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Qualcomm

Gauge Equivariant Mesh CNN

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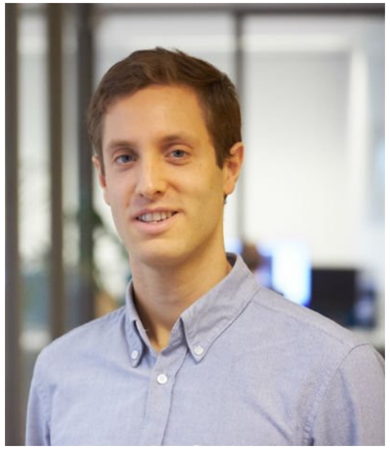

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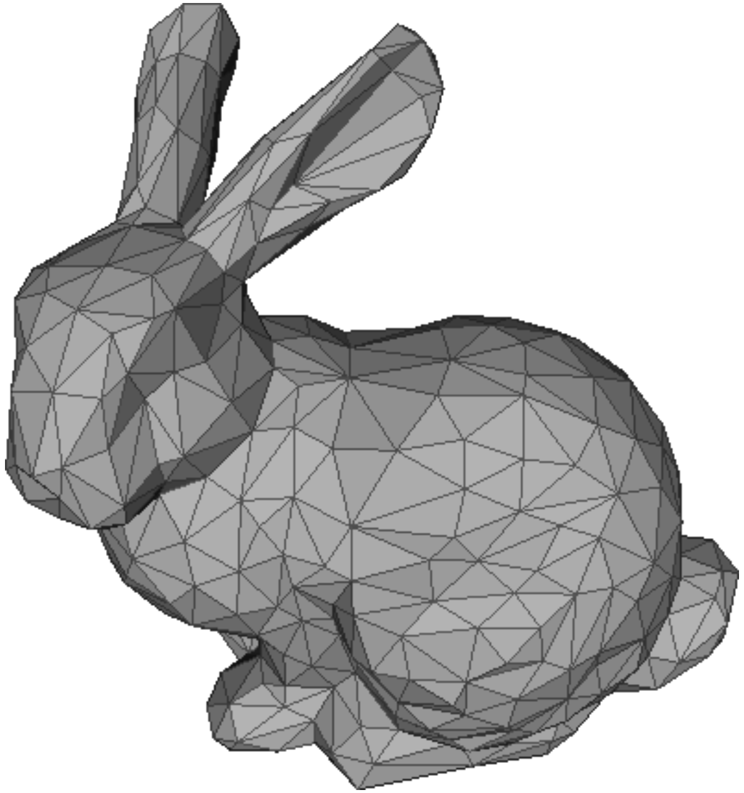
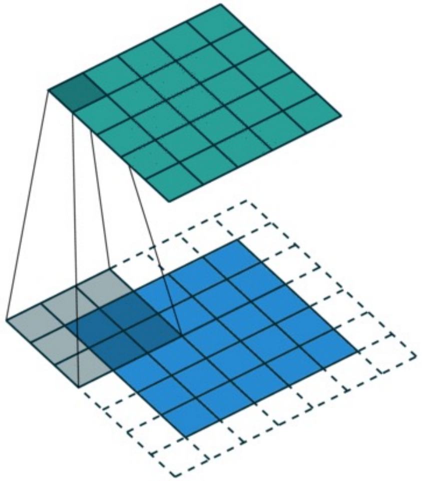
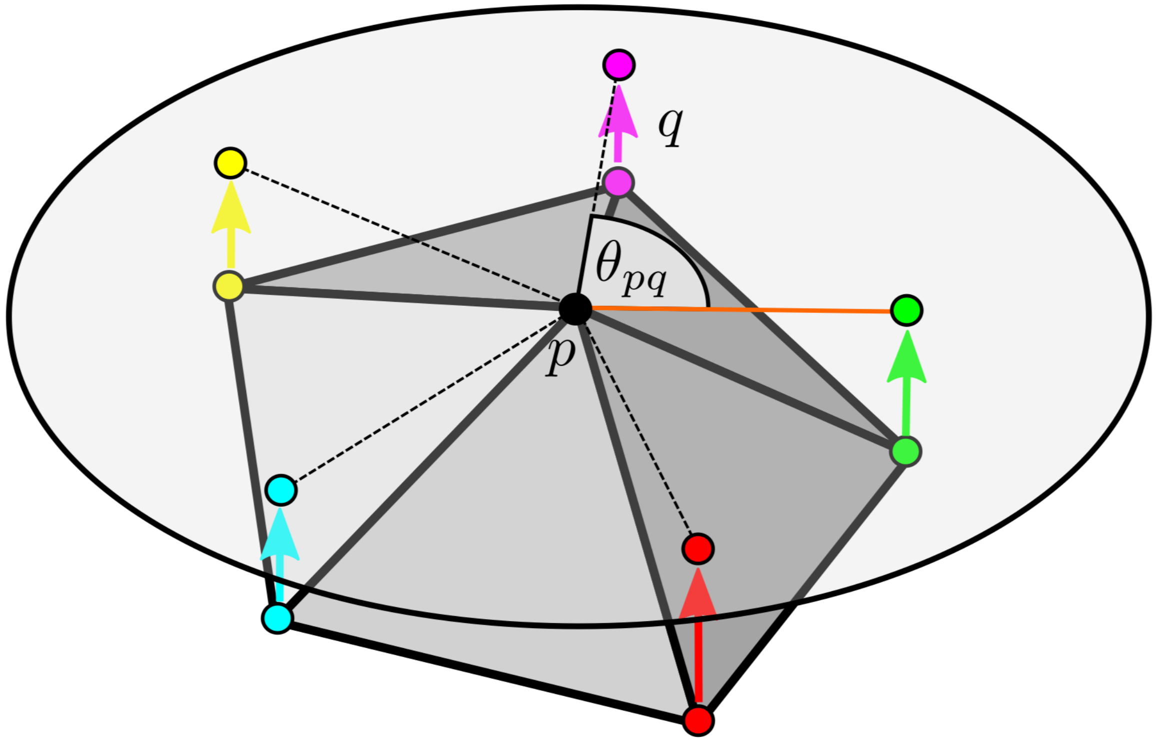
Objectives

- Mesh: discretization of curved surface
- Simple scalable CNN
- Expressive:
  - anisotropic filters
  - vector features
- No arbitrary choices



Convolutions on images and meshes





- Canonical relative (x, y) coordinates of neighbours

- Log map to tangent plane
- Polar coordinates
- What is $\theta = 0$?
- Choice of coordinates: gauge

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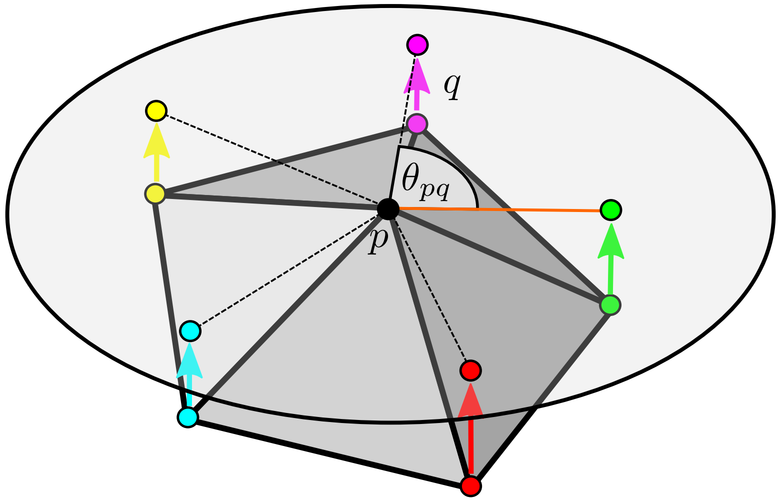
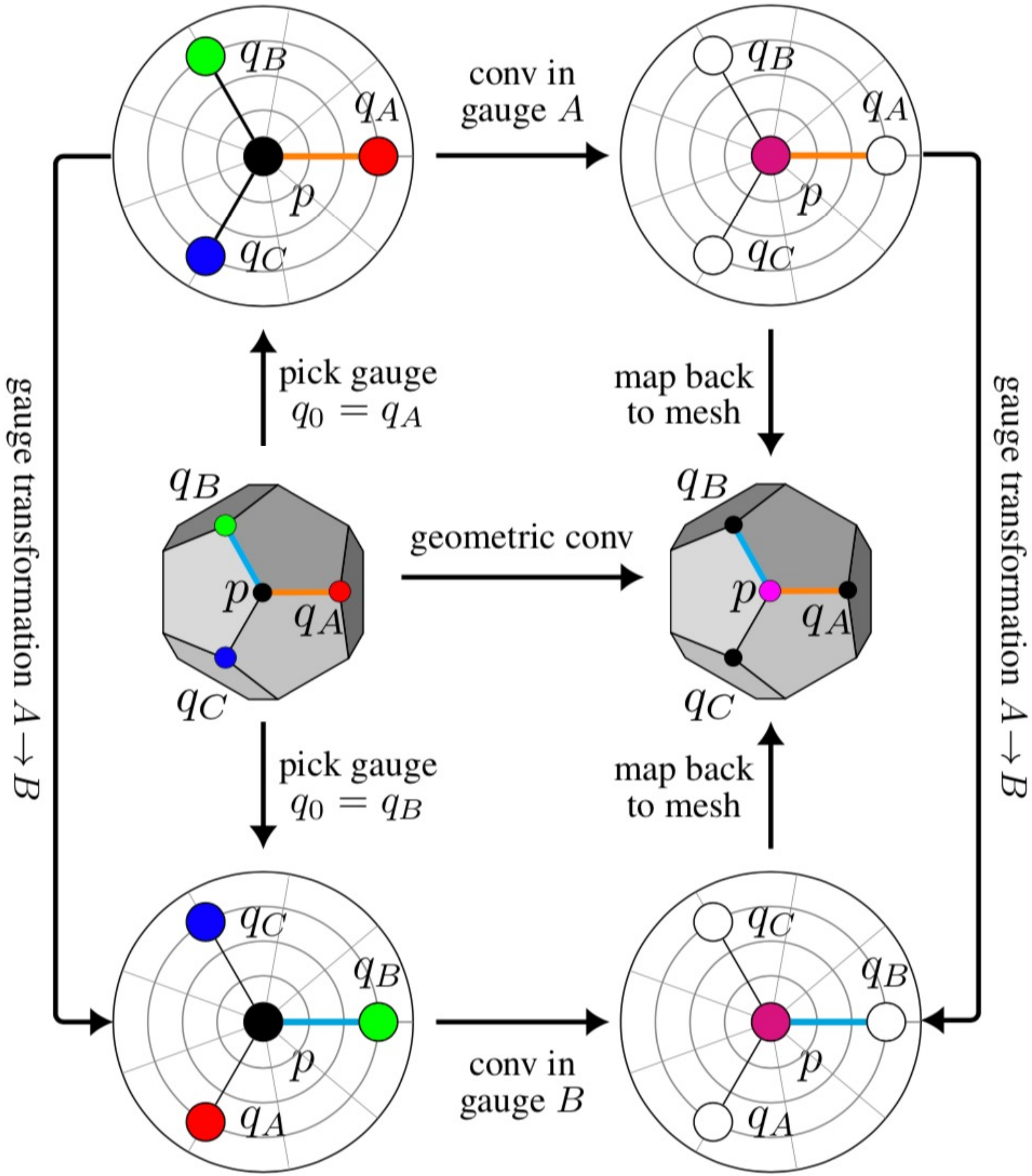
Gauge equivariance



- Gauge: choice of basis for each tangent plane
  - Reference neighbour
- Principal curvature direction?
- Gauge equivariance [Cohen et al. 2019]:
  - The same feature in different gauges has same output (up to rotation)

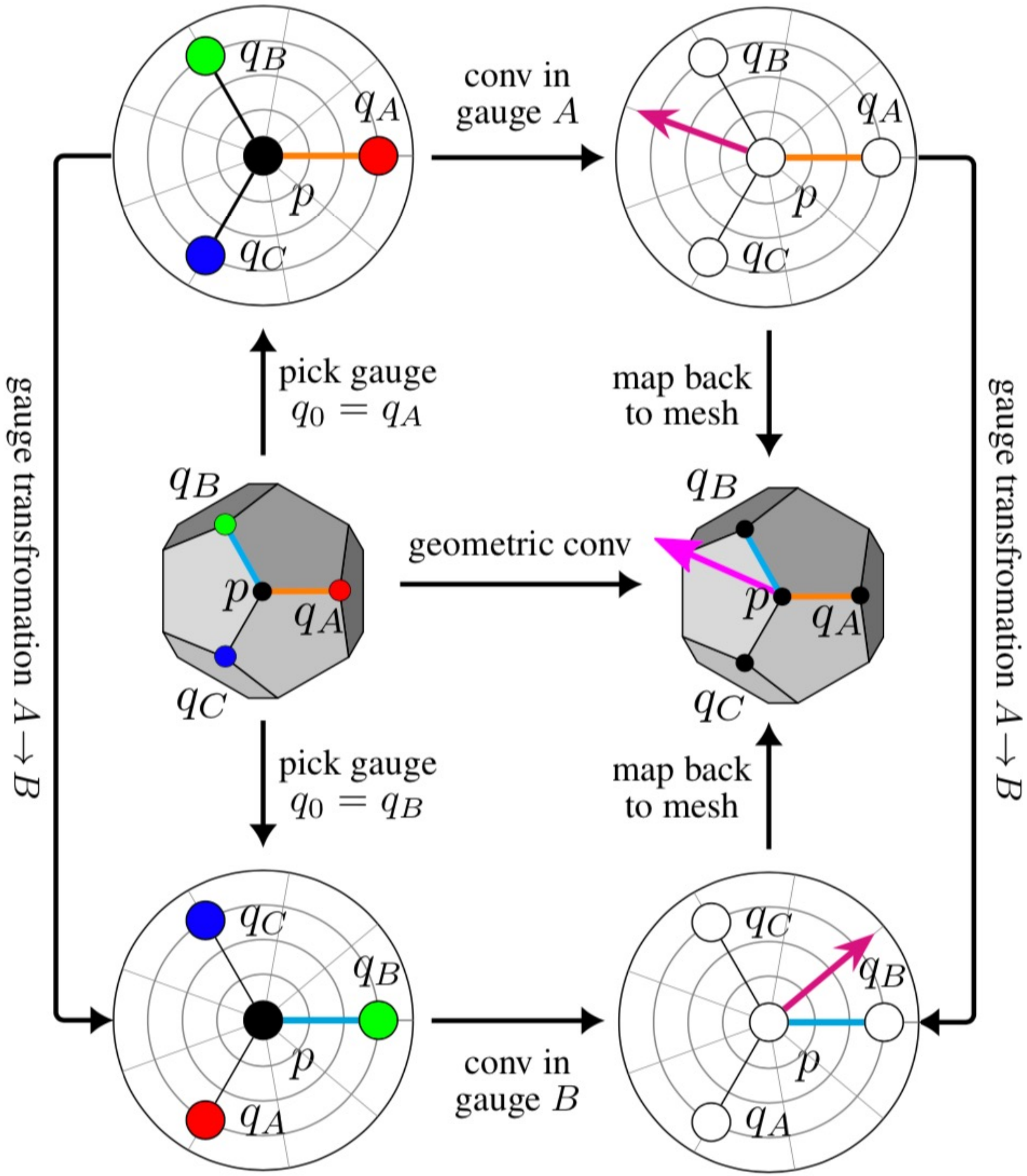
Attempt 1: Gauge equivariant convolutions on scalar fields

Scalar convolutions are isotropic



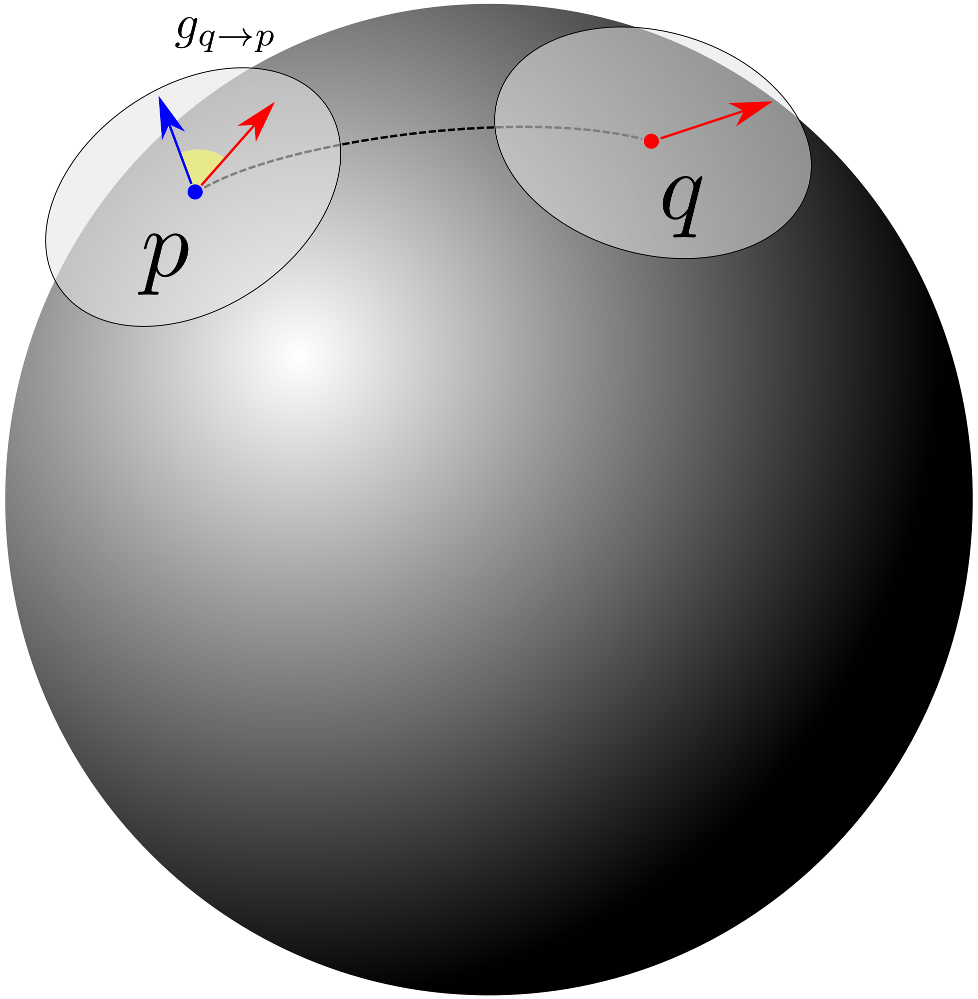
Attempt 2: Gauge equivariant convolutions on vector fields

Vector convolutions are anisotropic



Parallel Transport

- Tangent planes not parallel
- Parallel transport of geodesic
- Transport gauge-defining X-axis
- Angle $g_{q \to p}$
- Cheaply precomputed
- Any parallel transport by linearity

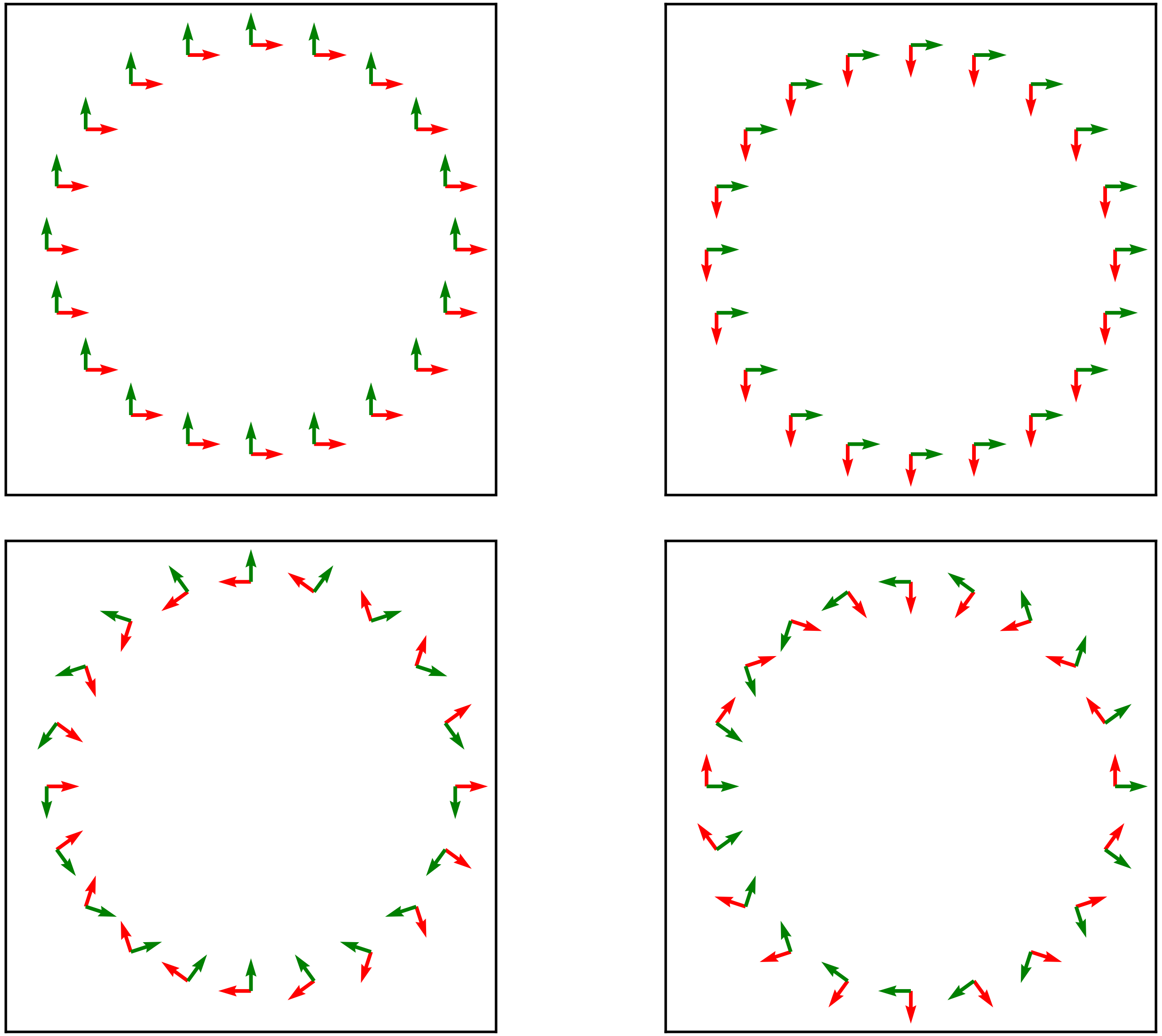


General Gauge Equivariant Convolution

- Two gauges are related by planar rotation $g \in SO(2)$
- Vertex feature: group representation $\rho(g) \in \mathbb{R}^{d \times d}$
  - E.g. scalar feature $\rho(g) = 1$
  - E.g. tangent vector feature $\rho(g) = \begin{pmatrix} \cos(g) & -\sin(g) \\ \sin(g) & \cos(g) \end{pmatrix}$
- Kernel $K(r, \theta) \in \mathbb{R}^{d' \times d}$
- Convolution: $(K \star f)_p = \sum_{q \in \mathcal{N}(p)} K\left(r_q, \theta_q\right) \rho(g_{q \to p}) f_q$
- Equivariance if: $\rho'(g) K(r, \theta) = K(r, \theta + g) \rho(g)$

Solutions to kernel constraint

- $\rho'(g)K(r,\theta) = K(r,\theta + g)\rho(g)$
- $K(r,\theta) = K(r)K(\theta)$
- $K(r)$ unconstrained
- Example: between tangent vectors
- Angular component $K(\theta) \in \mathbb{R}^{2\times 2}$
- Four solutions
- Precomputed
- Linearly combined with learnable parameters



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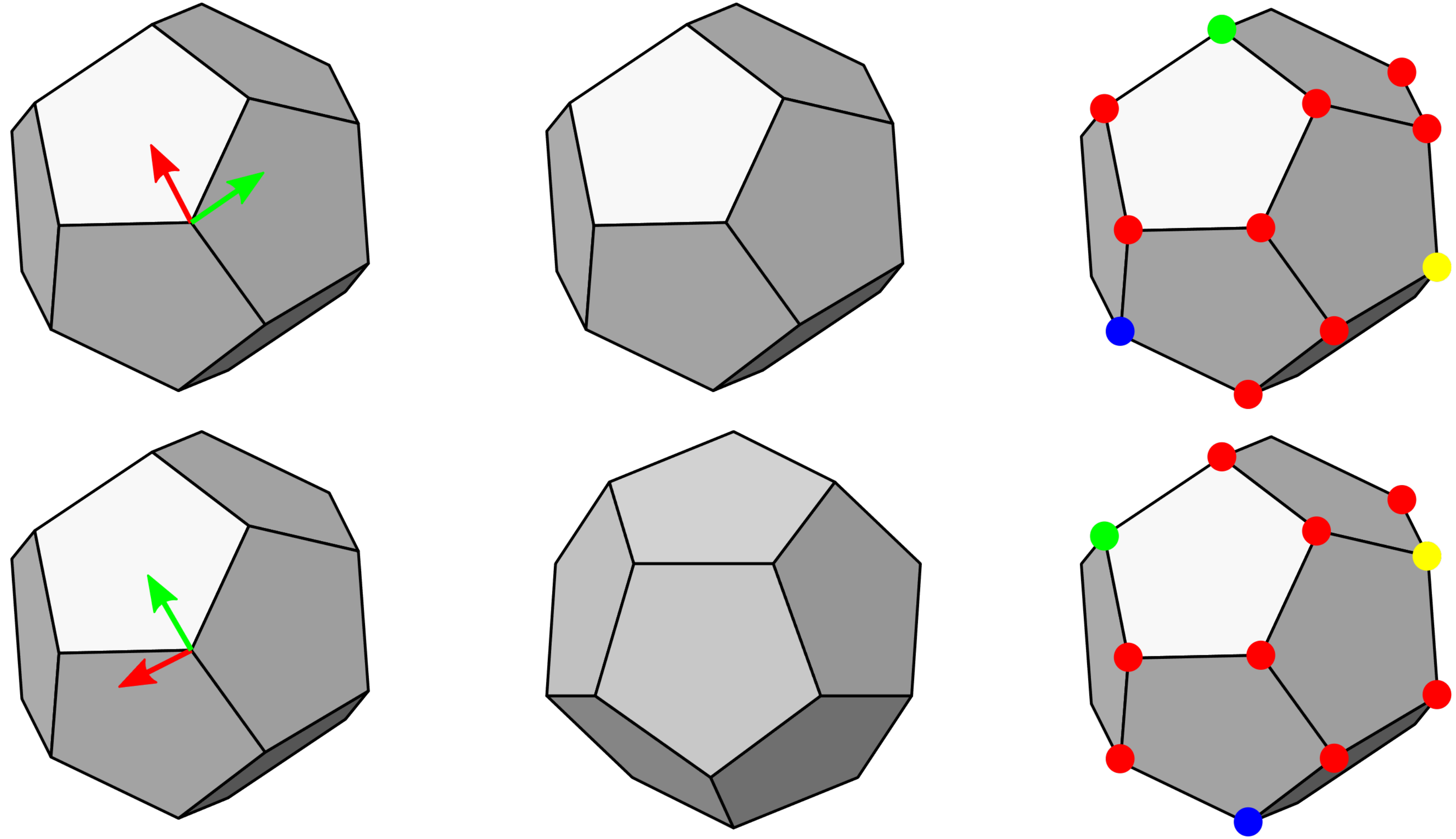
Implementation

- For each layer, pick an input and output representation
- Precomputation:
  - For each vertex, we define tangent plane
  - On each tangent plane, we pick gauge
  - For each pair of neighbours compute:
    - Log maps (r_q, θ_q)
    - Parallel transport matrices $\rho(g_{p\to q})$
    - Basis kernels $K_i(r_q, \theta_q)$
- During forward pass, combine basis kernels:

$$(K \star f)_p = \sum_i \sum_{q \in \mathcal{N}(p)} \alpha_i K_i(r_q, \theta_q) \rho(g_{q\to p}) f_q$$

Symmetry properties



Gauge Vertex coordinates Mesh isometries

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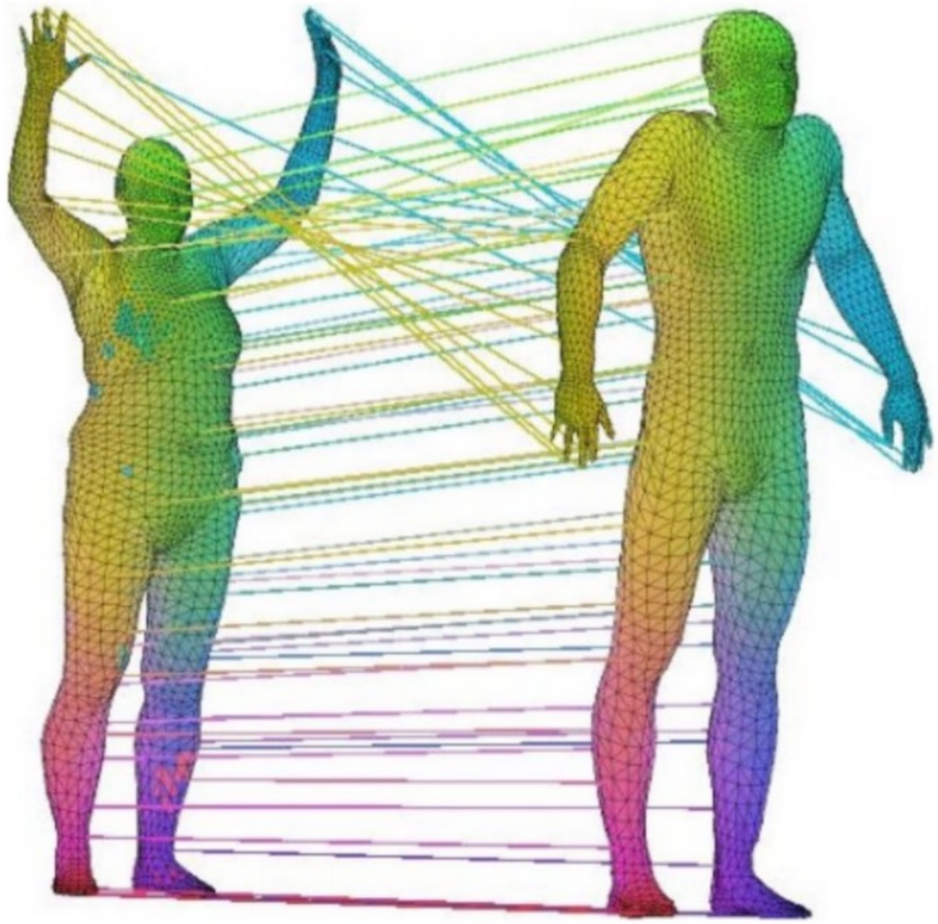
Experiment: Shape Correspondence



Model	Features	Accuracy
ACNN (Boscaini et al., 2016)	SHOT	62.4 %
Geodesic CNN (Masci et al., 2015)	SHOT	65.4 %
MoNet (Monti et al., 2016)	SHOT	73.8 %
FeaStNet (Verma et al., 2018)	XYZ	98.7%
ZerNet (Sun et al., 2018)	XYZ	96.9%
SpiralNet++ (Gong et al., 2019)	XYZ	99.8%
Graph CNN	XYZ	1.40 ± 0.5 %
Graph CNN	SHOT	23.80 ± 8%
GEM-CNN	XYZ	99.73 ± 0.04 %
GEM-CNN (broken symmetry)	XYZ	**99.89** ± 0.02 %

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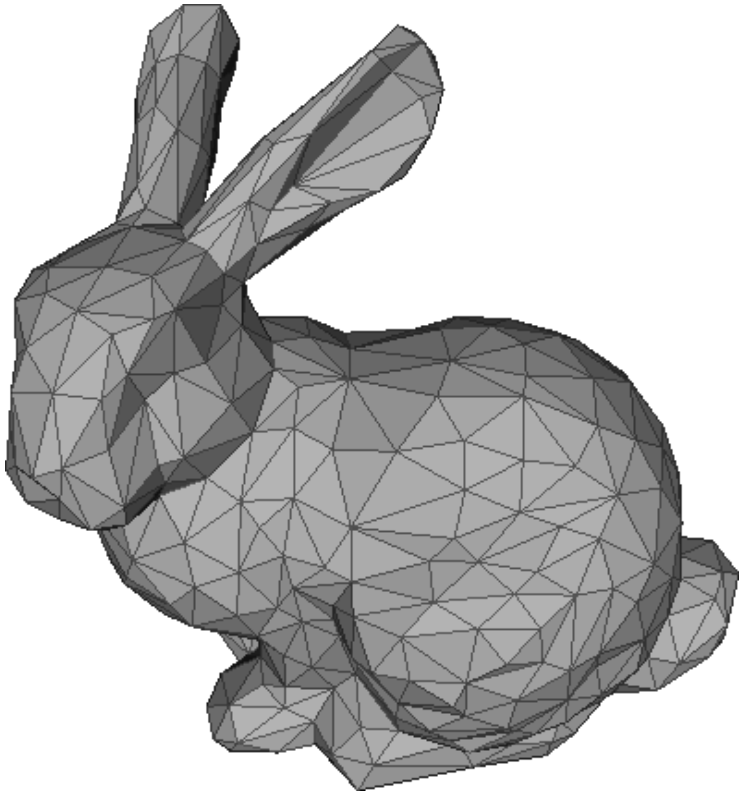
Takeaway

Gauge Equivariant Mesh CNN is:

- Simple
- Scalable
- Anisotropic ⇒ expressive
- Symmetry properties



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