Efficient Wasserstein Natural Gradients for Reinforcement Learning

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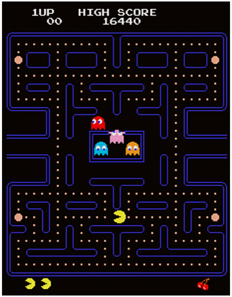
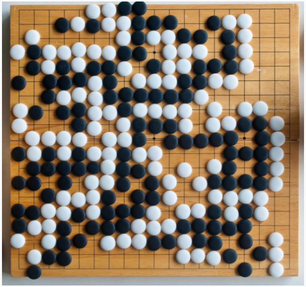
ICLR 2021

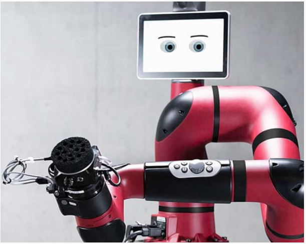
Gatsby Computational Neuroscience Unit

UNIVERSITY OF CAMBRIDGE

Motivation







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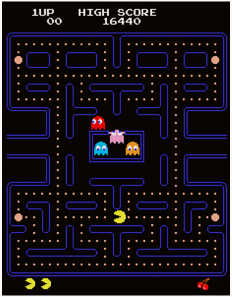

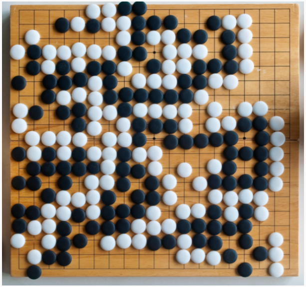

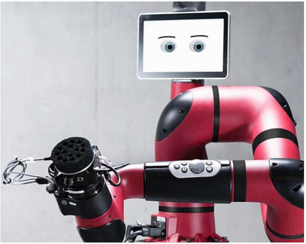

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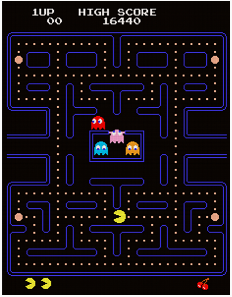

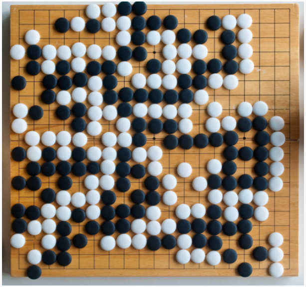

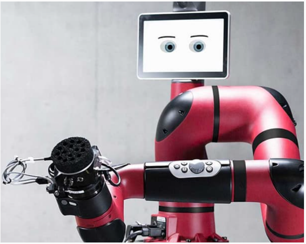

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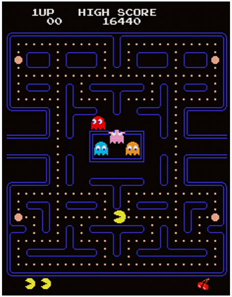

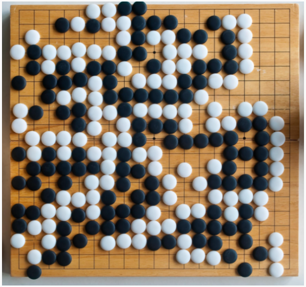

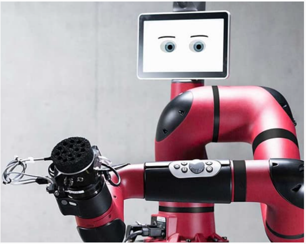

- Deep RL algorithms require $\sim 10^6$-10^9 samples for optimization
- Regularized policy optimization can improve sample efficiency
- The method of regularization can induce a geometry on the loss surface that is often underexploited

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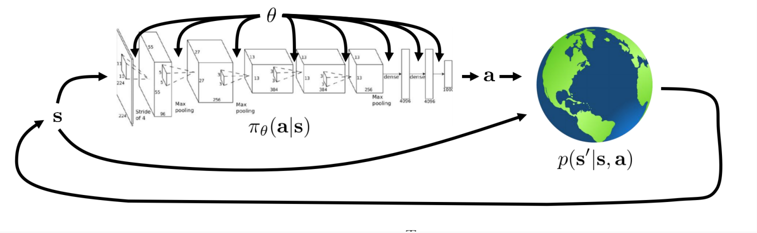
- WNG + policy gradients $\rightarrow$ WNPG
  WNG + evolution strategies $\rightarrow$ WNES [1]
- Improved results on challenging continuous control tasks.

RL Background



* figure adapted from http://rail.eecs.berkeley.edu/deeprlcourse/

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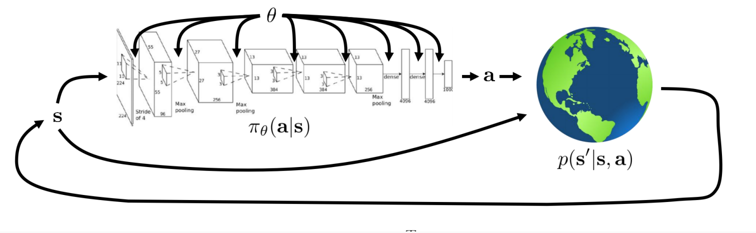

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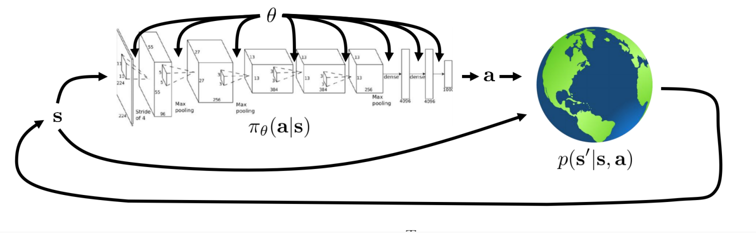

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- Running the policy in an episodic/finite horizon task of length T produces trajectories $\tau = (s_1, a_1, r_1, \ldots, s_T, a_T, r_T)$

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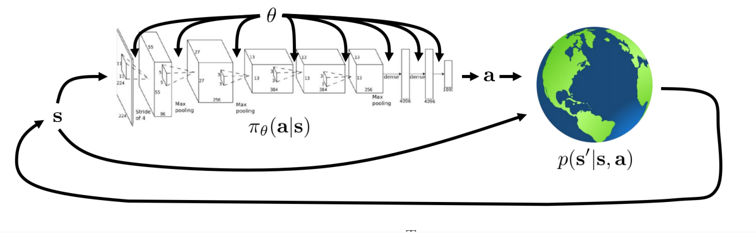

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- Maximize $\mathbb{E}_\pi [Z(\tau)] = \mathbb{E}_\pi \left[\sum_t \gamma^t r_t\right]$

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Regularized Policy Optimization

- Consider policy gradients on some parametric policy $\pi_\theta(a_t|s_t)$:

$$F(\theta) = \mathbb{E}_\pi \left[\sum_t \gamma^t r_t \right] \implies \nabla_\theta F = \mathbb{E}_\pi \left[Z(\tau) \sum_t \nabla_\theta \log \pi_\theta(a_t|s_t) \right]$$

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- $\Rightarrow$ We can make multiple updates with the same trajectories

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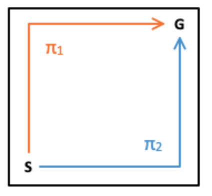

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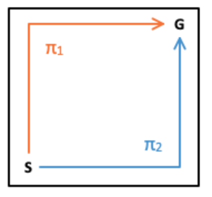
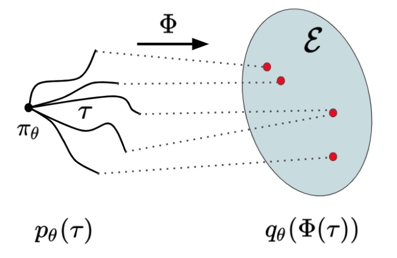


- How can we capture behavioral similarity? *Embed* trajectories [2]



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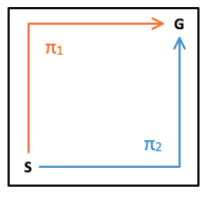

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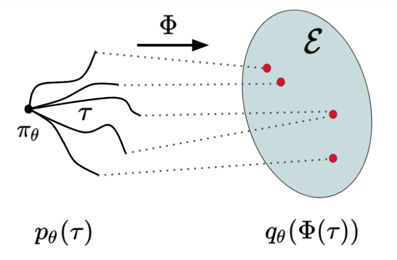

- How to compare? Measure WD between embedding distributions

Divergences Measures $\Rightarrow$ Natural Gradients

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- Euclidean gradient: Steepest direction wrt the L_2 distance

$$g^E = \underset{u}{\text{argmax}}\, F(\theta) + \nabla F^\top u - \frac{1}{2}\|u\|^2 \approx \underset{u}{\text{argmax}}\, F(\theta + u) - \frac{1}{2}\|u\|^2$$

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- Wasserstein natural gradient: Steepest direction wrt W_2

$$g^W = \operatorname*{argmax}_u F(\theta) + \nabla F^\mathsf{T} u - \frac{1}{2} u^\mathsf{T} G_W u \approx \operatorname*{argmax}_u F(\theta + u) - \frac{1}{2} W_2(\pi_{\theta+u}, \pi_\theta)$$

$$\Rightarrow g^W = {G_W}^{-1} \nabla F$$

Policy Optimization Using Behavioral Geometry

- The WD penalty captures a policy's global behavioral geometry [1]

$$\underset{\theta}{\operatorname{argmax}} F(\theta + u) + \frac{1}{2}\beta W_2^2(q_{\theta+u}, q_\theta) \tag{0.1}$$

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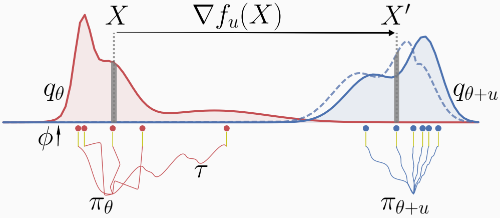
- The WNG captures the local behavior of a policy:

$$W_2^2(q_{\theta+u}, q_\theta) \simeq \mathbb{E}_{q_\theta}[\|\nabla f_u(X)\|^2] = u^\top G_W u \Rightarrow g^W = G_W^{-1}\nabla F \tag{0.2}$$



Wasserstein Natural Policy Gradients

- Apply behavioral WNG to policy gradients (PG):
  (1) $F(\theta)$ (WNPG) (2) $F(\theta) - \beta W_2(q_\theta, q_{\theta+u})$ (BG-WNPG)

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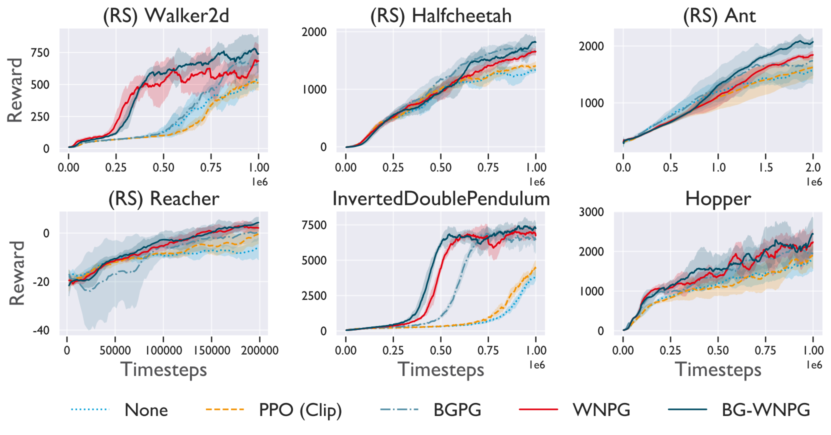

Wasserstein Natural Evolution Strategies

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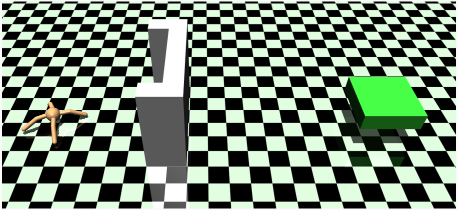

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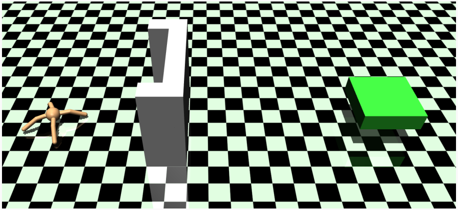

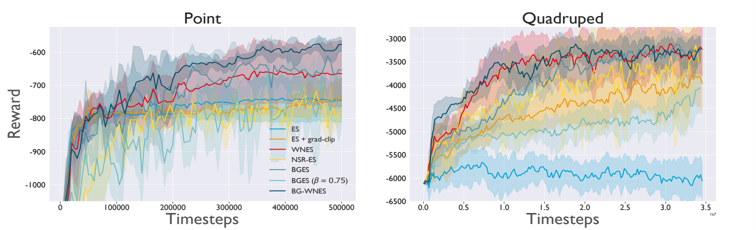

(a) (b)

[1] M. Arbel, A. Gretton, W. Li, and G. Montufar. Kernelized wasserstein natural gradient. In *International Conference on Learning Representations*, 2020.

[2] A. Pacchiano, J. Parker-Holder, Y. Tang, A. Choromanska, K. Choromanski, and M. I. Jordan. Learning to score behaviors for guided policy optimization. *arXiv preprint arXiv:1906.04349*, 2019.