## The Importance of Pessimism in Fixed-Dataset Policy Optimization

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Our goal is an algo with minimal worst-case suboptimality,

 $\operatorname{SubOpt}(\mathcal{O}(D)) = \mathbb{E}_{\rho}[\mathbf{v}_{\mathcal{M}}^{\pi_{\mathcal{M}}^{*}}] - \mathbb{E}_{\rho}[\mathbf{v}_{\mathcal{M}}^{\mathcal{O}(D)}].$ 

We consider "value-based" algorithms,

 $\mathcal{O}_{\mathrm{sub}}^{\mathrm{VB}}(D) := rg \max \mathbb{E}_{
ho}[\mathcal{E}_{\mathrm{sub}}(D,\pi)].$ 

These algorithms can be characterized by the choice of fixed point of  $E_{sub}$ . Suboptimality of these algos permits an "over/under decomposition",

**Theorem 1.** For any space  $\mathcal{X}$ , objective  $f : \mathcal{X} \to \mathbb{R}$ , and proxy objective  $\hat{f} : \mathcal{X} \to \mathbb{R}$ ,

$$f(x^*) - f(\hat{x}^*) \le \inf_{x \in \mathcal{X}} \left( [f(x^*) - f(x)] + [f(x) - \hat{f}(x)] \right) + \sup_{x \in \mathcal{X}} \left( \hat{f}(x) - f(x) \right)$$

where  $x^* := \arg \max_{x \in \mathcal{X}} f(x)$  and  $\hat{x}^* := \arg \max_{x \in \mathcal{X}} \hat{f}(x)$ . Furthermore, this bound is tight.

One important type of algo is "naive":

 $f_{\textit{naïve}}(\mathbf{v}^{\pi}) := A^{\pi}(\mathbf{r}_D + \gamma P_D \mathbf{v}^{\pi}). \qquad \qquad SUBOPT(\mathcal{O}_{\textit{naïve}}^{\textit{VB}}(D)) \leq \inf_{\pi} \left( \mathbb{E}_{\rho}[\mathbf{v}_{\mathcal{M}}^{\pi^{\star}} - \mathbf{v}_{\mathcal{M}}^{\pi}] + \mathbb{E}_{\rho}[\boldsymbol{\mu}_{D,\delta}^{\pi}] \right) + \sup_{\pi} \mathbb{E}_{\rho}[\boldsymbol{\mu}_{D,\delta}^{\pi}]$ 

This often leads to a large "sup" term. We can fix this by finding pessimistic fixed points, which let us choose the relative size of the two terms:

$$SUBOPT(\underline{\mathcal{O}}_{ua}^{VB}(D)) \leq \inf \left( \mathbb{E}_{\rho}[\mathbf{v}_{\mathcal{M}}^{\pi^*_{\mathcal{M}}} - \mathbf{v}_{\mathcal{M}}^{\pi}] + (1+\alpha) \cdot \mathbb{E}_{\rho}[\boldsymbol{\mu}_{D,\delta}^{\pi}] \right) + (1-\alpha) \cdot \left( \sup_{\pi} \mathbb{E}_{\rho}[\boldsymbol{\mu}_{D,\delta}^{\pi}] \right)$$
$$f_{ua}(\mathbf{v}^{\pi}) = A^{\pi}(\mathbf{r}_{D} + \gamma P_{D}\mathbf{v}^{\pi}) - \alpha \mathbf{u}_{D,\delta}^{\pi}$$

Implementing this algorithm requires implementing a valid uncertainty measure, which we don't know how to do right now with NNs. If we take "trivial uncertainty" of  $V_{max}$ , we get proximal algorithms:

$$f_{proximal}(\mathbf{v}^{\pi}) = A^{\pi}(\mathbf{r}_D + \gamma P_D \mathbf{v}^{\pi}) - \alpha \left(\frac{TV_{\mathcal{S}}(\pi, \hat{\pi}_D)}{(1 - \gamma)^2}\right)$$

The trivial uncertainty is the "worst" uncertainty, leading to a much looser bound; but it is, at least, implementable.

This work **provides formal justification** for the properties of **every "Offline RL" algorithm** in the literature, including:

BCQ, CRR, SPIBB, BEAR, CQL, KLC, BRAC, MBS-QI, MOREL, MOPO, and more.

		2	3	 1000
μ	99%	1%	1%	1%
n <sub>D</sub>	10000	1	1	1
$\mu_{D}$	98%	0%	100%	0%



