

# Robust Pruning at Initialization

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# Overparameterized Models

- Millions/Billions of parameters.
- Can we **reduce the size** of these models without major drop in the performance?
- Pruning **after** Training: train, prune, repeat...  
Very slow and requires excessive computational power.
- Pruning at Initialization?

- **Pruning**: apply a binary mask  $\delta$  to the weights. The pruned model is given by

$$y^l(x) = \mathcal{F}_l(\delta^l \circ W^l, y^{l-1}(x)) + B^l$$

- **Sensitivity** based pruning (SNIP, Lee et al. 2018): prune the weights at initialization based on  $|W \frac{\partial \mathcal{L}}{\partial W}|$ . Inspired from

$$\mathcal{L}_W \approx \mathcal{L}_{W=0} + W \frac{\partial \mathcal{L}}{\partial W}$$

# Ordered, Chaotic, and EOC Initializations

Assume  $W_{ij}^l \sim \mathcal{N}(0, \sigma_w^2/N_{l-1})$ ,  $B_i^l \sim \mathcal{N}(0, \sigma_b^2)$ .

- $q^l(x) = \text{var}(y_1^l(x)) \xrightarrow{l \rightarrow \infty} q$
- $C_l(x, x') = \text{corr}(y_1^l(x), y_1^l(x')) \xrightarrow{l \rightarrow \infty} ??$

Depending on the choice of  $(\sigma_b, \sigma_w)$ :

- **Ordered phase** where  $C_l(x, x') \rightarrow 1$  **exponentially quickly** [Schoenholz et al., 2017]
- **Chaotic phase** where  $C_l(x, x') \rightarrow c < 1$  **exponentially quickly** [Schoenholz et al., 2017]
- **Edge of Chaos (EOC)** where  $C_l(x, x') \rightarrow 1$  **polynomial rate** [Hayou et al., 2019]

# Sensitivity Based Pruning (SBP)

- **Critical sparsity**: sparsity level  $s_{cr}$  such that one layer at least is fully pruned.  $s_{cr}$  is random.

## Proposition (Initialization is crucial for SBP, Informal)

Assume  $W^l \in \mathbb{R}^{N \times N}$ , and let  $L$  be the depth.

- If  $(\sigma_b, \sigma_w) \in$  Ordered phase

$$\mathbb{E}[s_{cr}] = \mathcal{O} \left( \frac{\log(LN^2)}{L} + \frac{1}{\sqrt{LN^2}} \right)$$

- $(\sigma_b, \sigma_w) \in$  EOC, then the upper bound no longer holds.
- On the Ordered phase,  $\lim_{L \rightarrow \infty} \mathbb{E}[s_{cr}] = 0$ .
- Similar results can be proven for the Chaotic phase.

# Sensitivity Based Pruning (SBP)

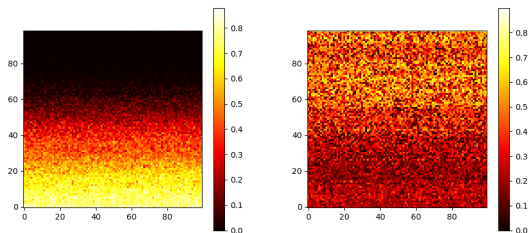


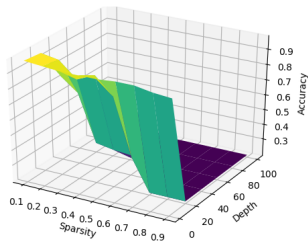
Figure: Percentage of weights kept after SBP. 100x100 FFNN,  $s = 70\%$ , Chaotic phase(left), EOC(right).

# Training the Sparse Network

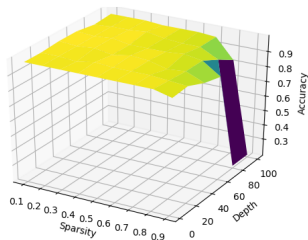
- After pruning, it might be difficult to train the sparse network...
- Putting the pruned network back on the EOC

$$y^l(x) = \rho_l \mathcal{F}_l(\delta^l \circ W^l, y^{l-1}(x)) + B^l$$

# Training the Sparse Architecture



Init Ordered Phase



Init EOC + ReScaling



# Training the Sparse Architecture

- Our algorithm **SBP-SR** yields SOTA (one shot pruning algorithms) performance for Deep ResNets.

Table: Classification accuracies on Tiny ImageNet for Resnet with varying depths

	ALGORITHM	85%	90%	95%
RESNET32	SBP-SR	<b>57.25 ± 0.09</b>	<b>55.67 ± 0.21</b>	50.63±0.21
	SNIP	56.92± 0.33	54.99±0.37	49.48±0.48
	GRASP	<b>57.25±0.11</b>	55.53±0.11	<b>51.34±0.29</b>
RESNET50	SBP-SR	<b>59.8±0.18</b>	<b>57.74±0.06</b>	<b>53.97±0.27</b>
	SNIP	58.91±0.23	56.15±0.31	51.19±0.47
	GRASP	58.46±0.29	57.48±0.35	52.5±0.41
RESNET104	SBP-SR	<b>62.84±0.13</b>	<b>61.96±0.11</b>	<b>57.9±0.31</b>
	SNIP	59.94±0.34	58.14±0.28	54.9±0.42
	GRASP	61.1±0.41	60.14±0.38	56.36±0.51

For more details, check our paper

Robust Pruning at Initialization. ICLR 2021. *S. Hayou, J.F. Ton, A. Doucet, Y.W. Teh.*

- S.S. Schoenholz, J. Gilmer, S. Ganguli, and J. Sohl-Dickstein. Deep information propagation. *5th International Conference on Learning Representations*, 2017.
- S. Hayou, A. Doucet, and J. Rousseau. On the impact of the activation function on deep neural networks training. *ICML*, 2019.