Topology-Aware Segmentation Using Discrete Morse Theory

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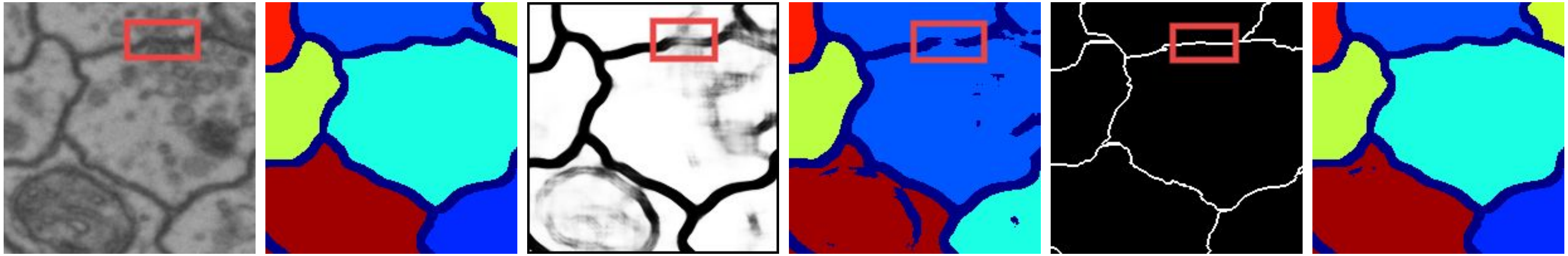
Importance of correct topology for image segmentation

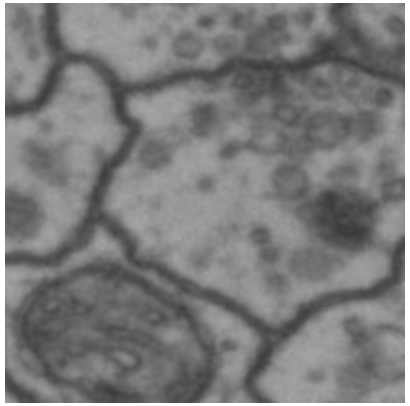
- Existing methods optimize w.r.t. per-pixel accuracy
- Topological errors:
  - broken connection, missing components
- Structural errors damage downstream analysis



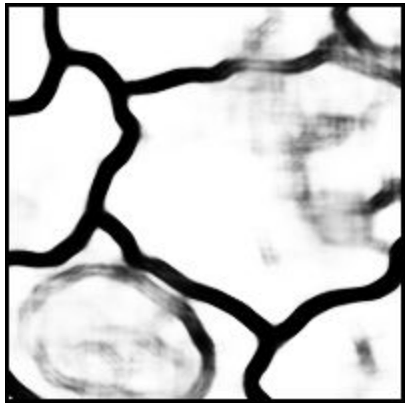
Input Image | Ground Truth | Likelihood Map | With Topo Errors | Structures | Our Method

Why Discrete Morse Theory

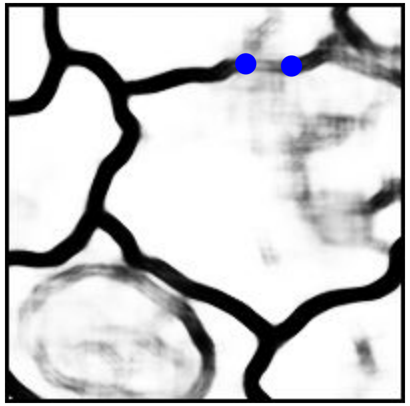
- Fix topological errors with persistent homology:
  - [Hu et al. NeurIPS’19] – Topological loss by matching persistence diagram



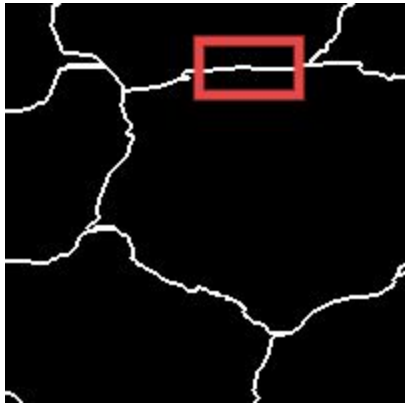
Input Image



Likelihood Map



Critical Points



DMT Structures

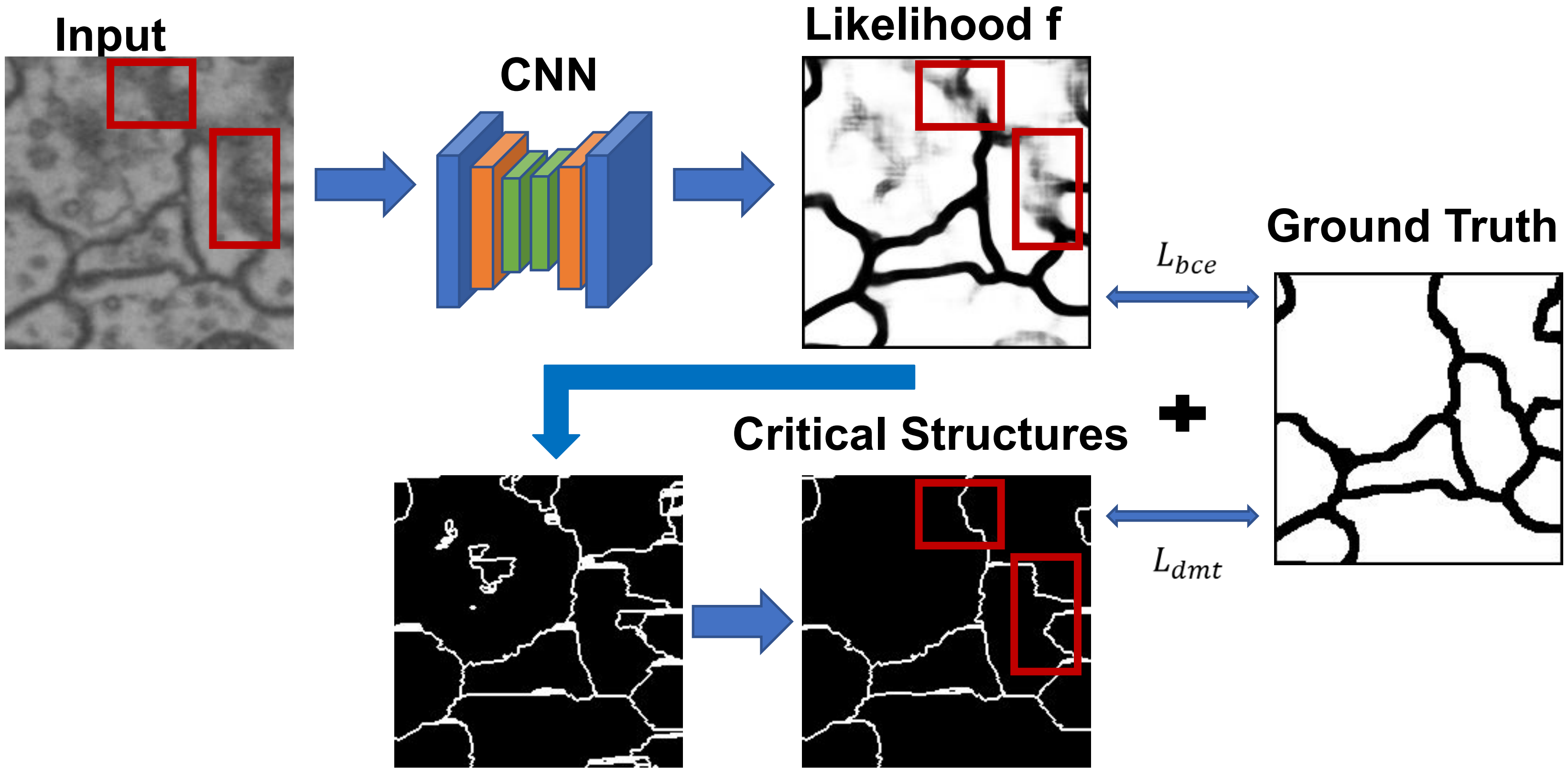
Not efficient enough!

Summary of Contributions

- Our contributions:
  - **DMT loss:** capturing the critical structures of the training data
  - DMT-based **loss function** for end-to-end training of neural networks
  - Efficiency: converging faster than topological loss

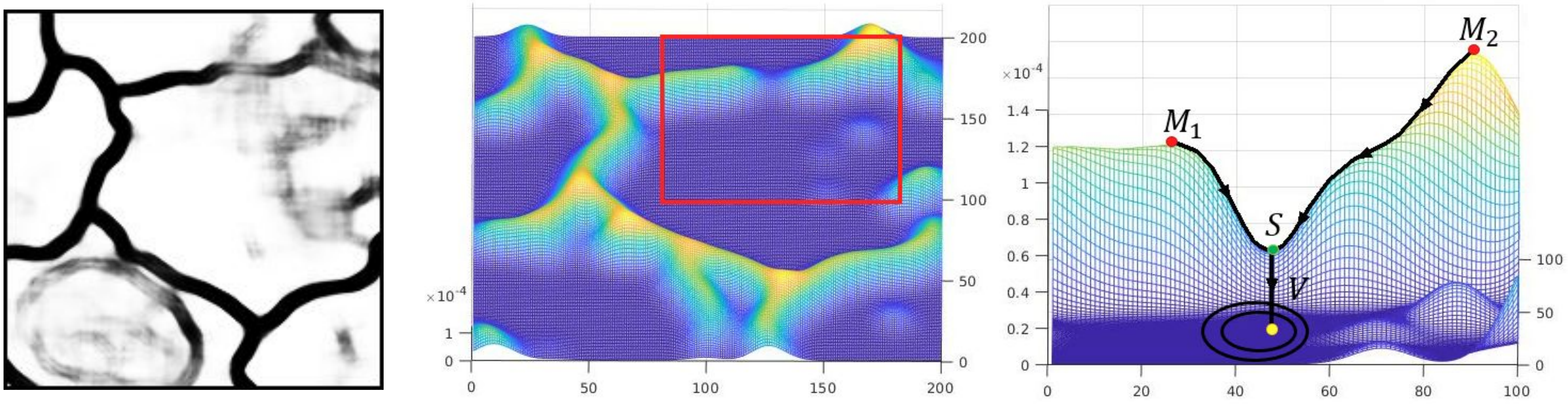
DMT Loss

- loss function – train the model to be topology-preserving
  - Identity the critical structures instead of critical points



Overview of the proposed method

Morse theory

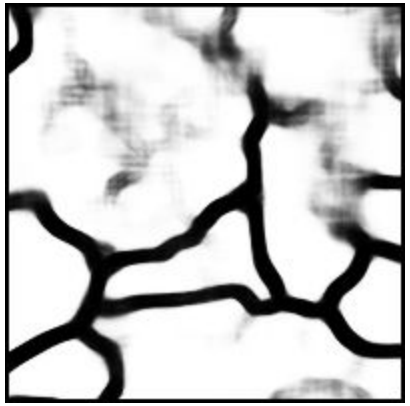


Likelihood Map **Density Map** **Density Map for highlighted region**

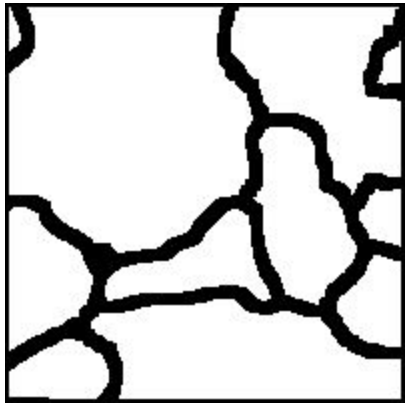
Gradient: $\nabla f(x) = [\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_d}]^T$

Critical Points (minimum, maximum, saddle): $\nabla f(x) = 0$

Persistence-based structure pruning



Likelihood Map



Ground Truth

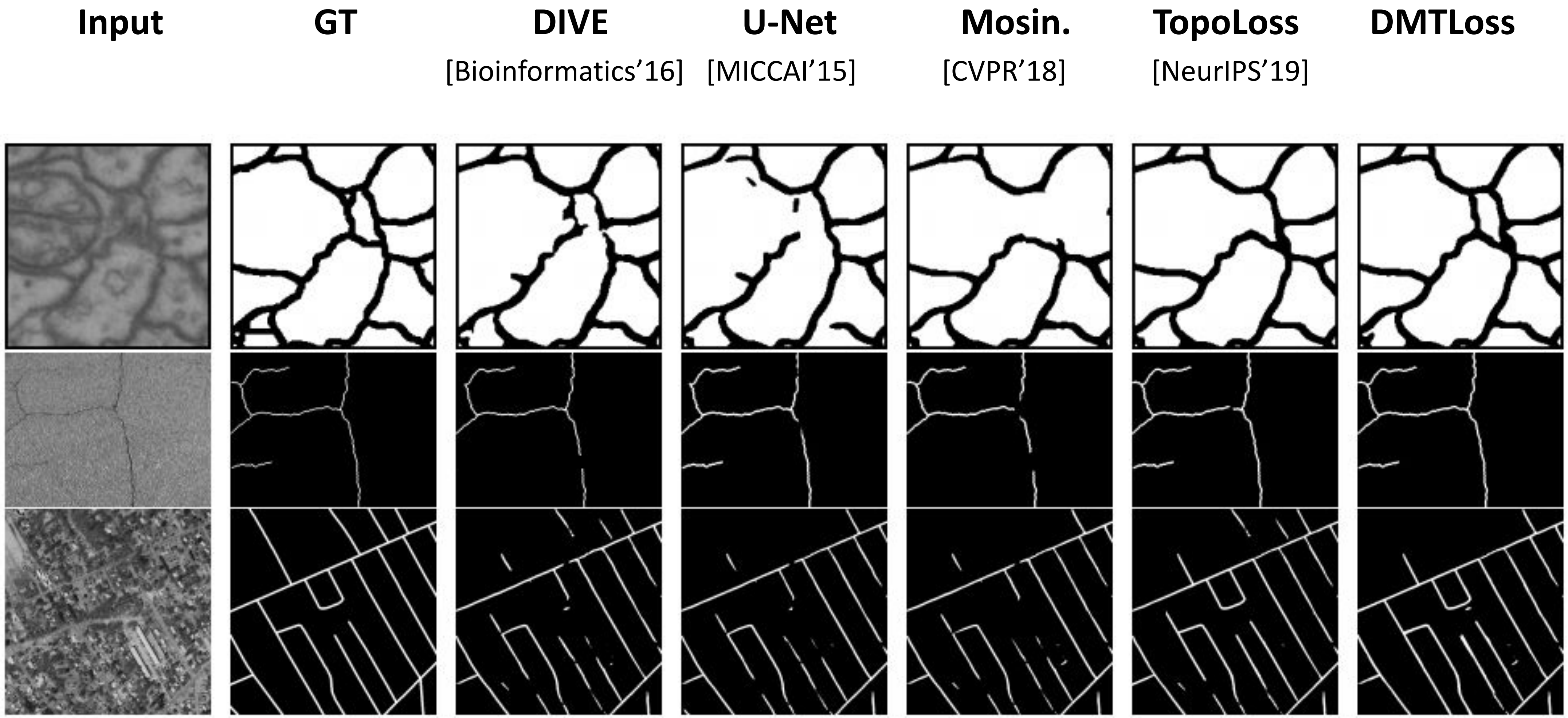
Improperly pruned structures

Properly pruned structures

Approximation

Approximate $S_2(\epsilon)$ by $\widehat{S_2}(\epsilon)$ using spanning tree:

Qualitative Results



Quantitative Results for 2D datatest

- Per-pixel error, DICE score, Betti number error, Adjusted Rand Index, Variation of Information

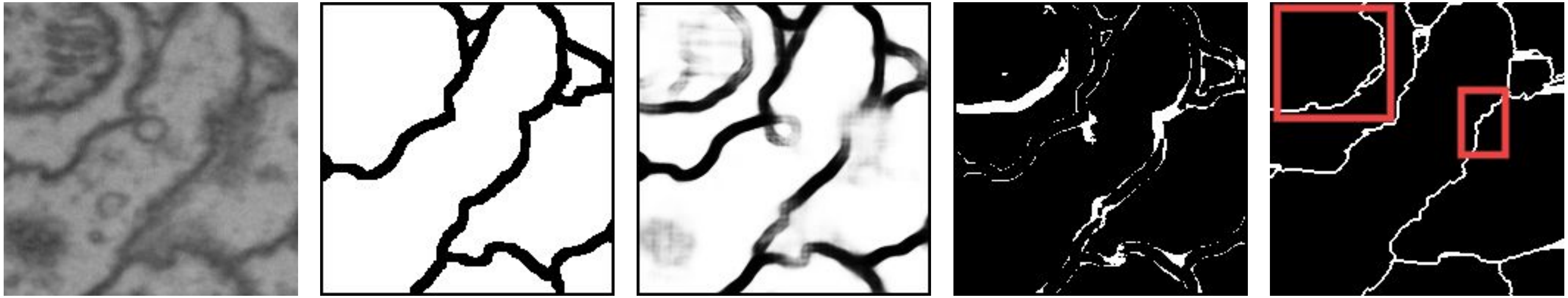
Method	Accuracy	DICE	ARI	VOI	Betti Error
			ISBI13		
DIVE	0.9642 ± 0.0018	0.9658 ± 0.0020	0.6923 ± 0.0134	2.790 ± 0.025	3.875 ± 0.326
U-Net	0.9631 ± 0.0024	0.9649 ± 0.0057	0.7031 ± 0.0256	2.583 ± 0.078	3.463 ± 0.435
Mosin.	0.9578 ± 0.0029	0.9623 ± 0.0047	0.7483 ± 0.0367	1.534 ± 0.063	2.952 ± 0.379
TopoLoss	0.9569 ± 0.0031	$\mathbf{0.9689} \pm \mathbf{0.0026}$	0.8064 ± 0.0112	1.436 ± 0.008	$\mathbf{1.253} \pm \mathbf{0.172}$
DMT	0.9625 ± 0.0027	$\mathbf{0.9712} \pm \mathbf{0.0047}$	$\mathbf{0.8289} \pm \mathbf{0.0189}$	$\mathbf{1.176} \pm \mathbf{0.052}$	$\mathbf{1.102} \pm \mathbf{0.203}$
			CREMI		
DIVE	0.9498 ± 0.0029	0.9542 ± 0.0037	0.6532 ± 0.0247	2.513 ± 0.047	4.378 ± 0.152
U-Net	0.9468 ± 0.0048	0.9523 ± 0.0049	0.6723 ± 0.0312	2.346 ± 0.105	3.016 ± 0.253
Mosin.	0.9467 ± 0.0058	0.9489 ± 0.0053	0.7853 ± 0.0281	1.623 ± 0.083	1.973 ± 0.310
TopoLoss	0.9456 ± 0.0053	0.9596 ± 0.0029	0.8083 ± 0.0104	1.462 ± 0.028	$\mathbf{1.113} \pm \mathbf{0.224}$
DMT	0.9475 ± 0.0031	$\mathbf{0.9653} \pm \mathbf{0.0019}$	$\mathbf{0.8203} \pm \mathbf{0.0147}$	$\mathbf{1.089} \pm \mathbf{0.061}$	$\mathbf{0.982} \pm \mathbf{0.179}$
			CrackTree		
DIVE	0.9854 ± 0.0052	0.6530 ± 0.0017	0.8634 ± 0.0376	1.570 ± 0.078	1.576 ± 0.287
U-Net	0.9821 ± 0.0097	0.6491 ± 0.0029	0.8749 ± 0.0421	1.625 ± 0.104	1.785 ± 0.303
Mosin.	0.9833 ± 0.0067	0.6527 ± 0.0010	0.8897 ± 0.0201	1.113 ± 0.057	1.045 ± 0.214
TopoLoss	0.9826 ± 0.0084	0.6732 ± 0.0041	$\mathbf{0.9291} \pm \mathbf{0.0123}$	0.997 ± 0.011	$\mathbf{0.672} \pm \mathbf{0.176}$
DMT	0.9842 ± 0.0041	$\mathbf{0.6811} \pm \mathbf{0.0047}$	$\mathbf{0.9307} \pm \mathbf{0.0172}$	$\mathbf{0.901} \pm \mathbf{0.081}$	$\mathbf{0.518} \pm \mathbf{0.189}$
			Road		
DIVE	0.9734 ± 0.0077	0.6743 ± 0.0051	0.8201 ± 0.0128	2.368 ± 0.203	3.598 ± 0.783
U-Net	0.9786 ± 0.0052	0.6612 ± 0.0016	0.8189 ± 0.0097	2.249 ± 0.175	3.439 ± 0.621
Mosin.	0.9754 ± 0.0043	0.6673 ± 0.0044	0.8456 ± 0.0174	1.457 ± 0.096	2.781 ± 0.237
TopoLoss	0.9728 ± 0.0063	0.6903 ± 0.0038	0.8671 ± 0.0068	1.234 ± 0.037	$\mathbf{1.275} \pm \mathbf{0.192}$
DMT	0.9744 ± 0.0049	$\mathbf{0.7056} \pm \mathbf{0.0022}$	$\mathbf{0.8819} \pm \mathbf{0.0104}$	$\mathbf{1.092} \pm \mathbf{0.129}$	$\mathbf{0.995} \pm \mathbf{0.301}$

Quantitative Results for 3D datatest

- Per-pixel error, DICE score, Betti number error, Adjusted Rand Index, Variation of Information

Method	Accuracy	DICE	ARI	VOI	Betti Error
ISBI13					
3D DIVE	0.9723 ± 0.0021	0.9681 ± 0.0043	0.8719 ± 0.0189	1.208 ± 0.149	2.375 ± 0.419
3D U-Net	0.9746 ± 0.0025	0.9701 ± 0.0012	$\mathbf{0.8956} \pm \mathbf{0.0391}$	1.123 ± 0.091	1.954 ± 0.585
MALA	0.9701 ± 0.0018	0.9699 ± 0.0013	$\mathbf{0.8945} \pm \mathbf{0.0481}$	0.901 ± 0.106	1.103 ± 0.207
3D TopoLoss	0.9689 ± 0.0031	0.9752 ± 0.0045	$\mathbf{0.9043} \pm \mathbf{0.0283}$	0.792 ± 0.086	0.972 ± 0.245
DMT	0.9701 ± 0.0026	$\mathbf{0.9803} \pm \mathbf{0.0019}$	$\mathbf{0.9149} \pm \mathbf{0.0217}$	$\mathbf{0.634} \pm \mathbf{0.086}$	$\mathbf{0.812} \pm \mathbf{0.134}$
CREMI					
3D DIVE	0.9503 ± 0.0061	0.9641 ± 0.0011	0.8514 ± 0.0387	1.219 ± 0.103	2.674 ± 0.473
3D U-Net	0.9547 ± 0.0038	0.9618 ± 0.0026	0.8322 ± 0.0315	1.416 ± 0.097	2.313 ± 0.501
MALA	0.9472 ± 0.0027	0.9583 ± 0.0023	0.8713 ± 0.0286	1.109 ± 0.093	1.114 ± 0.309
3D TopoLoss	0.9523 ± 0.0043	0.9672 ± 0.0010	0.8726 ± 0.0194	1.044 ± 0.128	1.076 ± 0.206
DMT	0.9529 ± 0.0031	$\mathbf{0.9731} \pm \mathbf{0.0045}$	$\mathbf{0.9013} \pm \mathbf{0.0202}$	$\mathbf{0.891} \pm \mathbf{0.099}$	$\mathbf{0.726} \pm \mathbf{0.187}$
3Dircadb					
3D DIVE	0.9618 ± 0.0054	0.6097 ± 0.0034	/	/	4.571 ± 0.505
3D U-Net	0.9632 ± 0.0009	0.5898 ± 0.0025	/	/	4.131 ± 0.483
MALA	0.9546 ± 0.0033	0.5719 ± 0.0043	/	/	2.982 ± 0.105
3D TopoLoss	0.9561 ± 0.0019	0.6138 ± 0.0029	/	/	2.245 ± 0.255
DMT	0.9587 ± 0.0023	$\mathbf{0.6257} \pm \mathbf{0.0021}$	/	/	$\mathbf{1.415} \pm \mathbf{0.305}$

Comparison with reweighted cross entropy loss



Input Image | Ground Truth | Likelihood map | Pixels identified by reweighted CE | Highlighted Structures by DMT

Method	Accuracy	Betti Error
DMT	0.9475	0.982
Reweighted CE	0.9481	2.753

Conclusions

- DMT loss identifies critical structures that are relevant to image topology and fixes them once at a time.
- Could be incorporated into any segmentation backbones to train the model to be topology-preserving.
- Works for both 2D and 3D images with rich structures.

Thank you for your attention!

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