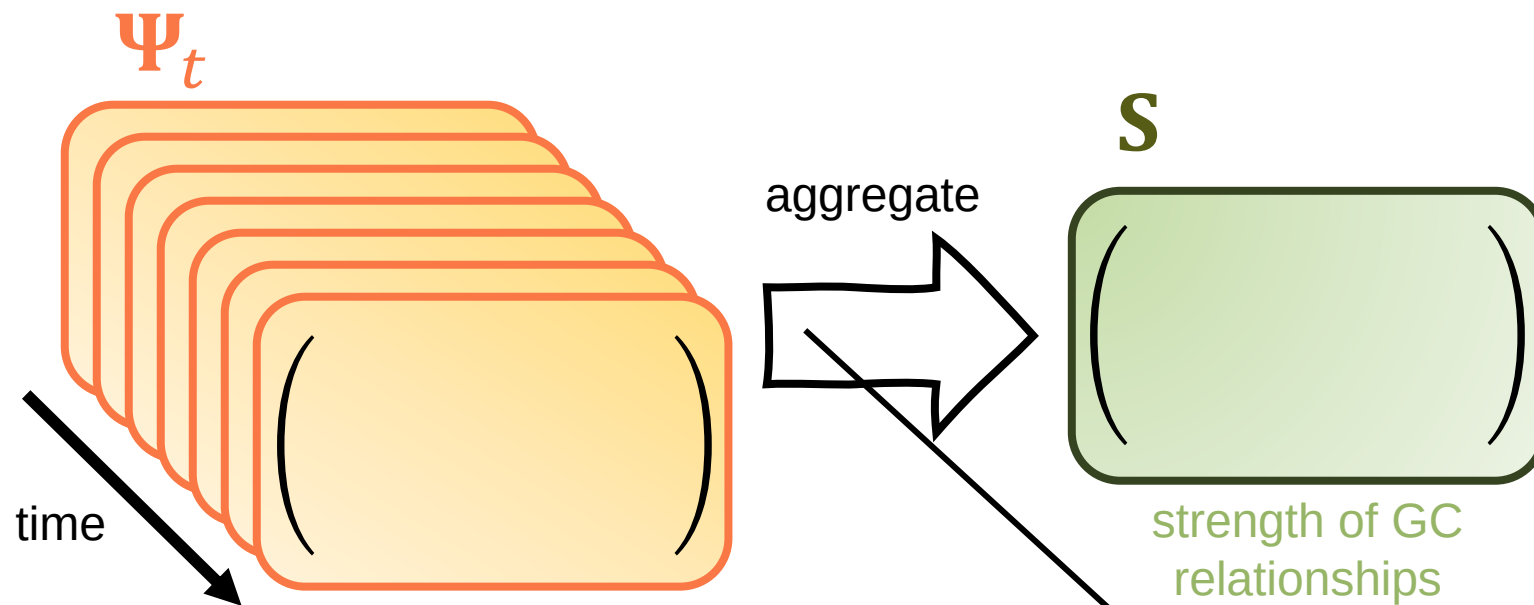
$$\underbrace{\sum_{t=K+1}^T \ell(\mathbf{x}_t, \hat{\mathbf{x}}_t)}_{\text{forecasting loss}} + \lambda \underbrace{\sum_{t=K+1}^T R(\Psi_t)}_{\text{sparsity-inducing penalty}} + \gamma \sum_{t=K+1}^{T-1} \|\Psi_{t+1} - \Psi_t\|_2^2$$

Loss Function

$$\underbrace{\sum_{t=K+1}^T \ell(\mathbf{x}_t, \hat{\mathbf{x}}_t)}_{\text{forecasting loss}} + \lambda \underbrace{\sum_{t=K+1}^T R(\Psi_t)}_{\text{sparsity-inducing penalty}} + \gamma \underbrace{\sum_{t=K+1}^{T-1} \|\Psi_{t+1} - \Psi_t\|_2^2}_{\text{smoothing penalty}}$$

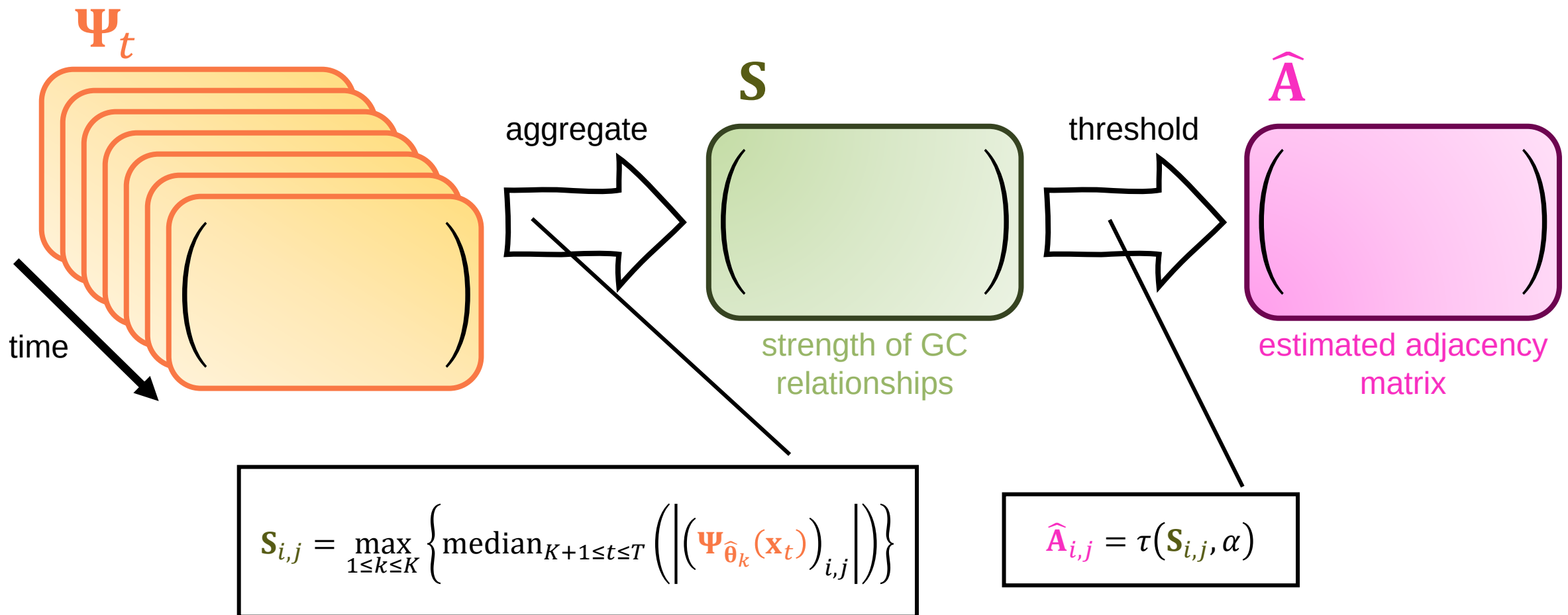
Inference Framework





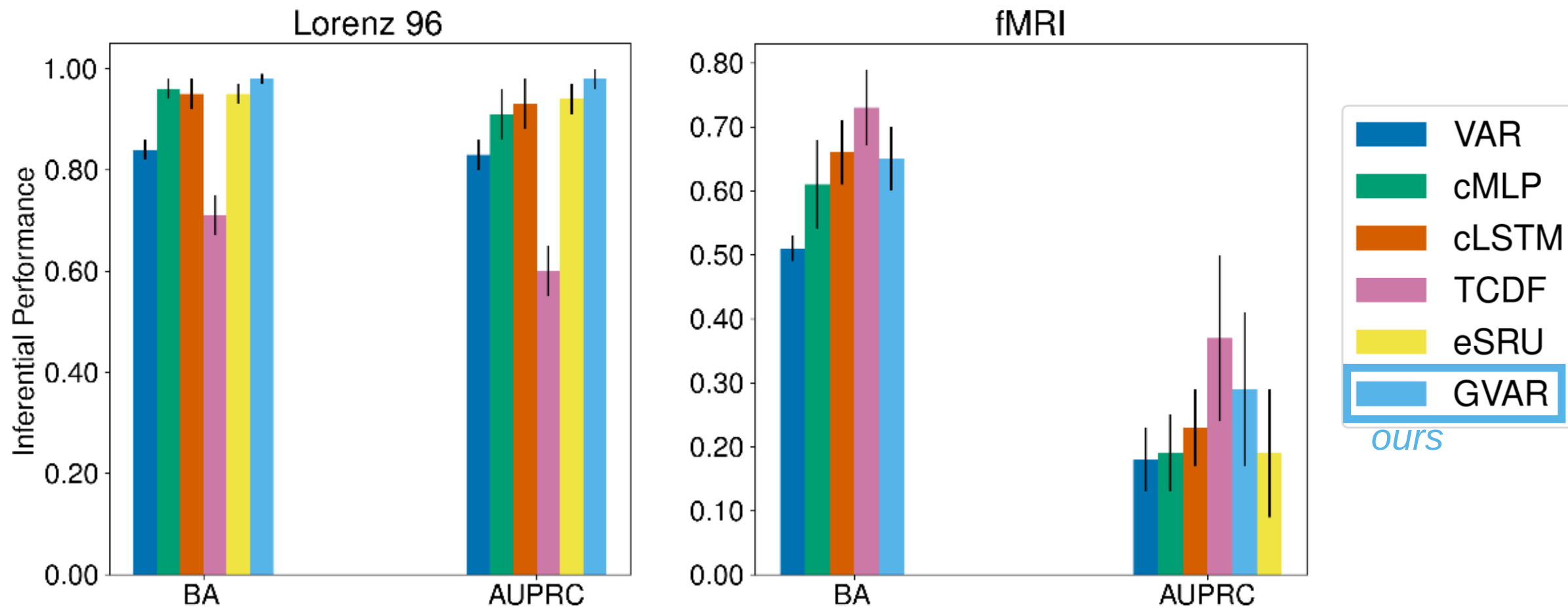
$$s_{i,j} = \max_{1 \leq k \leq K} \left\{ \text{median}_{K+1 \leq t \leq T} \left(\left| \left(\Psi_{\hat{\theta}_k}(\mathbf{x}_t) \right)_{i,j} \right| \right) \right\}$$

Inference Framework

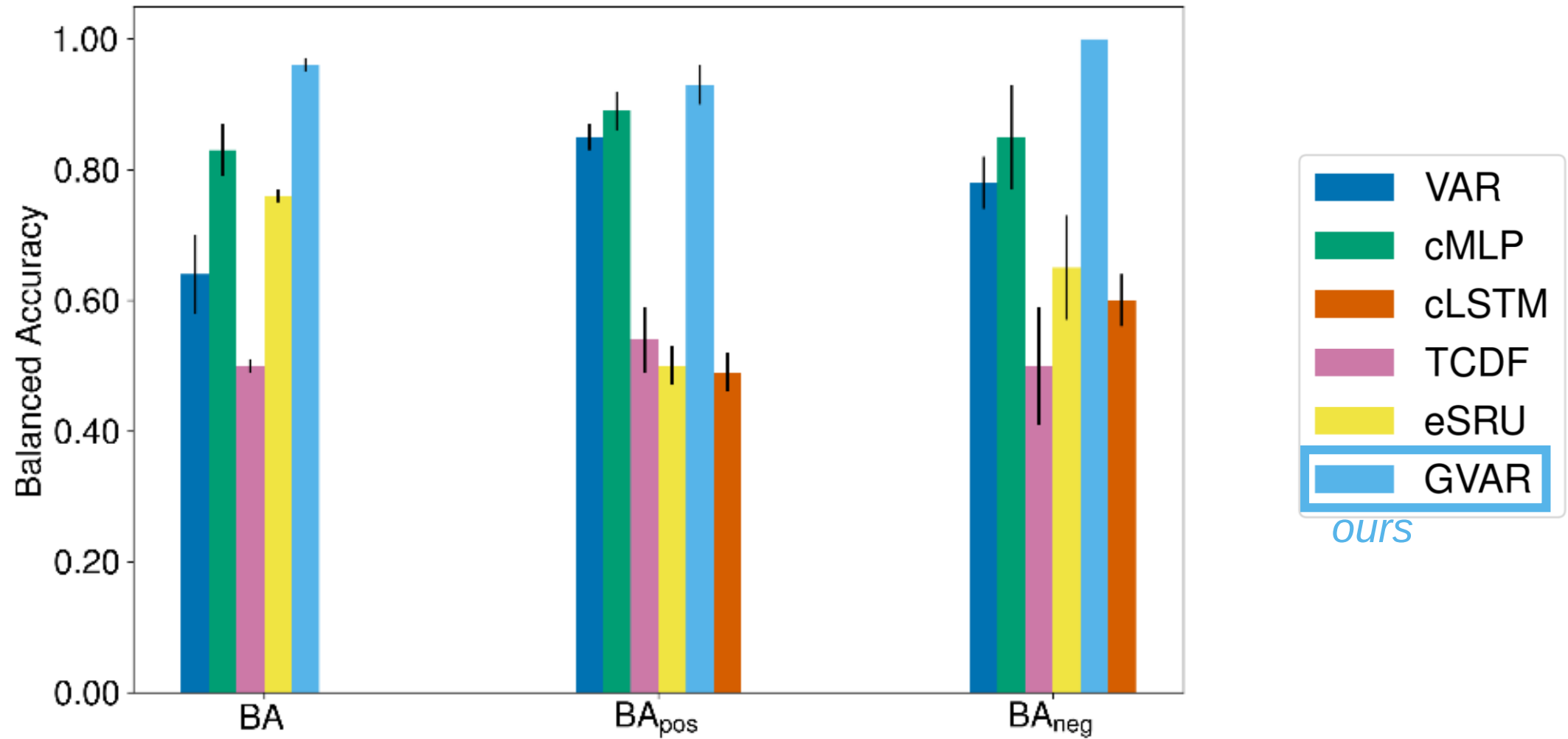


Empirical Results

Inferring Relationships



Detecting Effect Signs



- Extension of self-explaining neural networks to autoregressive setting

- Extension of self-explaining neural networks to autoregressive setting
- Interpretable inference framework for nonlinear multivariate Granger causality

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- Comprehensive experiments on simulated data

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Paper:



Code:

