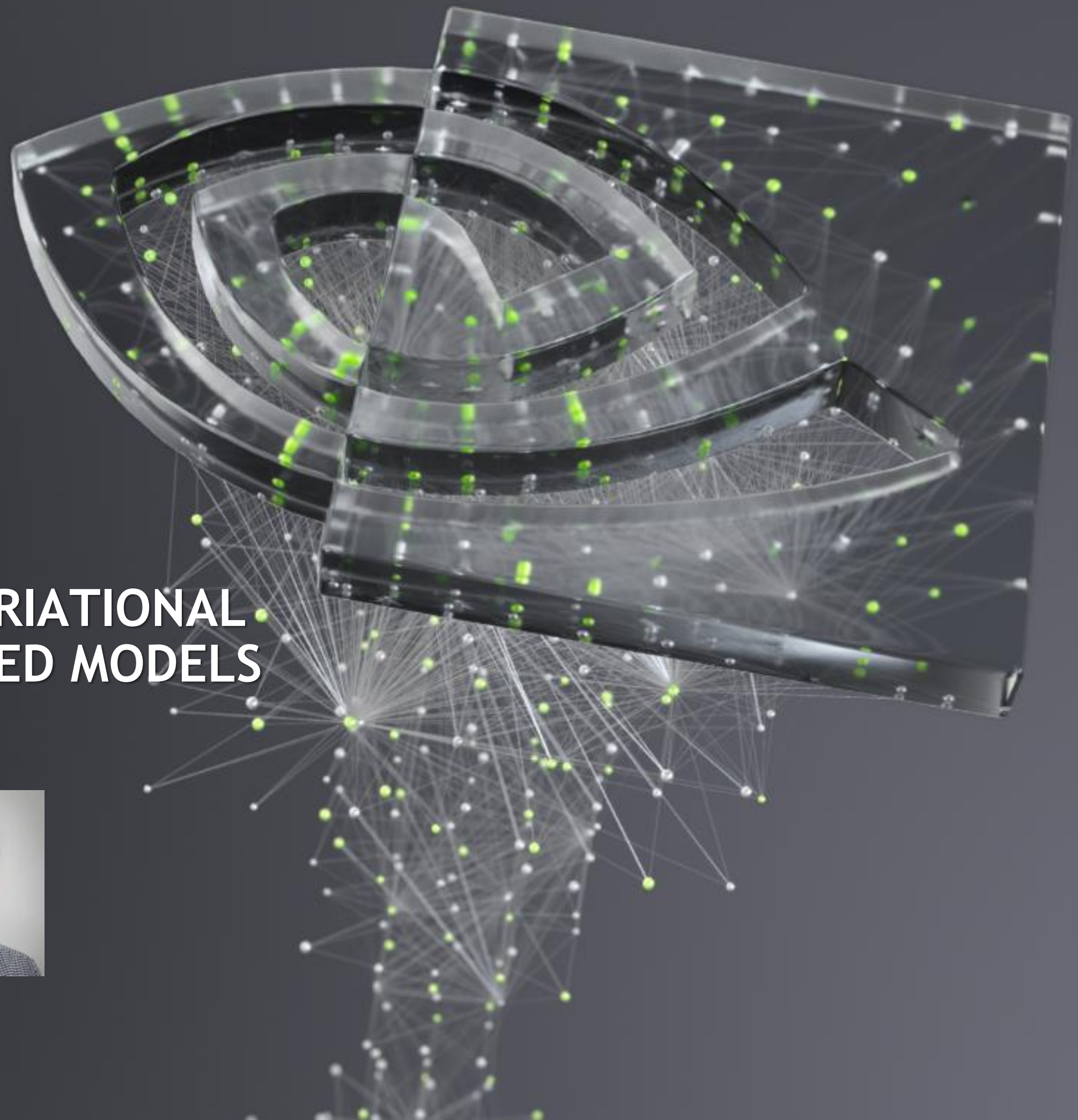




# VAEBM: A SYMBIOSIS BETWEEN VARIATIONAL AUTOENCODERS AND ENERGY-BASED MODELS

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# VARIATIONAL AUTO-ENCODERS (VAES)

## A quick recap

- Assume data distribution is modeled by  $p(x) = \int p(x|z)p(z) dz$ 
  - $p(x|z)$  is the decoder distribution, and  $p(z)$  is the prior
  - Training would be easy if we have access to the posterior  $p(z|x)$ , but this is intractable in general
- Resort to use variational inference, where a variational posterior  $q(z|x)$  is introduced as an approximation to the true posterior, resulting in the variational lower bound:

$$\log p(x) \geq \underbrace{\mathbb{E}_{q(z|x)}[\log p(x|z)]}_{\text{Reconstruction Term}} - \underbrace{KL(q(z|x) || p(z))}_{\text{KL Regularization Term}}$$

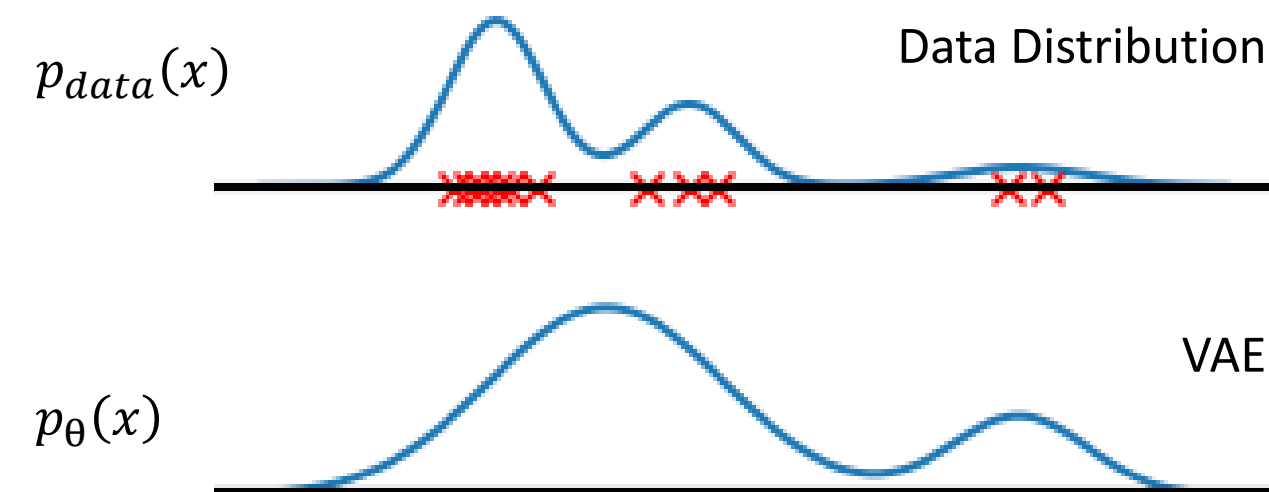
- Recently, large VAEs such as NVAE\* and VDVAE\*\* with carefully designed network structures and hierarchical latent variables achieve impressive results in likelihood modeling, but their sample qualities are still limited.

\* NVAE: A Deep Hierarchical Variational Autoencoder, Vahdat and Kautz.

\*\* Very Deep VAEs Generalize Autoregressive Models and Can Outperform Them on Images, Child

# WHAT'S WRONG WITH VAEs?

VAEs tend to assign high probabilities to non-data like regions!



NVAE

$t = 1.$





# ENERGY-BASED MODELS (EBMS)

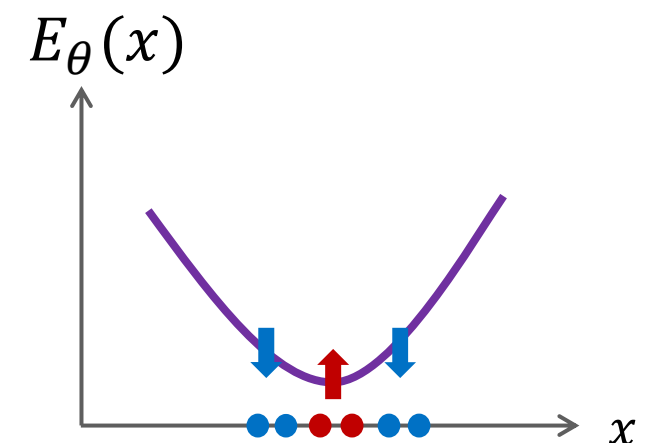
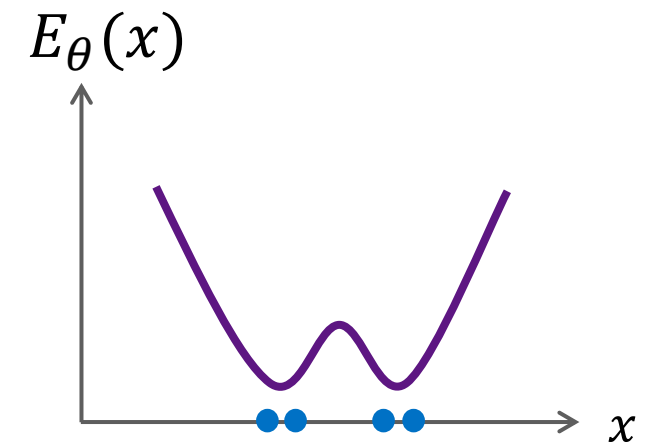
## A quick introduction

- Assume data distribution is modeled by  $p_{\text{EBM}}(x) = \frac{1}{Z} e^{-E_{\theta}(x)}$ 
  - where  $E_{\theta}: \mathcal{X} \rightarrow \mathbb{R}$  is an energy function implemented by neural networks
  - $Z$  is the normalization constant

- Maximum likelihood training:

$$\nabla_{\theta} \mathbb{E}_{x \sim p_{\text{data}}(x)} [\log p_{\text{EBM}}(x)] = - \underbrace{\mathbb{E}_{x \sim p_{\text{data}}(x)} [\nabla_{\theta} E_{\theta}(x)]}_{\text{Training Samples}} + \underbrace{\mathbb{E}_{x \sim p_{\text{EBM}}(x)} [\nabla_{\theta} E_{\theta}(x)]}_{\text{Model Samples}}$$

- Sampling from model** is often done by Markov chain Monte Carlo (MCMC) sampling



# VAES VS. EBMS

## A comparison

### Energy-based Models (EBMs):

- 😊 Explicitly push down the densities of non-data like regions
- 😊 Almost no constrain on the energy function (unlike normalizing flows)
- 😞 Slow sampling during training and test due to expensive MCMC steps

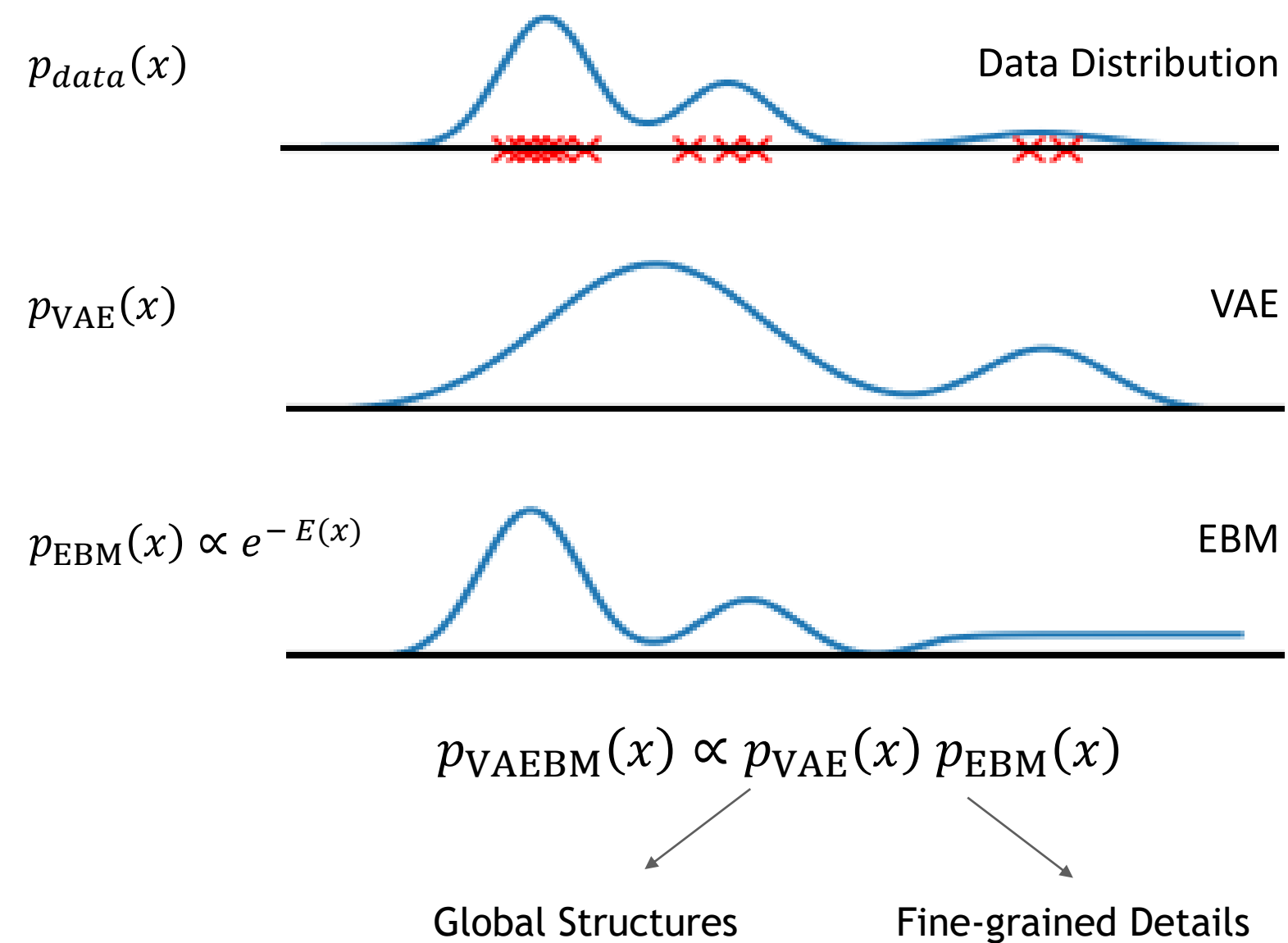
### Variational Autoencoders (VAEs):

- 😊 Fast sampling, easy train
- 😊 Latent embedding allows fast traversal in data space
- 😞 High probabilities for non-data-like regions in the data space

VAEBM: A symbiotic composition of VAEs and EBMs

# VAEBM

## The basic idea



By taking the product of the densities of a VAE and an EBM, we want the VAE to **capture the global structures of data**, and the EBM to **refine the distribution** by pushing down the densities of non-data-like regions.

# VAEBM

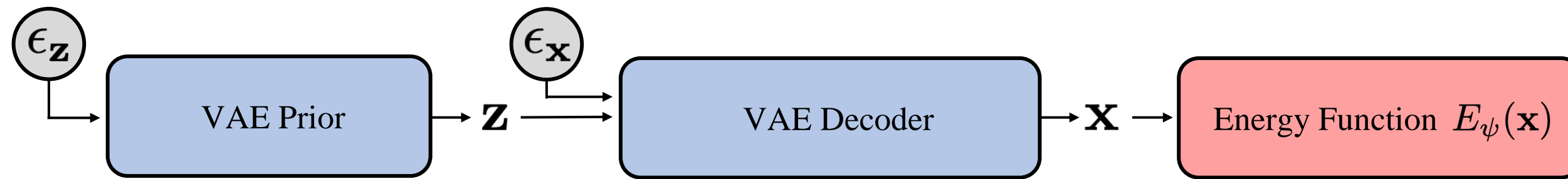
## Conceptual Visualization



$$p_{\text{VAEBM}}(x) \propto p_{\text{VAE}}(x) p_{\text{EBM}}(x)$$

# VAEBM

## Training



$$p_{\text{VAEBM}}(x) = \frac{1}{Z} p_{\text{VAE}}(x) p_{\text{EBM}}(x)$$

$$\log p_{\text{VAEBM}}(x) = \underbrace{\log p_{\text{VAE}}(x)}_{\text{Stage (1) train VAE}} + \underbrace{\log p_{\text{EBM}}(x)}_{\text{Stage (2) train EBM}} - \log Z$$

Stage (1) train VAE

Stage (2) train EBM

easy with  
reparam. trick  
😊

requires  
MCMC sampling  
😞

→  
 $x = T(\epsilon_z, \epsilon_x)$

Run MCMC  
in the  $\epsilon$  space\*  
😊



# VAEBM

Two stage training



Stage (1) train VAE

Stage (2) train EBM

## A symbiotic composition:

- VAE learns the overall mode structure
- VAE provides re-parametrization for MCMC sampling from EBM
- EBM helps VAE to exclude non-data-like regions
- MCMC steps are expensive, but VAEBM requires very few training epochs for EBM



# CIFAR10

Use NVAE as the base VAE

NVAE ( $t = 1$ )



VAEBM

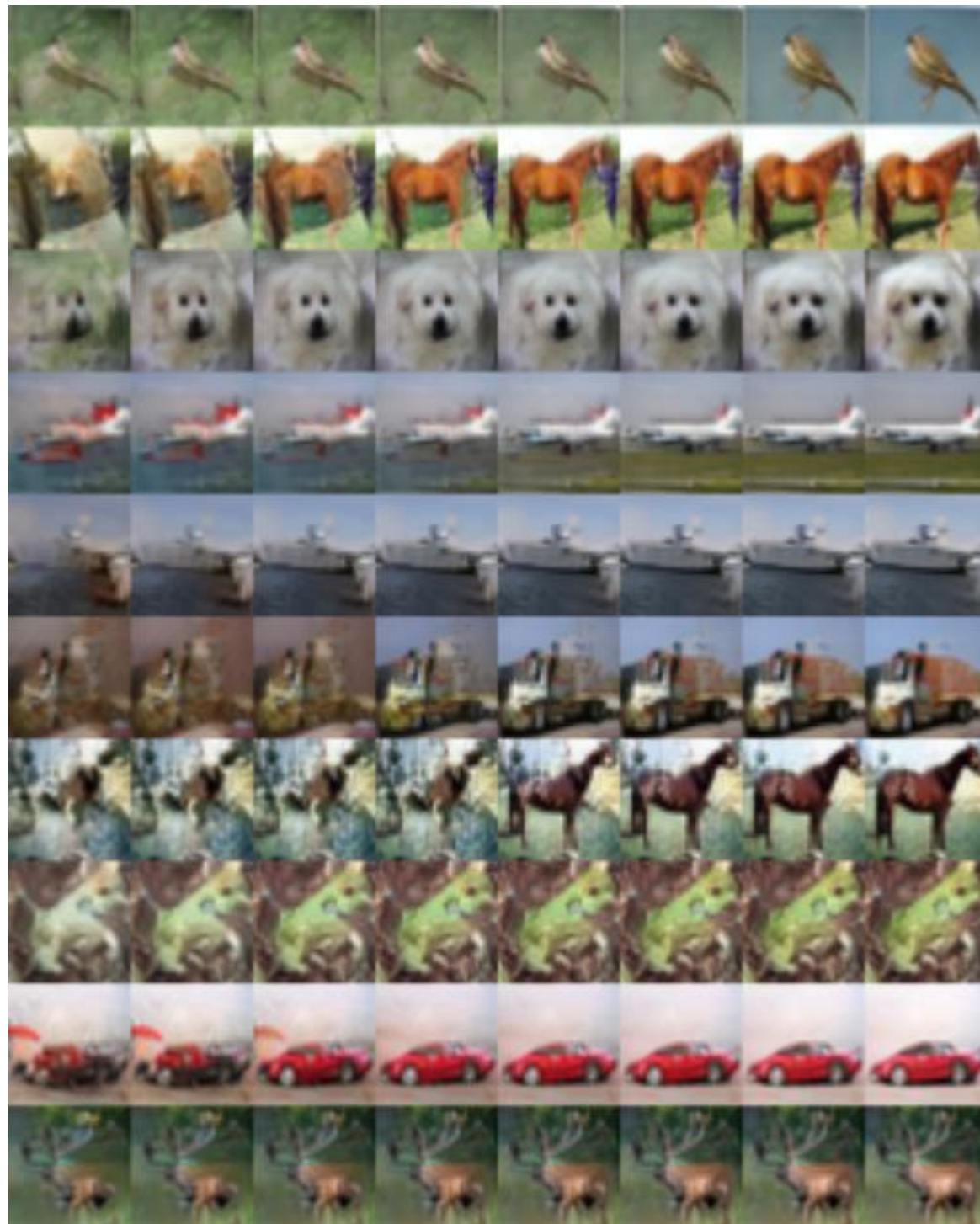




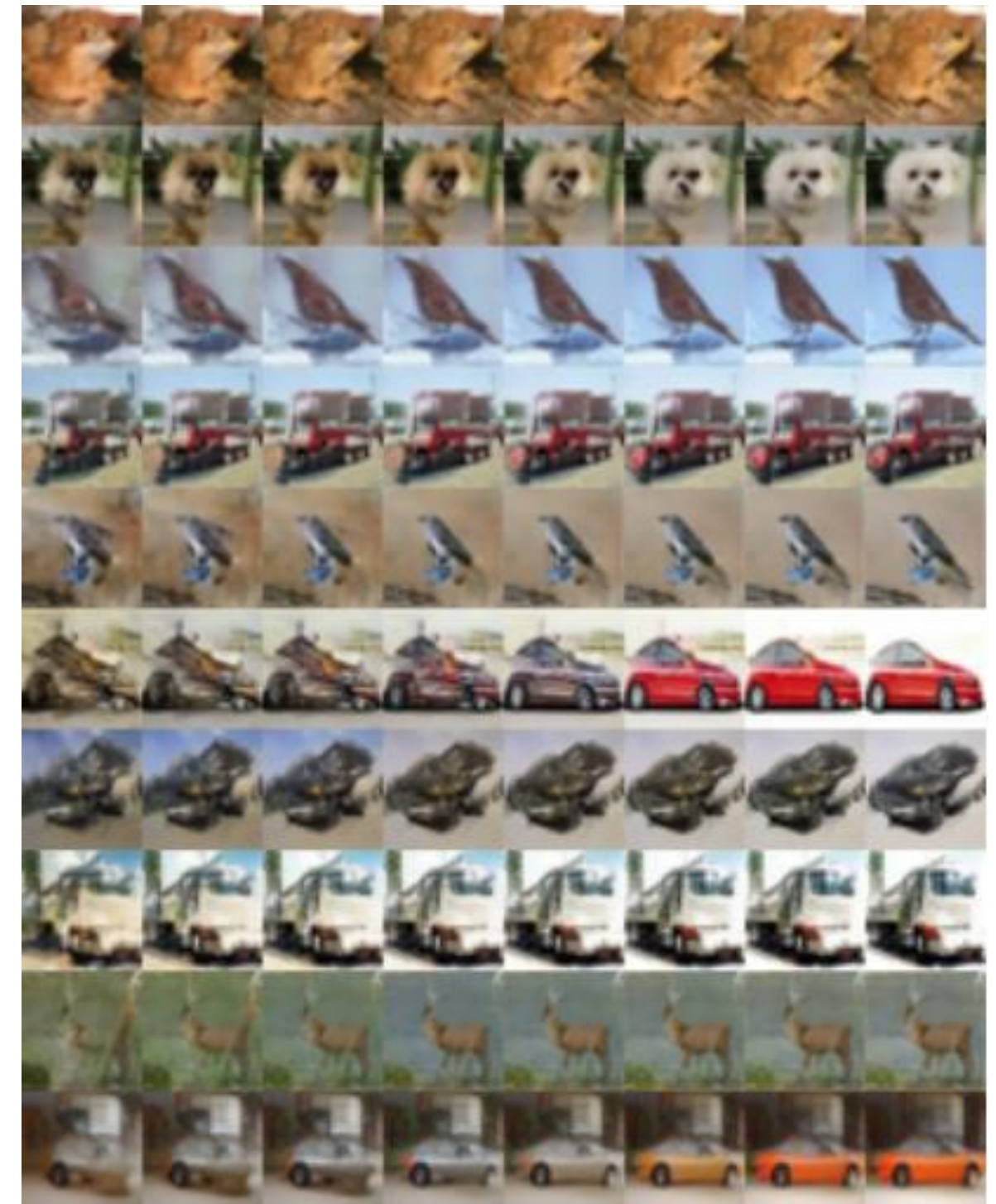
# 16 MCMC STEPS

## CIFAR-10

NVAE  $\xrightarrow{\text{16-step MCMC}}$  VAEBM



NVAE  $\xrightarrow{\text{16-step MCMC}}$  VAEBM





# QUANTITATIVE RESULTS

CIFAR-10 (unconditional)

	Model	IS↑	FID↓
<b>Ours</b>	VAEBM w/o persistent chain	8.21	12.26
	VAEBM w/ persistent chain	8.43	12.19
<b>EBMs</b>	IGEBM (Du & Mordatch, 2019)	6.02	40.58
	EBM with short-run MCMC (Nijkamp et al., 2019b)	6.21	-
	F-div EBM (Yu et al., 2020a)	8.61	30.86
	FlowCE (Gao et al., 2020)	-	37.3
	FlowEBM (Nijkamp et al., 2020)	-	78.12
	GEBM (Arbel et al., 2020)	-	23.02
	Divergence Triangle (Han et al., 2020)	-	30.1
<b>Other Likelihood Models</b>	Glow (Kingma & Dhariwal, 2018)	3.92	48.9
	PixelCNN (Oord et al., 2016b)	4.60	65.93
	NVAE (Vahdat & Kautz, 2020)	5.51	51.67
	VAE with EBM prior (Pang et al., 2020)	-	70.15
<b>Score-based Models</b>	NCSN (Song & Ermon, 2019)	8.87	25.32
	NCSN v2 (Song & Ermon, 2020)	-	31.75
	Multi-scale DSM (Li et al., 2019)	8.31	31.7
	Denoising Diffusion (Ho et al., 2020)	9.46	3.17
<b>GAN-based Models</b>	SNGAN (Miyato et al., 2018)	8.22	21.7
	SNGAN+DDLs (Che et al., 2020)	9.09	15.42
	SNGAN+DCD (Song et al., 2020)	9.11	16.24
	BigGAN (Brock et al., 2018)	9.22	14.73
	StyleGAN2 w/o ADA (Karras et al., 2020a)	8.99	9.9

VAEBM is 12x faster than NCSN (Song & Ermon)



# QUALITATIVE RESULTS

## Other datasets

FID: NVAE 14.7 → VAEBM 5.3



(a) CelebA 64

FID: NVAE 41.3 → VAEBM 13.5



(b) LSUN Church 64



FID: NVAE 45.1 → VAEBM 20.4

(c) CelebA HQ 256



# OUT OF DISTRIBUTION DETECTION

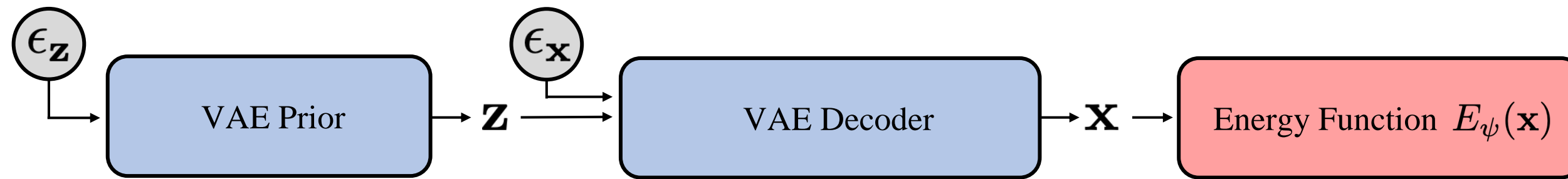


Table 6: Table for AUROC $\uparrow$  of  $\log p(\mathbf{x})$  computed on several OOD datasets. In-distribution dataset is CIFAR-10. Interp. corresponds to linear interpolation between CIFAR-10 images.

		SVHN	Interp.	CIFAR100	CelebA
<b>Unsupervised Training</b>	NVAE (Vahdat & Kautz, 2020)	0.42	0.64	0.56	0.68
	Glow (Kingma & Dhariwal, 2018)	0.05	0.51	0.55	0.57
	IGEBM (Du & Mordatch, 2019)	0.63	<b>0.7</b>	0.5	0.7
	Divergence Traingle (Han et al., 2020)	0.68	-	-	0.56
	VAEBM (ours)	<b>0.83</b>	<b>0.7</b>	<b>0.62</b>	<b>0.77</b>
<b>Supervised Training</b>	JEM (Grathwohl et al., 2020a)	0.67	0.65	0.67	0.75
	HDGE (Liu & Abbeel, 2020)	0.96	0.82	0.91	0.8

# SUMMARY

## VAEBM: A symbiotic composition of VAE & EBM



$$p_{\text{VAEBM}}(x) \propto p_{\text{VAE}}(x) p_{\text{EBM}}(x)$$

- A two-stage training is proposed
- The experimental results show that the EBM component can improve the generative quality of VAEs by a large margin
- VAE helps with MCMC sampling from the EBM component
- We showed out of distribution detection results and studied mode coverage properties
- Codes will be available at [github.com/NVlabs/VAEBM](https://github.com/NVlabs/VAEBM)