

Gauge Equivariant Mesh CNN

Pim de Haan^{*12}, Maurice Weiler^{*2}, Taco Cohen¹, Max Welling¹²

¹ Qualcomm AI Research

² University of Amsterdam QUVA Lab

³ University of Amsterdam

Pim de Haan
Research Associate

Qualcomm Technologies Netherlands B.V.

Collaboration



Pim de Haan*
Qualcomm AI Research
University of Amsterdam



Maurice Weiler*
QUVA lab
University of Amsterdam



Taco Cohen
Qualcomm AI Research

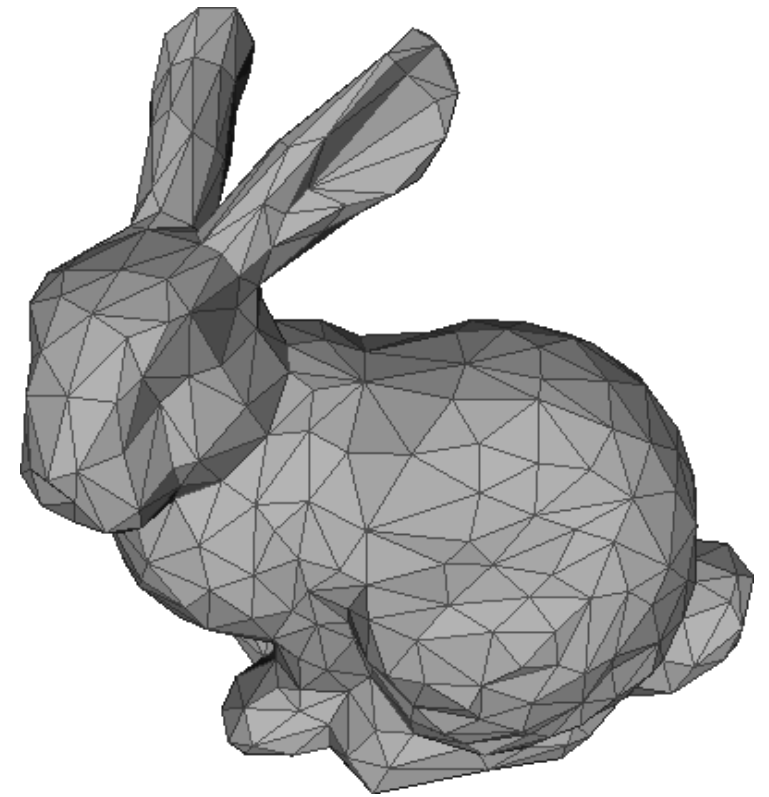


Max Welling
Qualcomm AI Research
University of Amsterdam

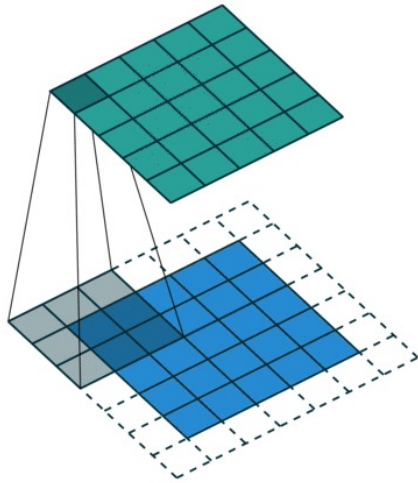
*Equal Contribution

Objectives

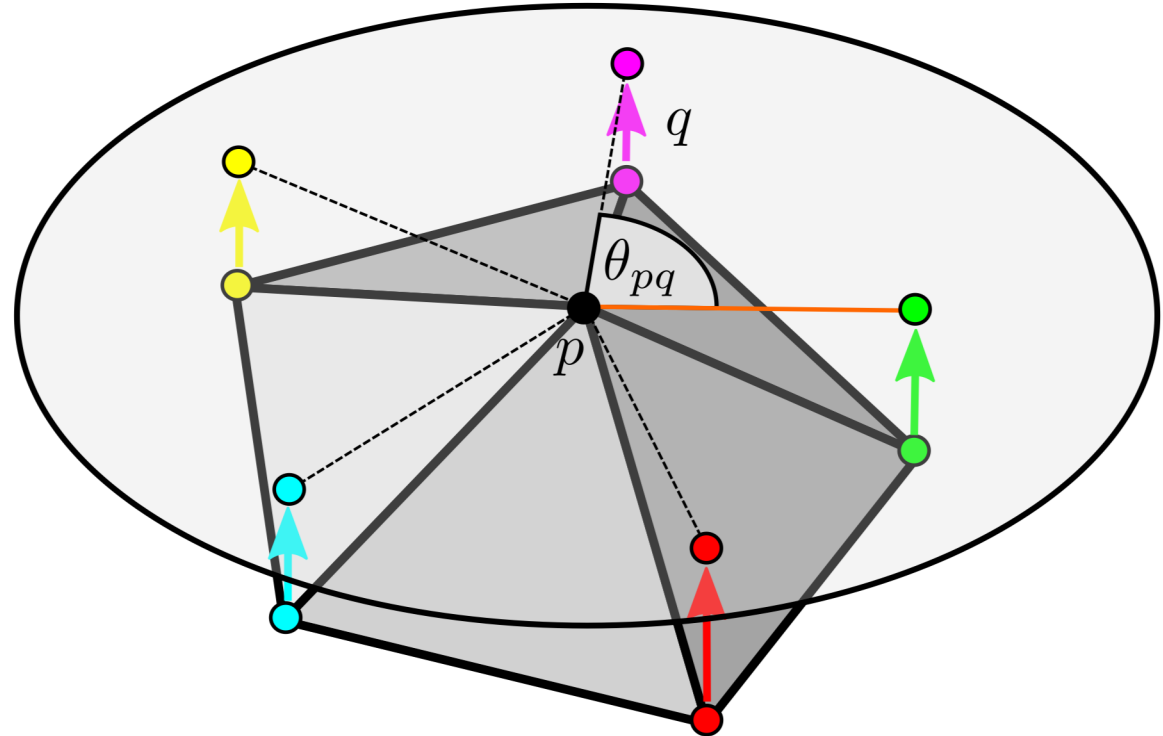
- Mesh: discretization of curved surface
- Simple scalable CNN
- Expressive:
 - anisotropic filters
 - vector features
- No arbitrary choices



Convolutions on images and meshes



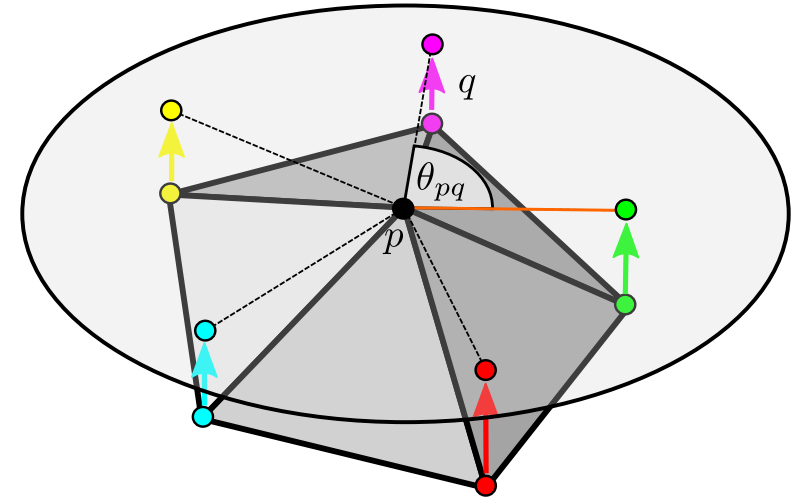
- Canonical relative (x, y) coordinates of neighbours



- Log map to tangent plane
- Polar coordinates
- What is $\theta = 0$?
- Choice of coordinates: gauge

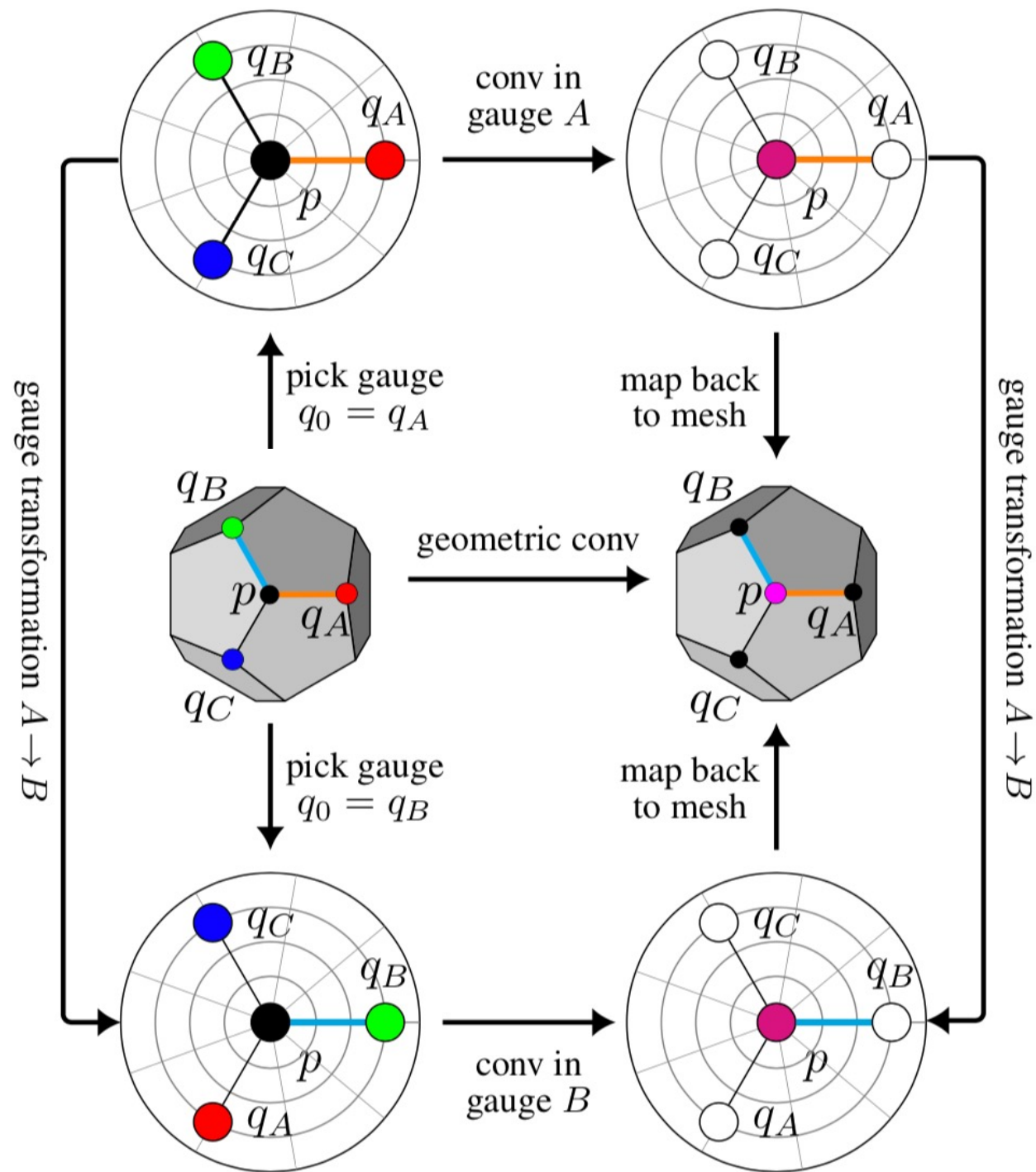
Gauge equivariance

- Gauge: choice of basis for each tangent plane
 - Reference neighbour
- Principal curvature direction?
- Gauge equivariance [Cohen et al. 2019]:
 - The same feature in different gauges has same output (up to rotation)



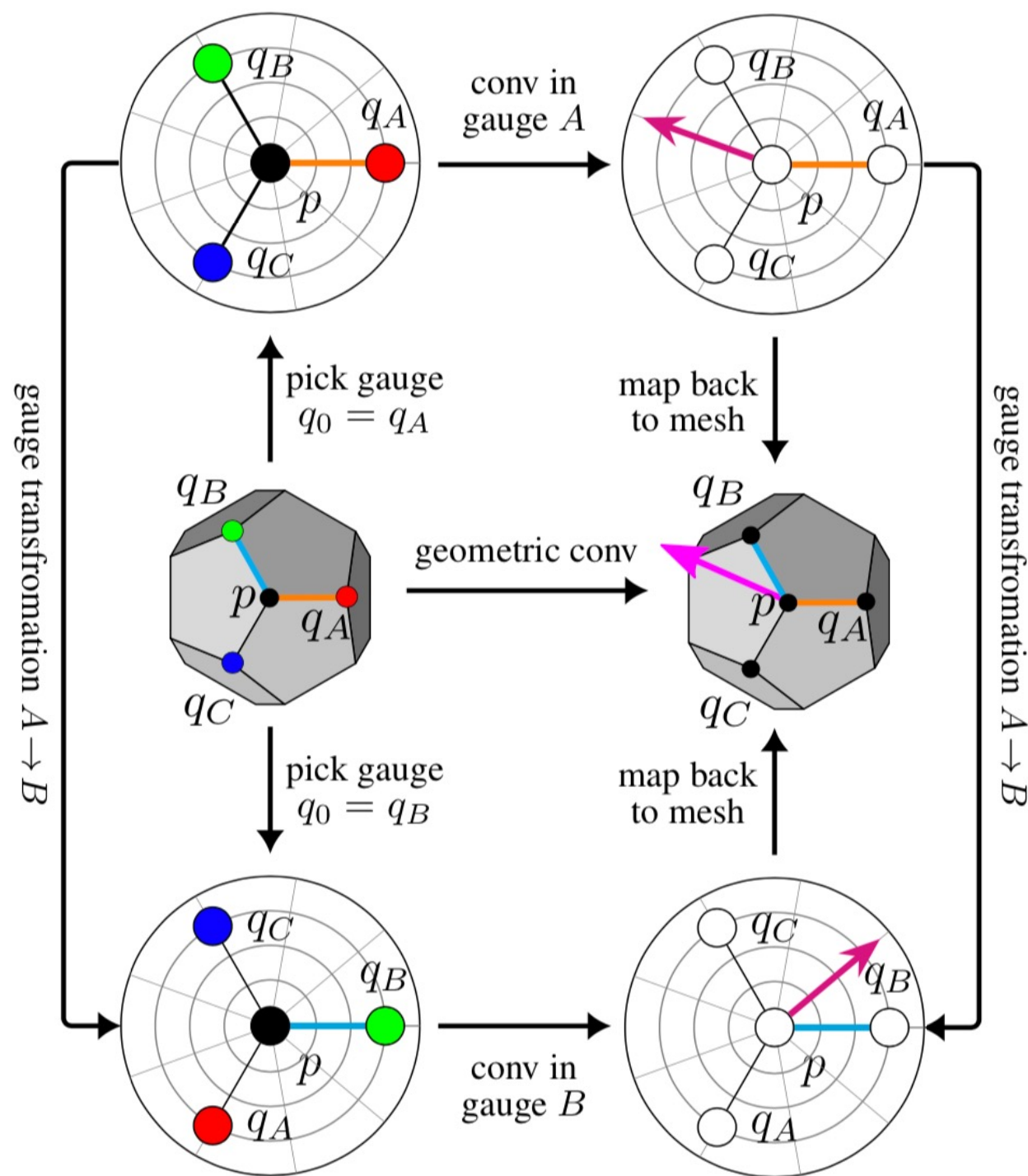
Attempt 1: Gauge equivariant convolutions on scalar fields

Scalar convolutions are isotropic



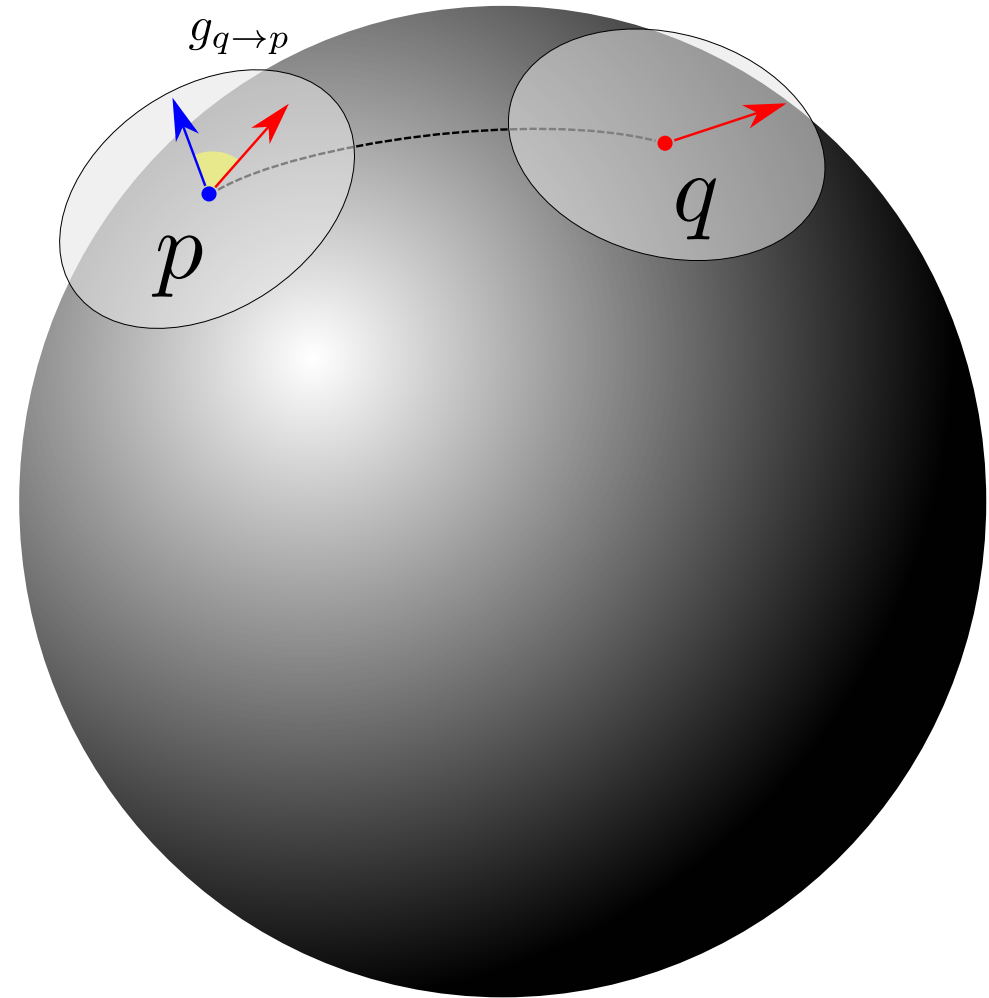
Attempt 2: Gauge equivariant convolutions on vector fields

Vector convolutions are anisotropic



Parallel Transport

- Tangent planes not parallel
- Parallel transport of geodesic
- Transport gauge-defining X-axis
- Angle $g_{q \rightarrow p}$
- Cheaply precomputed
- Any parallel transport by linearity

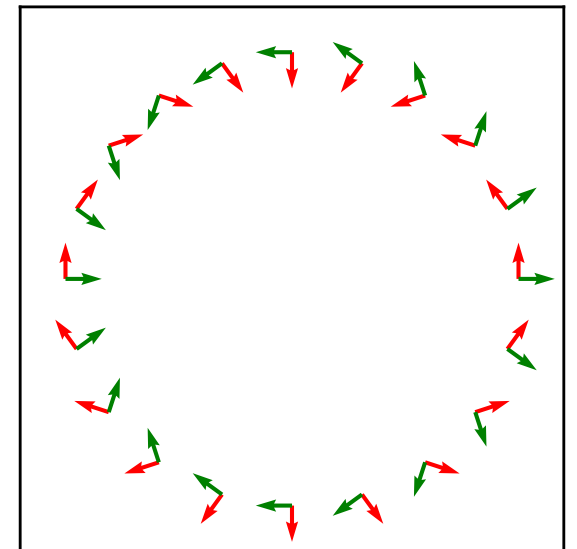
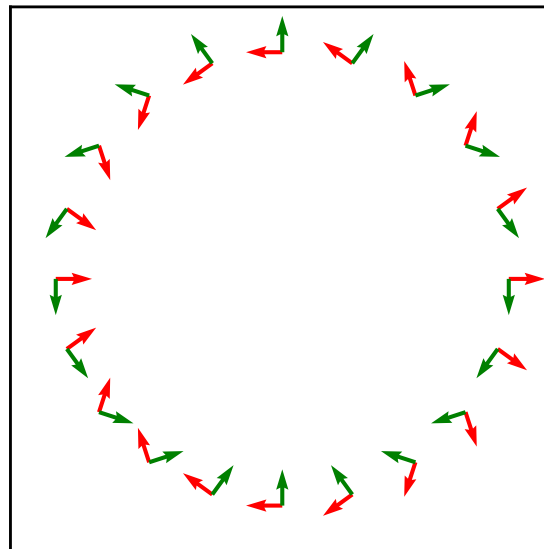
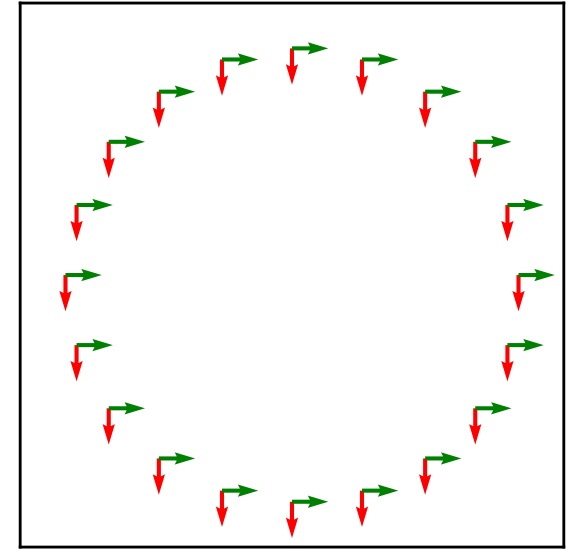
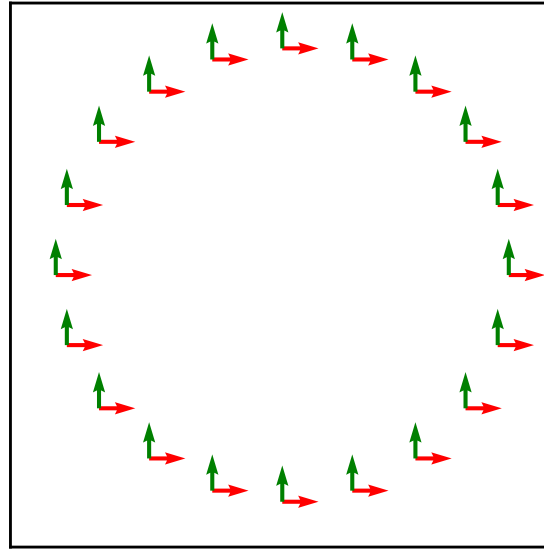


General Gauge Equivariant Convolution

- Two gauges are related by planar rotation $g \in SO(2)$
- Vertex feature: group representation $\rho(g) \in \mathbb{R}^{d \times d}$
 - E.g. scalar feature $\rho(g) = 1$
 - E.g. tangent vector feature $\rho(g) = \begin{pmatrix} \cos(g) & -\sin(g) \\ \sin(g) & \cos(g) \end{pmatrix}$
- Kernel $K(r, \theta) \in \mathbb{R}^{d' \times d}$
- Convolution: $(K \star f)_p = \sum_{q \in \mathcal{N}(p)} K(r_q, \theta_q) \rho(g_{q \rightarrow p}) f_q$
- Equivariance if: $\rho'(g) K(r, \theta) = K(r, \theta + g) \rho(g)$

Solutions to kernel constraint

- $\rho'(g)K(r, \theta) = K(r, \theta + g)\rho(g)$
- $K(r, \theta) = K(r)K(\theta)$
- $K(r)$ unconstrained
- Example: between tangent vectors
- Angular component $K(\theta) \in \mathbb{R}^{2 \times 2}$
- Four solutions
- Precomputed
- Linearly combined with learnable parameters

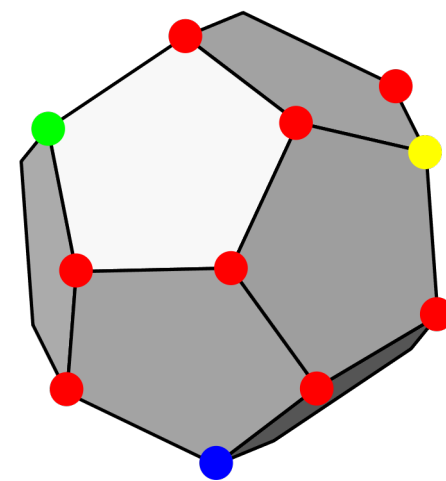
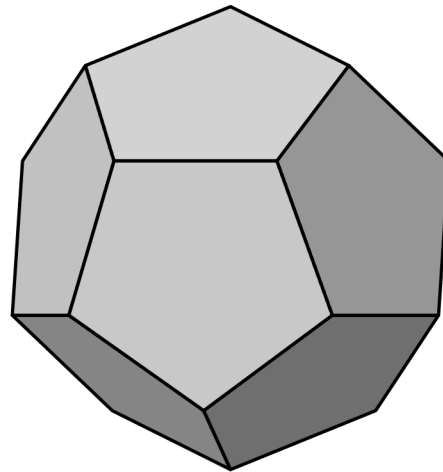
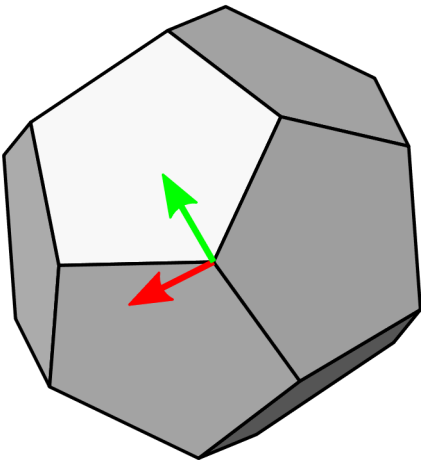
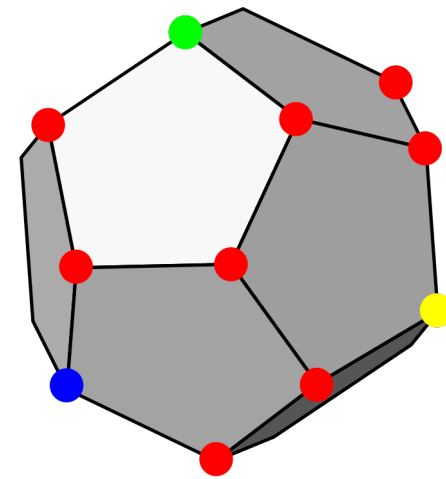
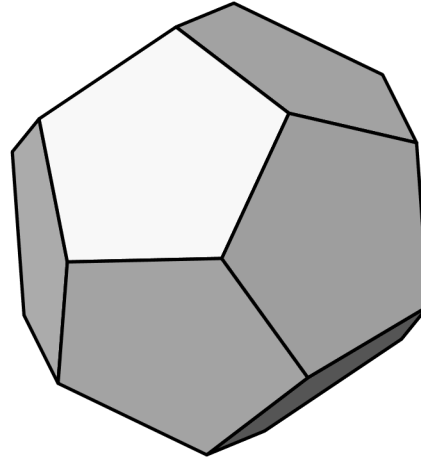
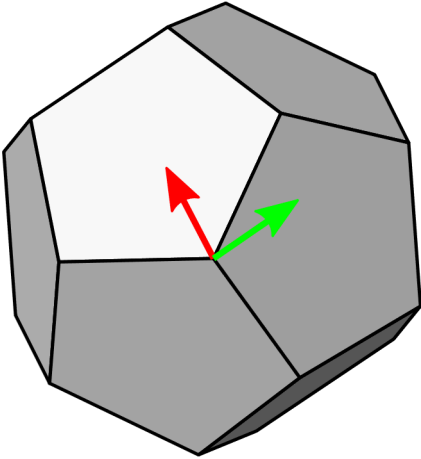


Implementation

- For each layer, pick an input and output representation
- Precomputation:
 - For each vertex, we define tangent plane
 - On each tangent plane, we pick gauge
 - For each pair of neighbours compute:
 - Log maps (r_q, θ_q)
 - Parallel transport matrices $\rho(g_{p \rightarrow q})$
 - Basis kernels $K_i(r_q, \theta_q)$
- During forward pass, combine basis kernels:

$$(K \star f)_p = \sum_i \sum_{q \in \mathcal{N}(p)} \alpha_i K_i(r_q, \theta_q) \rho(g_{q \rightarrow p}) f_q$$

Symmetry properties

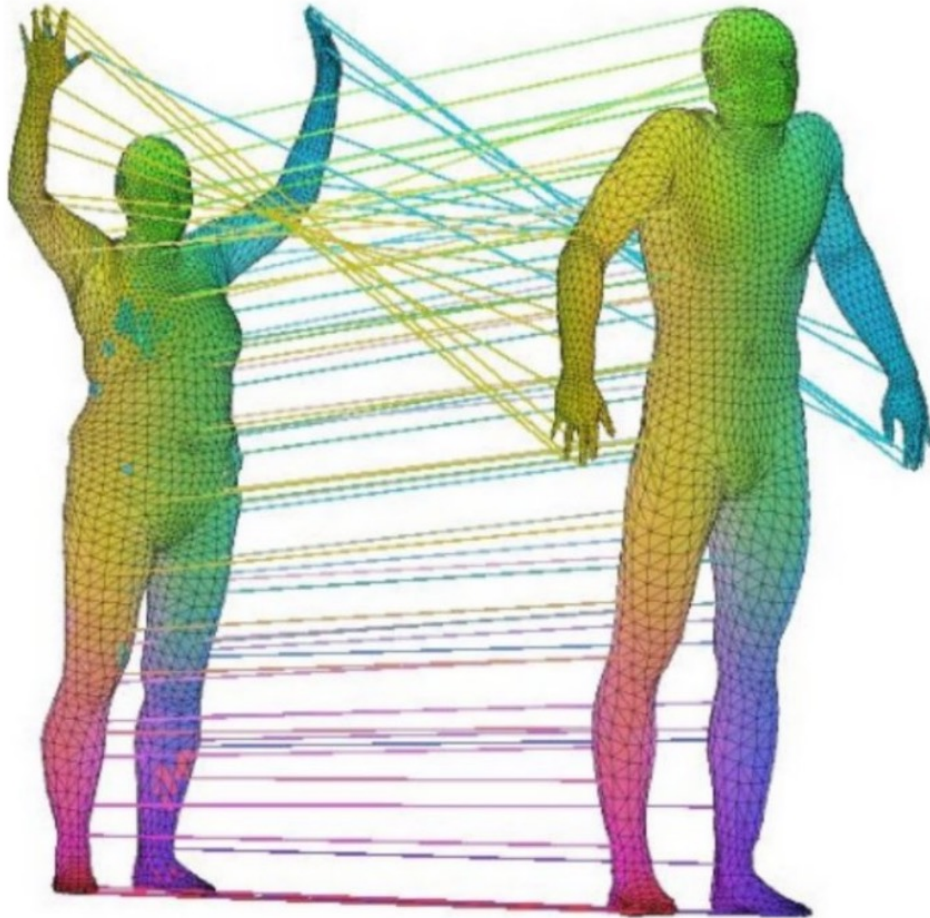


Gauge

Vertex coordinates

Mesh isometries

Experiment: Shape Correspondence

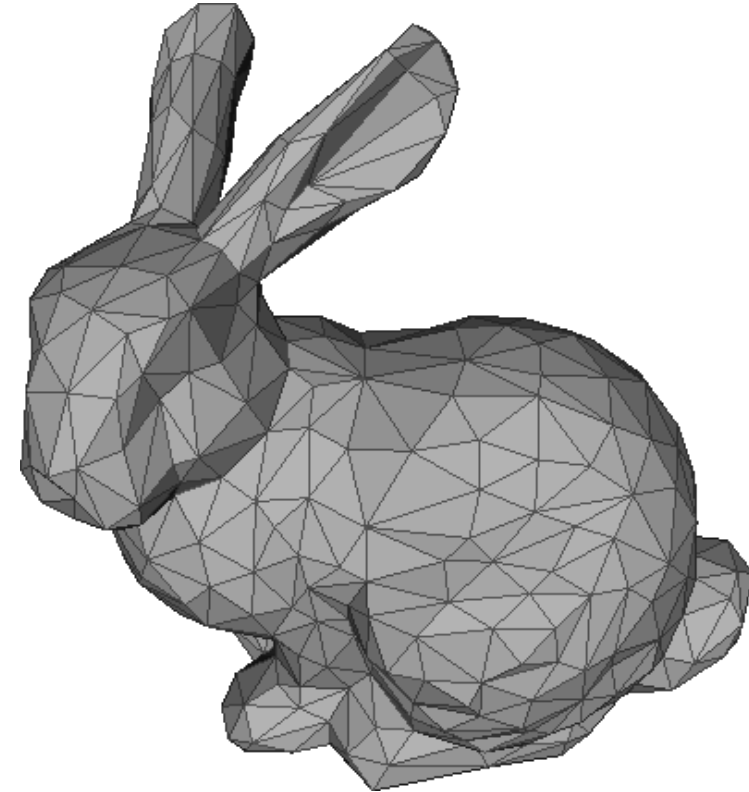


Model	Features	Accuracy
ACNN (Boscaini et al., 2016)	SHOT	62.4 %
Geodesic CNN (Masci et al., 2015)	SHOT	65.4 %
MoNet (Monti et al., 2016)	SHOT	73.8 %
FeaStNet (Verma et al., 2018)	XYZ	98.7%
ZerNet (Sun et al., 2018)	XYZ	96.9%
SpiralNet++ (Gong et al., 2019)	XYZ	99.8%
Graph CNN	XYZ	1.40 ± 0.5 %
Graph CNN	SHOT	23.80 ± 8 %
GEM-CNN	XYZ	99.73 ± 0.04 %
GEM-CNN (broken symmetry)	XYZ	99.89 ± 0.02 %

Takeaway





Gauge Equivariant Mesh CNN is:

- Simple
- Scalable
- Anisotropic \Rightarrow expressive
- Symmetry properties





Thank you

Follow us on:    

For more information, visit us at:

www.qualcomm.com & www.qualcomm.com/blog

Nothing in these materials is an offer to sell any of the components or devices referenced herein.

©2018-2021 Qualcomm Technologies, Inc. and/or its affiliated companies. All Rights Reserved.

Qualcomm is a trademark or registered trademark of Qualcomm Incorporated. Other products and brand names may be trademarks or registered trademarks of their respective owners.

References in this presentation to “Qualcomm” may mean Qualcomm Incorporated, Qualcomm Technologies, Inc., and/or other subsidiaries or business units within the Qualcomm corporate structure, as applicable. Qualcomm Incorporated includes our licensing business, QTL, and the vast majority of our patent portfolio. Qualcomm Technologies, Inc., a subsidiary of Qualcomm Incorporated, operates, along with its subsidiaries, substantially all of our engineering, research and development functions, and substantially all of our products and services businesses, including our QCT semiconductor business.