



Geometric Transformers for Protein Interface Contact Prediction

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- 1: Overview
- 2: Preliminaries
- 3: DeepInteract & The Geometric Transformer
- 4: Results & Conclusions

Protein Complexes

Interacting Protein Chains

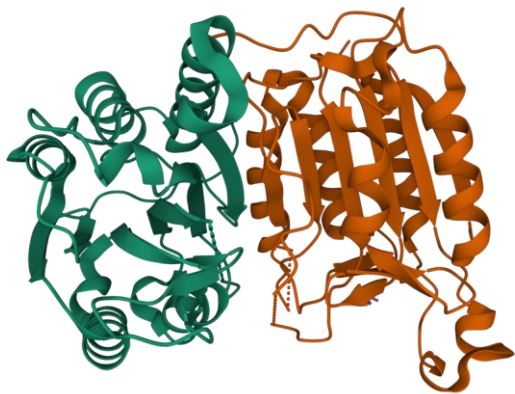


PDB ID: 3H11

- Proteins drive **fundamental** processes in all known forms of life
- They often **interact** with one another to form new macromolecules (i.e., protein *complexes*) with new functions

Contact Prediction at Protein Interfaces

Interacting Protein Chains

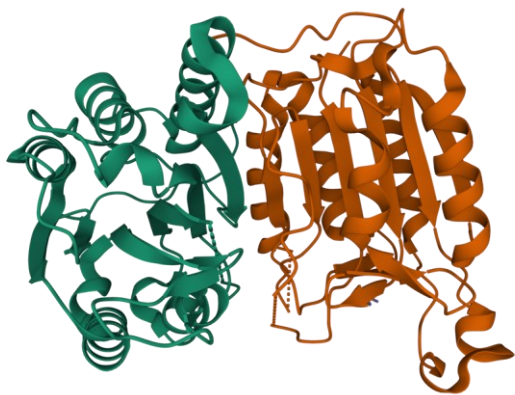


PDB ID: 3H11

- Here, we want to predict **which** of a protein's atoms will be the *contact* points between two protein *chains*
- The number of *inter-chain* (i.e., *interface*) contact points is often very **low**

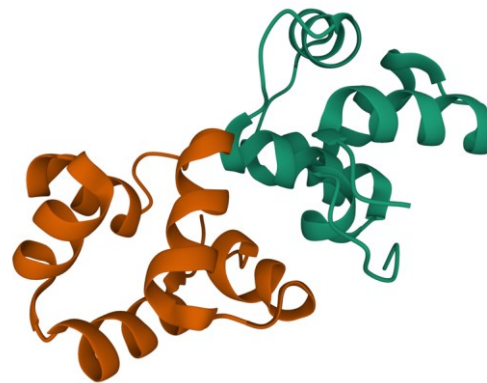
Contact Prediction at Protein Interfaces

Interacting Protein Chains



PDB ID: 3H11

Inter-Chain Contact Points



PDB ID: 6TRI

Contact Prediction at Protein Interfaces

Interacting Protein Chains



PDB ID: 3H11

Inter-Chain Contact Points



PDB ID: 6TRI

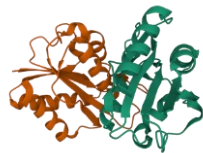
Question

- Knowing a protein chain's 3D structure can provide **detailed** molecular information about the geometry of its constituent parts and about the **intra-chain** interactions **between its atoms**



PDB ID: 4HEQ; Chain A

- Q. Can we utilize graph-based **self-attention** and the rich **geometric** information available in 3D protein structures to more precisely predict which atoms serve as contact points **between chains**?



PDB ID: 4HEQ; Chains A & B

What We Know So Far

- There are many works that have approached interface contact prediction by representing protein chains as **flat** (i.e., 2D) feature tensors or graphs to serve as inputs for neural networks such as CNNs and geometry-**agnostic** GNNs
- Nonetheless, **no works** on interface contact prediction have leveraged two novel ideas:
 - (1) Geometric deep learning to evolve protein chains' **geometric representations**
 - (2) **Node-local graph-based self-attention** similar to that of [VSPUJGKP 17] and [DB 21]

What We Show

- We introduce the new **Geometric Transformer**, a graph-based Transformer model trained to evolve representations of 3D protein chains in an SE(3)-invariant manner (e.g., to simplify its learning landscape) 😊
- This model yields new **state-of-the-art** results for protein interface contact prediction
- The Geometric Transformer also outlines **an alternative means of message-passing** and information processing on geometric graphs
 - Examples:
 - (1) Treating edges as pseudo-nodes and message passing with them like nodes
 - (2) Enabling the network to learn to gate geometric features much like previous work with RNNs
 - Adapting insights from [LWLLZOJ 22] to model large biomolecules such as proteins

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Structural Foundations

- As many of you may already be aware, Computational Biology (CB) is focused on the study of biological entities and concepts **through the lense of computation**
- Geometric Deep Learning (GDL), on the other hand, is a branch of machine learning dedicated to the study, analysis, and representation learning of objects **with some underlying structure**

Inductive Priors (1)

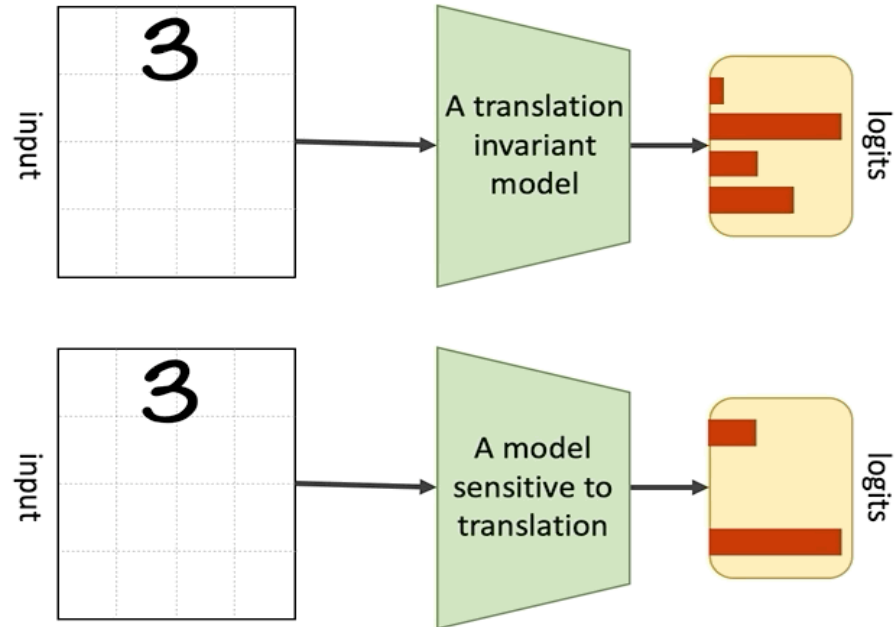
- One of my goals for this talk is for us all to begin thinking more intentionally about the **inductive priors (i.e., the architectural assumptions)** we can bring to the sets of problems we are interested in solving
- One such example can be found in the use of convolutional neural networks (CNNs) for image-based deep learning

Inductive Priors (2)

- In [Geometric Deep Learning](#), Bronstein et al. (2021) argue that CNNs have become popular for such image-based tasks because of the **translation equivariance** shared-weight neural networks such as CNNs possess
- The symmetry-preserving properties these neural networks have indicate that their learned feature representations **transform in response** to equivalent transformations on the domain of their input data

Geometric Deep Learning

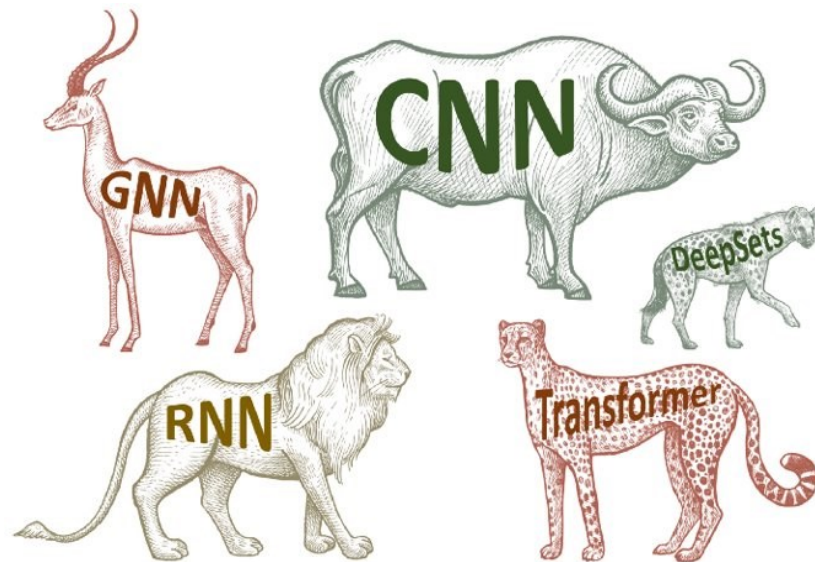
- Translation **Equivariance** in CNNs



- Credit: Samira Abnar

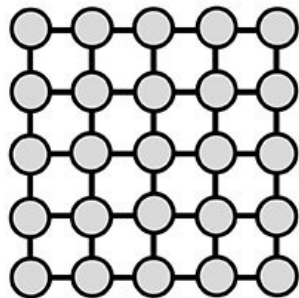
Geometric Deep Learning

- Graph neural networks (**GNNs**) can be seen as one of many existing geometric deep learning algorithms
- Others, for example, include Recurrent Neural Networks (**RNNs**) and **DeepSets**

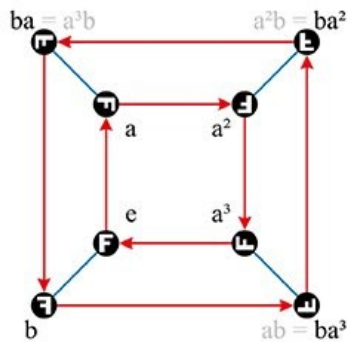


- Credit: Michael Bronstein

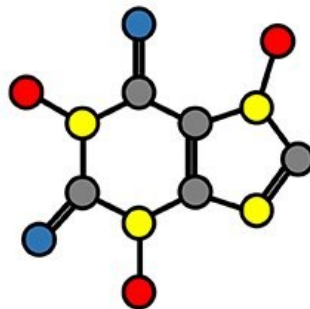
The 4G of Geometric Deep Learning



Grids



Groups



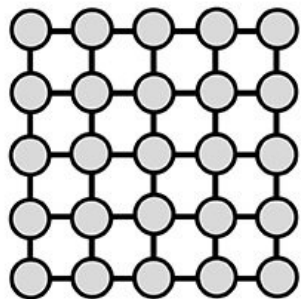
Graphs



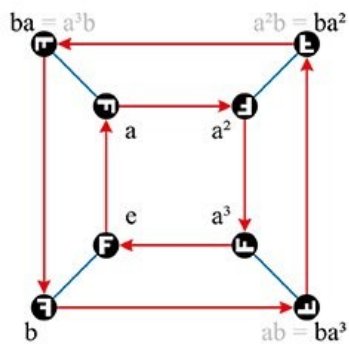
"Gauges"

"Erlangen Programme" of Deep Learning

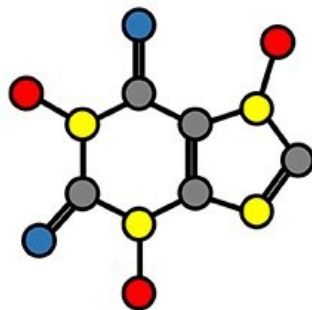
The 4G of Geometric Deep Learning



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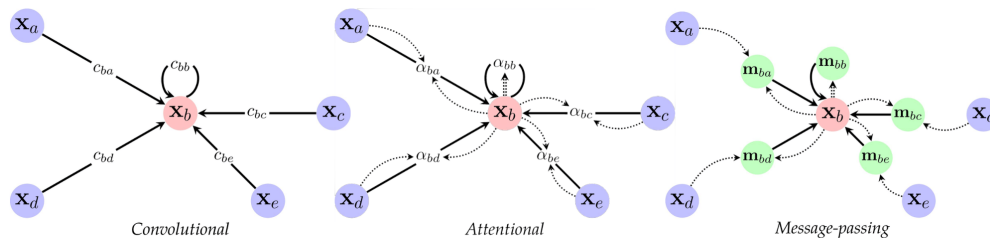
"Gauges"

"Erlangen Programme" of Deep Learning

Graph Neural Networks and their Variations

- It can often be helpful to group graph neural networks into **three** related categories

The three “flavours” of GNN layers



- Credit: Petar Veličković

$$\mathbf{h}_i = \phi \left(\mathbf{x}_i, \bigoplus_{j \in \mathcal{N}_i} c_{ij} \psi(\mathbf{x}_j) \right)$$

$$\mathbf{h}_i = \phi \left(\mathbf{x}_i, \bigoplus_{j \in \mathcal{N}_i} a(\mathbf{x}_i, \mathbf{x}_j) \psi(\mathbf{x}_j) \right)$$

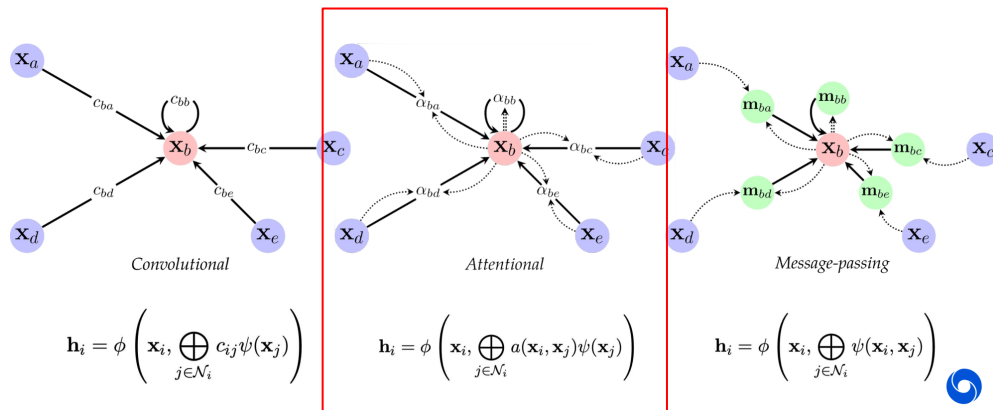
$$\mathbf{h}_i = \phi \left(\mathbf{x}_i, \bigoplus_{j \in \mathcal{N}_i} \psi(\mathbf{x}_i, \mathbf{x}_j) \right)$$



Graph Neural Networks and their Variations

- It can often be helpful to group graph neural networks into **three** related categories

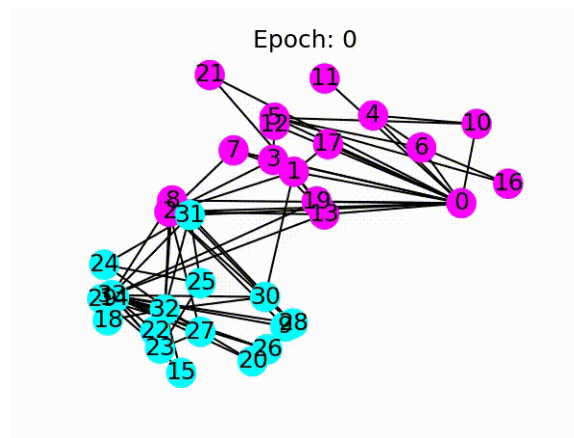
The three “flavours” of GNN layers



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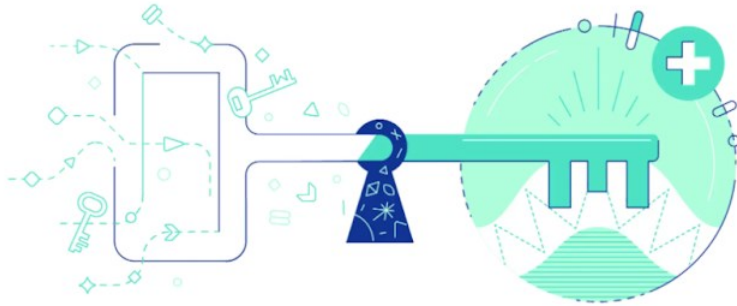
An Example of Graph Geometric Deep Learning

- Labeled nodes: #0 and #33
- Calculate losses only from the label nodes
- Doesn't require a lot of labeled samples
- Feature-less graph
- Use embedding weights to represent nodes
- (i.e., converting node ID to vector of weights)

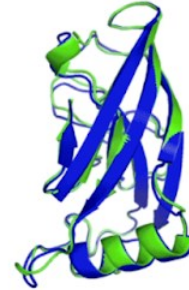


References: [KW 17] & [MCC 22]

Geometric Deep Learning in Computational Biology

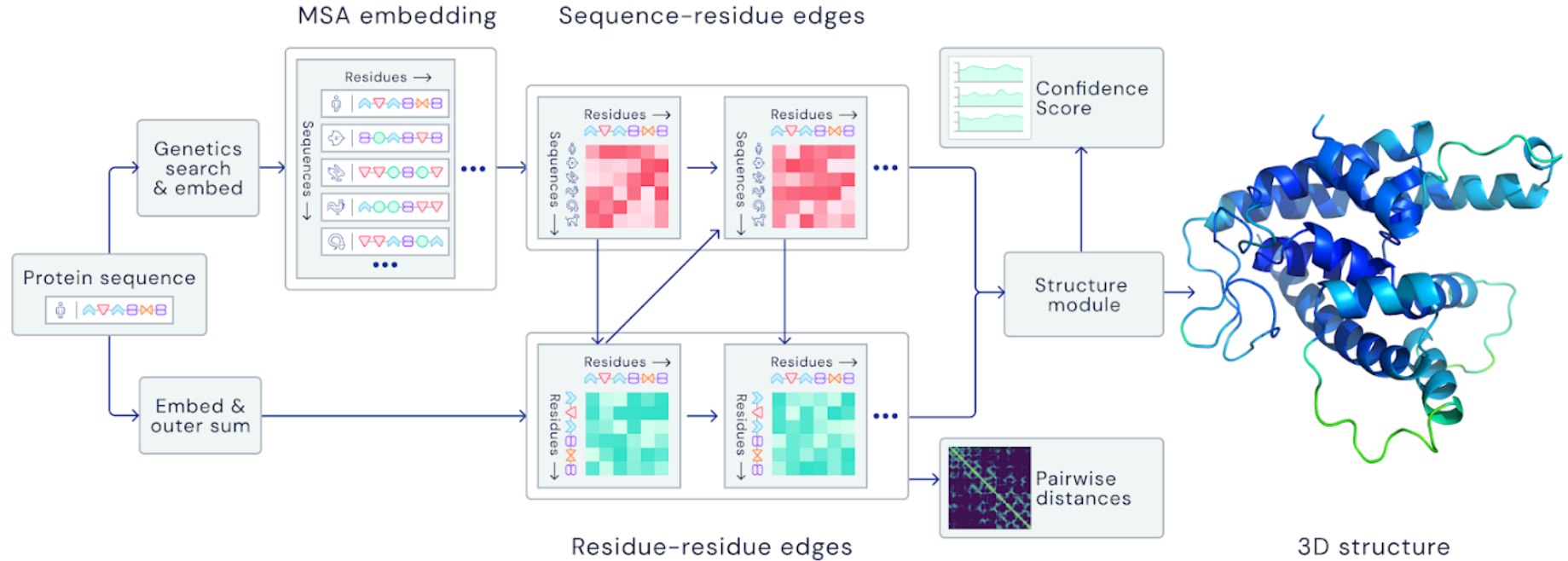


T1037 / 6vr4
90.7 GDT
(RNA polymerase domain)

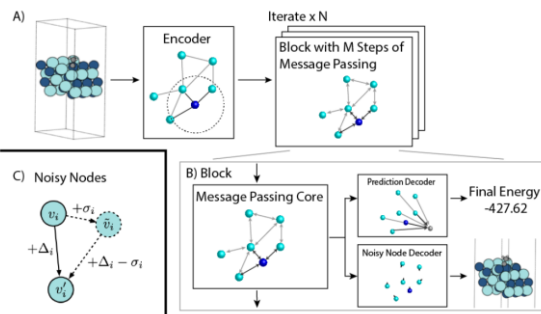
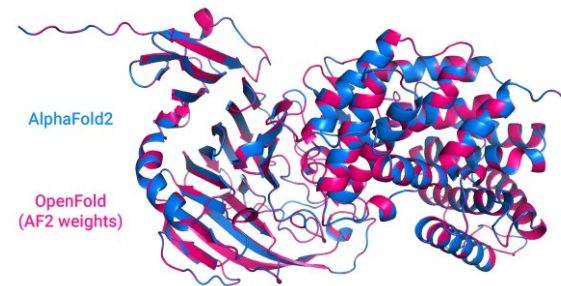
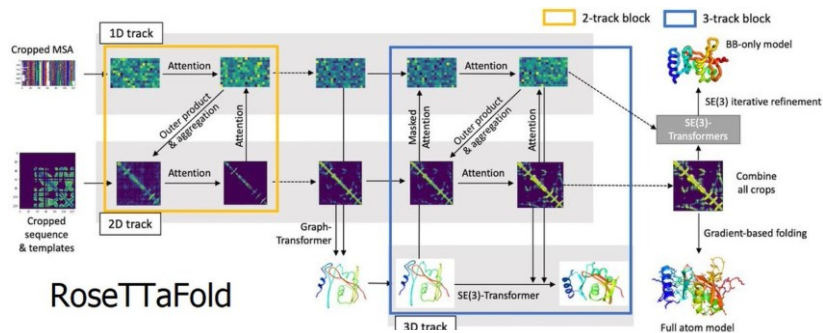


T1049 / 6y4f
93.3 GDT
(adhesin tip)

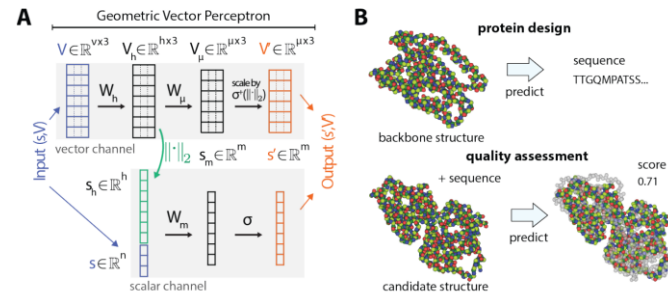
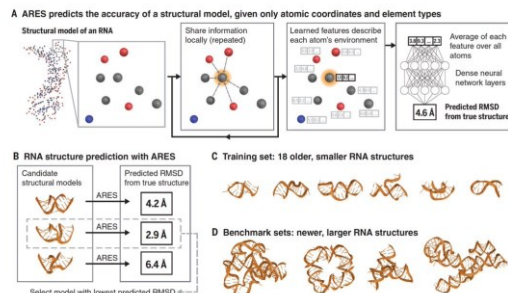
Geometric Deep Learning in Computational Biology



Geometric Deep Learning in Computational Biology

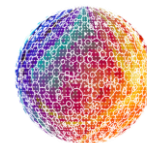


Geometric deep learning of RNA structure



ATOM3D

Graphein



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Notation & Problem Formulation

- For each protein **chain**, define a graph $\mathbb{G} : \langle \mathbb{N}, \mathbb{E}, k \rangle$.
 - \mathbb{N} : Atoms in the chain (**simplified to the number of Ca atoms**/amino acid residues in the chain)
 - \mathbb{E} : Edges between Ca atoms
 - k : Number of nearest neighbors to which to connect each Ca atom
 - In this work, we let $k = 20$ similar to [FBSB 17] and [IGBJ 19]
- Our learning task then is three-fold:
 - 1. Learn **new** node-level representations $h_{\mathbb{A}} \in \mathbb{R}^{\mathbb{A} \times k}$ and $h_{\mathbb{B}} \in \mathbb{R}^{\mathbb{B} \times k}$ **for a chain pair**.
 - 2. Channel-wise interleave $h_{\mathbb{A}}$ and $h_{\mathbb{B}}$ into interaction tensor $\mathbb{I} \in \mathbb{R}^{\mathbb{A} \times \mathbb{B} \times 2\mathbb{C}}$, where $\mathbb{A} \in \mathbb{R}$ and $\mathbb{B} \in \mathbb{R}$ are the numbers of amino acid residues in the first and second input protein chains, respectively, and $\mathbb{C} \in \mathbb{R}$ is the number of hidden channels in both $h_{\mathbb{A}}$ and $h_{\mathbb{B}}$.
 - 3. Convolve over $\mathbb{I} \in \mathbb{R}^{\mathbb{A} \times \mathbb{B} \times 2\mathbb{C}}$ to predict interface residue-residue **contact probabilities**.

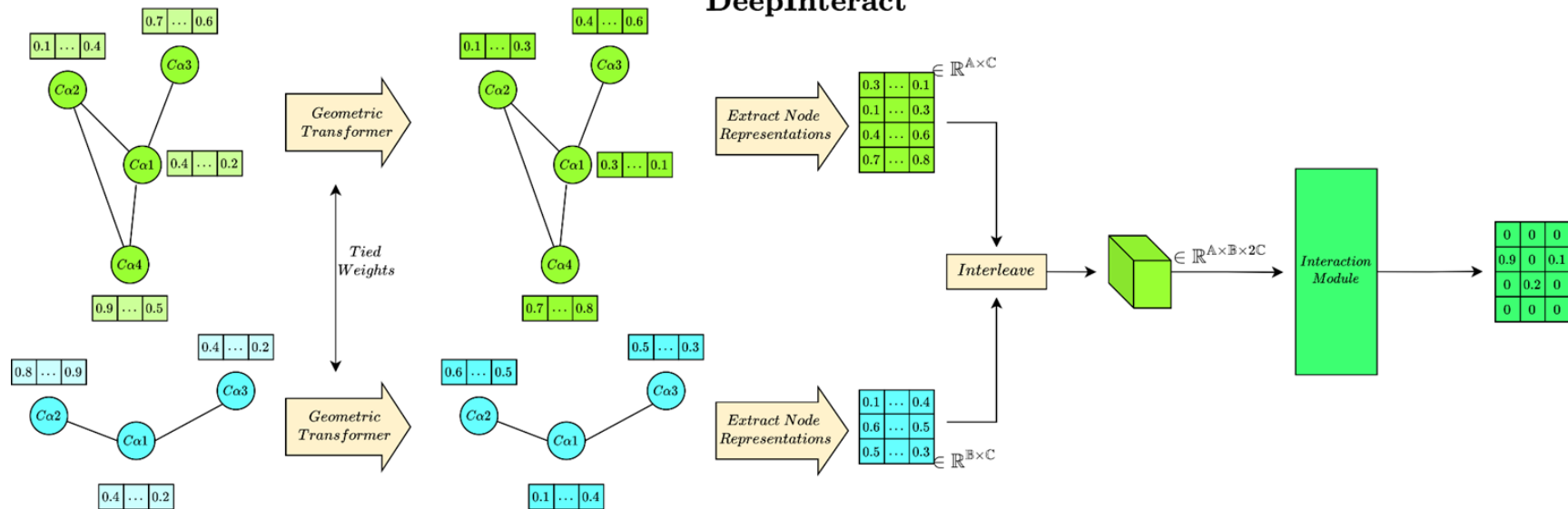
Datasets

- We derive our training and validation datasets as well as our first test dataset from the enhanced version of the Database of Interacting Protein Structures (DIPS) [TBSD 19], referred to as DIPS-Plus [MCSC 21]
 - After preprocessing all proteins in DIPS-Plus, we are left with 15,618 and 3,548 pairs of protein chains for training and validation, respectively
 - Our first test dataset is comprised of 32 randomly-chosen homodimers and heterodimers from DIPS-Plus (16 of each type, respectively)
- Our second test dataset consists of 14 homodimers and 5 heterodimers from CASP-CAPRI 13-14 [LBNV 19] [LBMN 21]
- Finally, our third test dataset includes the traditional 55 chain pairs for interface contact prediction from Docking Benchmark 5 (DB5) [VMVP 15]

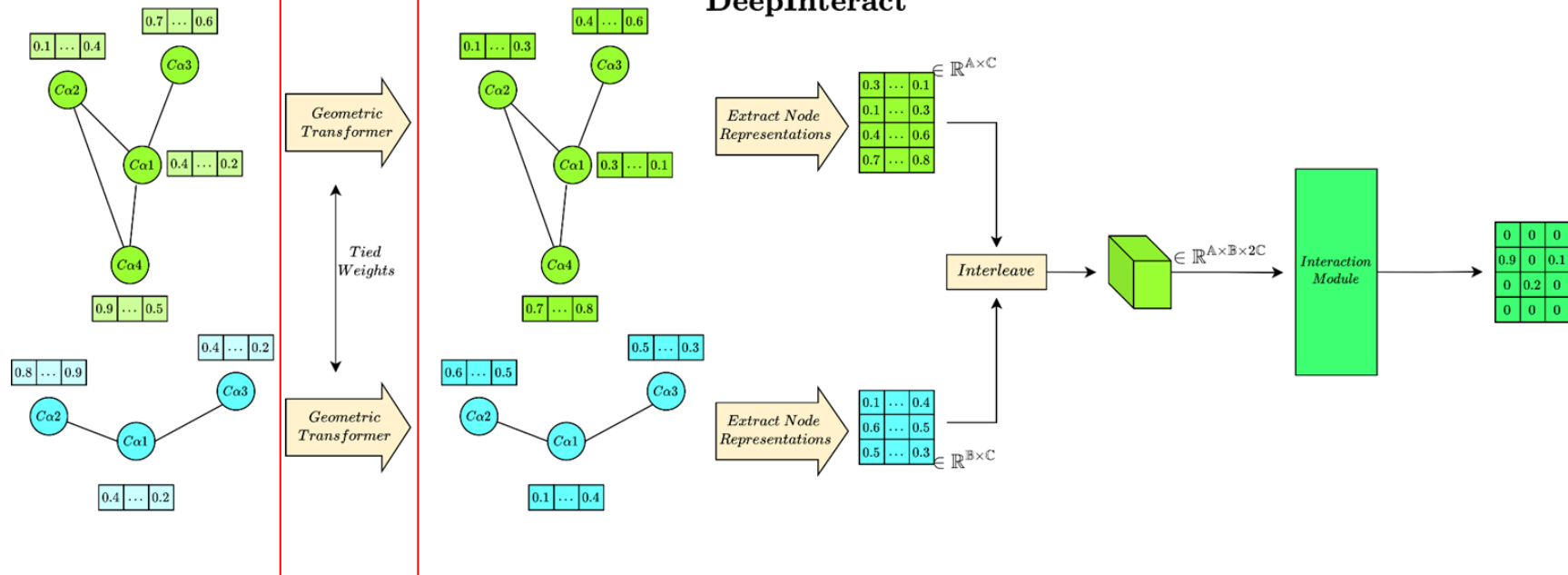
Key Ideas behind DeepInteract

- Can we use **graph self-attention** and **geometric feature gating** (GFG) to **evolve** an initial set of geometric protein descriptors?
- Are these new residue representations **useful** for a deep CNN to more easily distinguish between interacting and non-interacting inter-chain pairs of residues?
- Can **alternative** forms of message passing enable **better learning** over 3D molecular graphs?

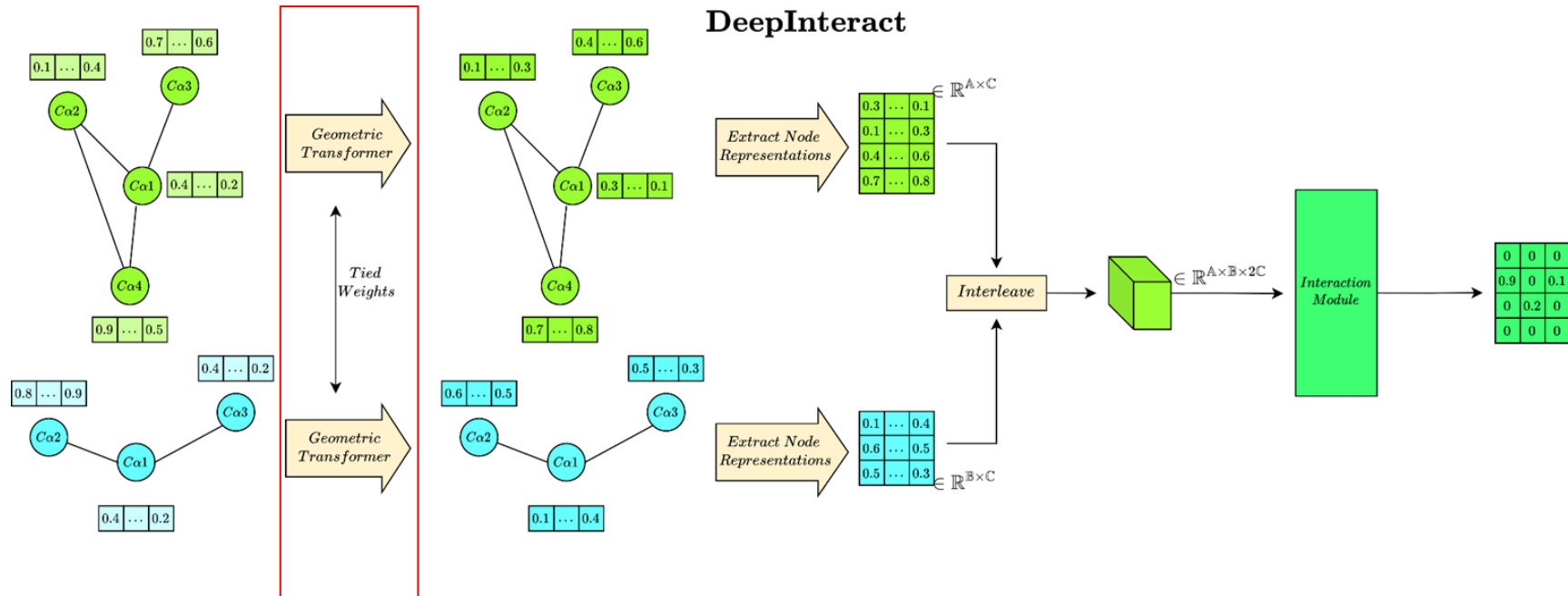
DeepInteract



DeepInteract

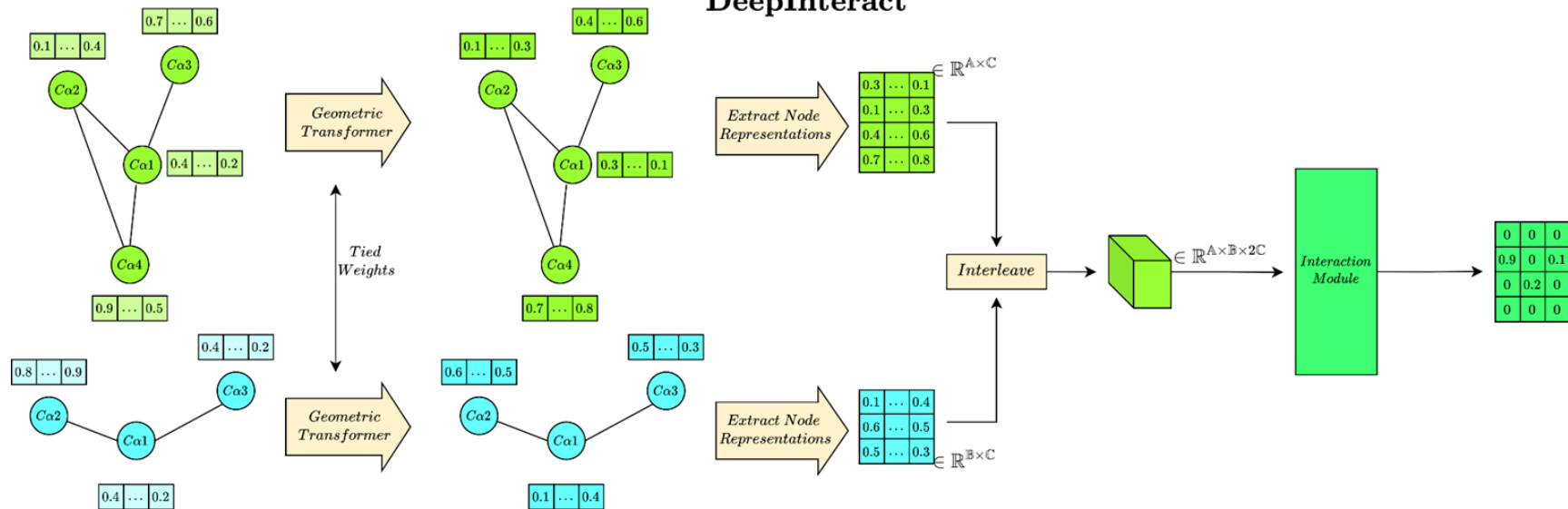


DeepInteract

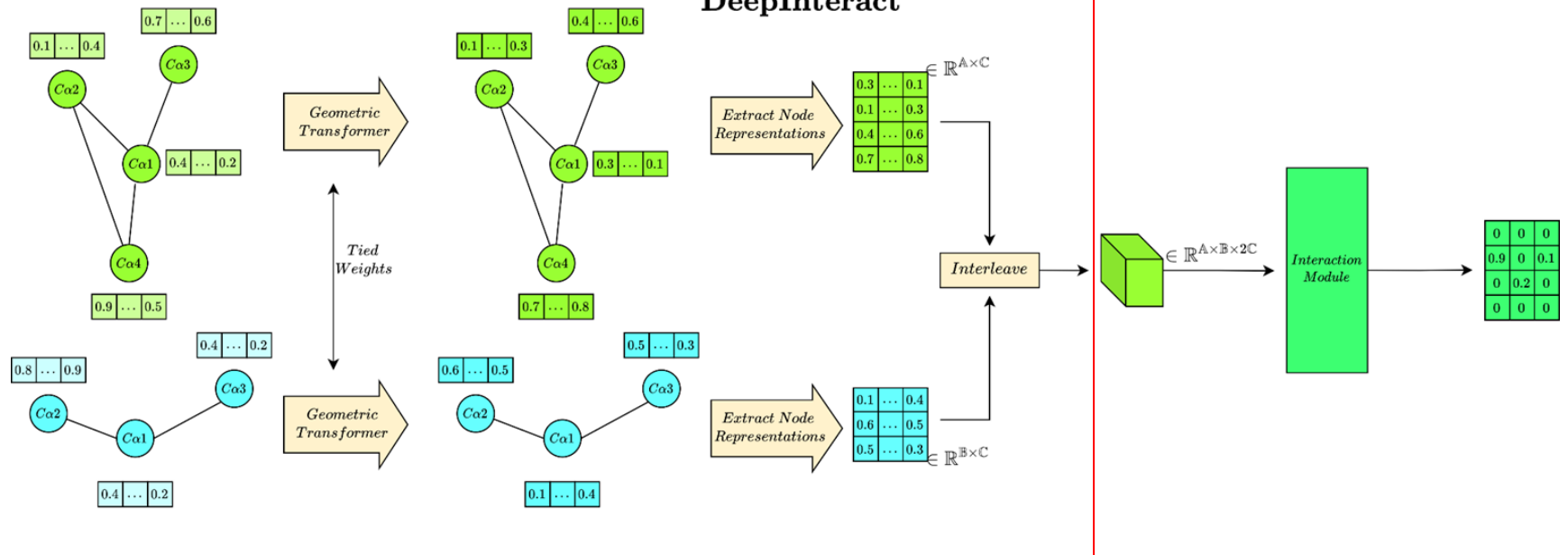


The Geometric Transformer here
learns new atom (i.e., node) representations

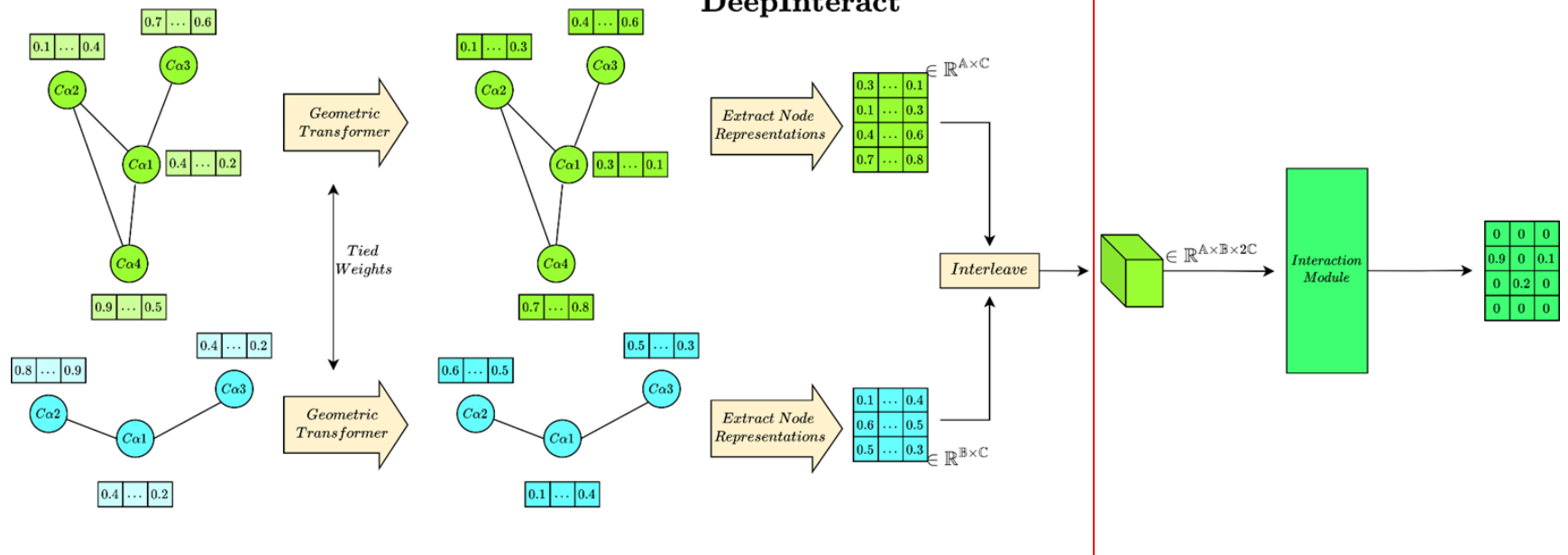
DeepInteract



DeepInteract

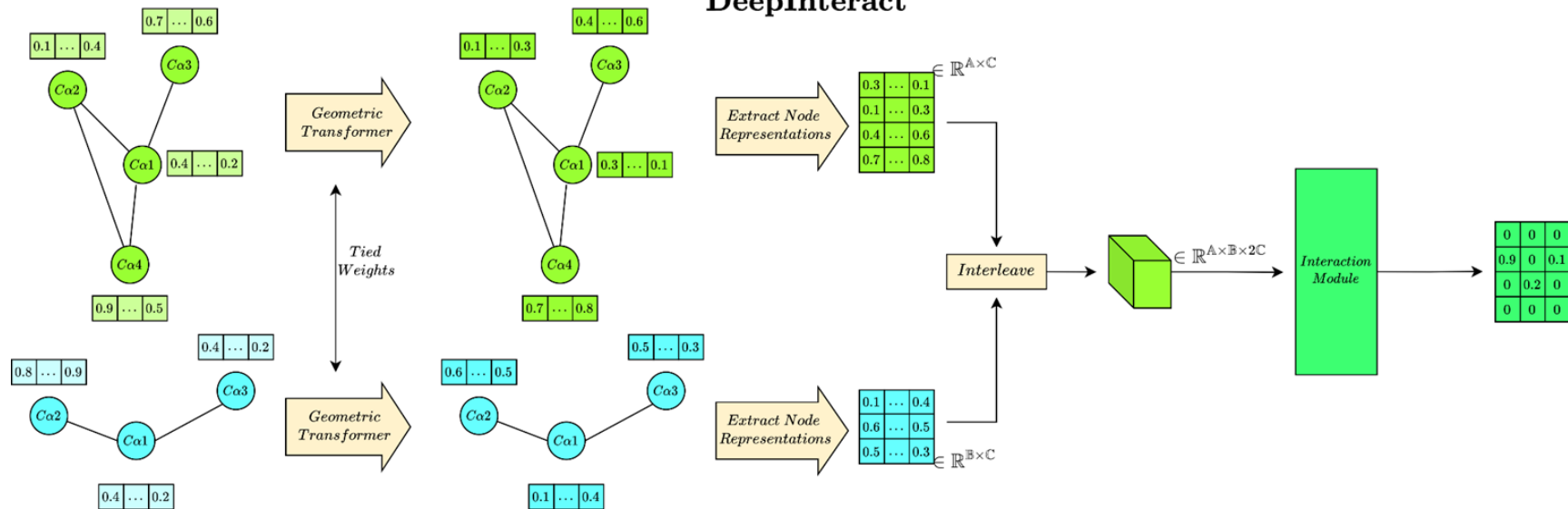


DeepInteract

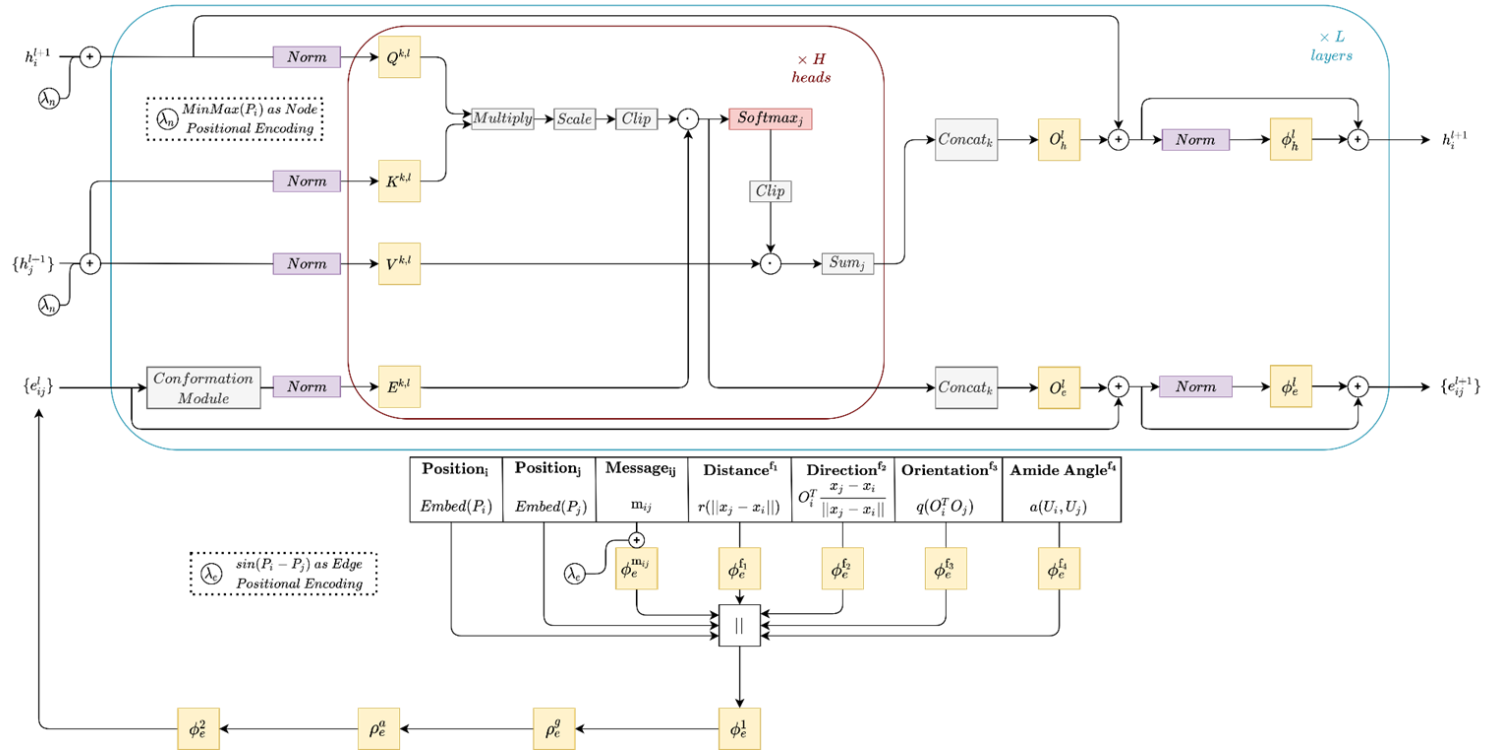


We then feed the new atom representations into a deep CNN to get contact probabilities as output

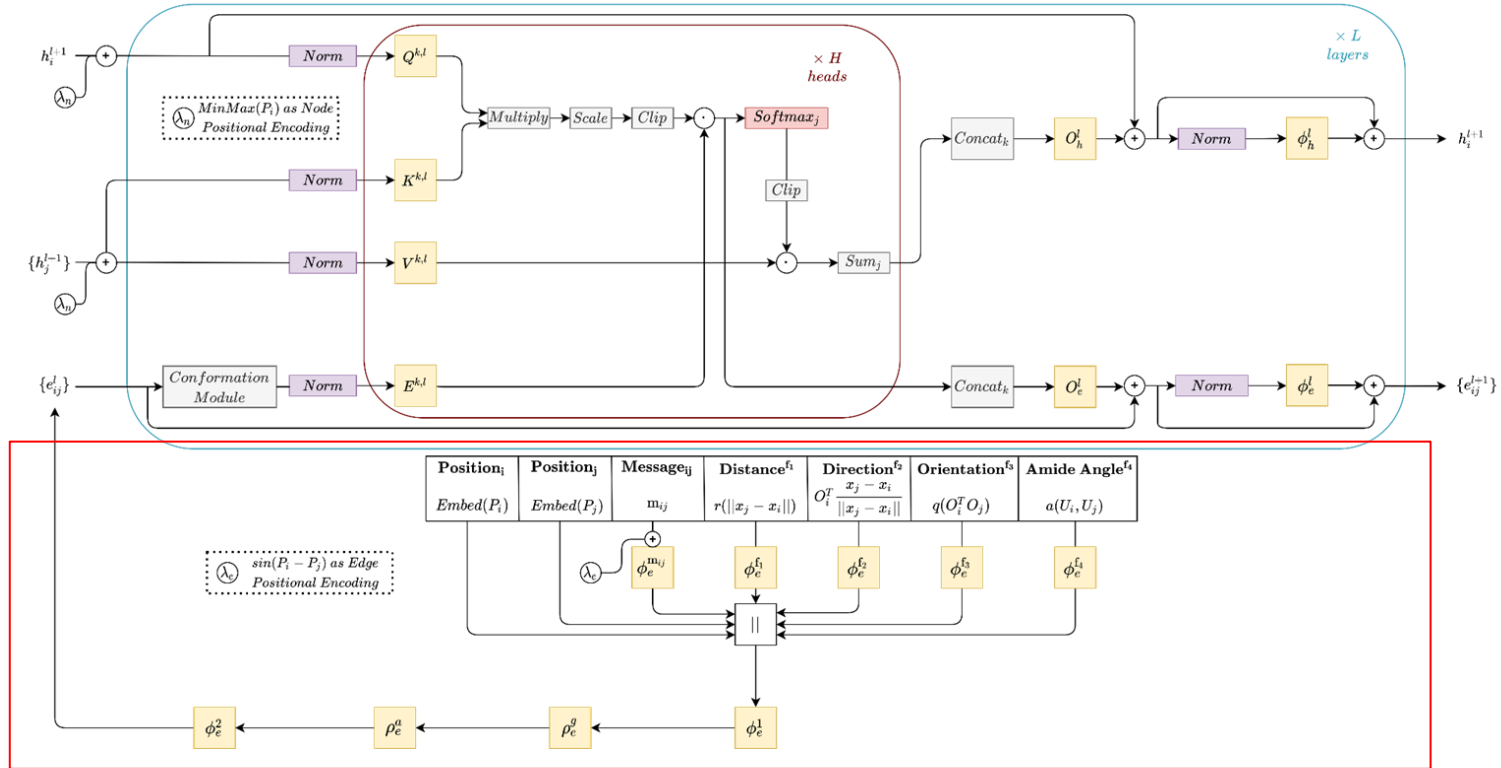
DeepInteract



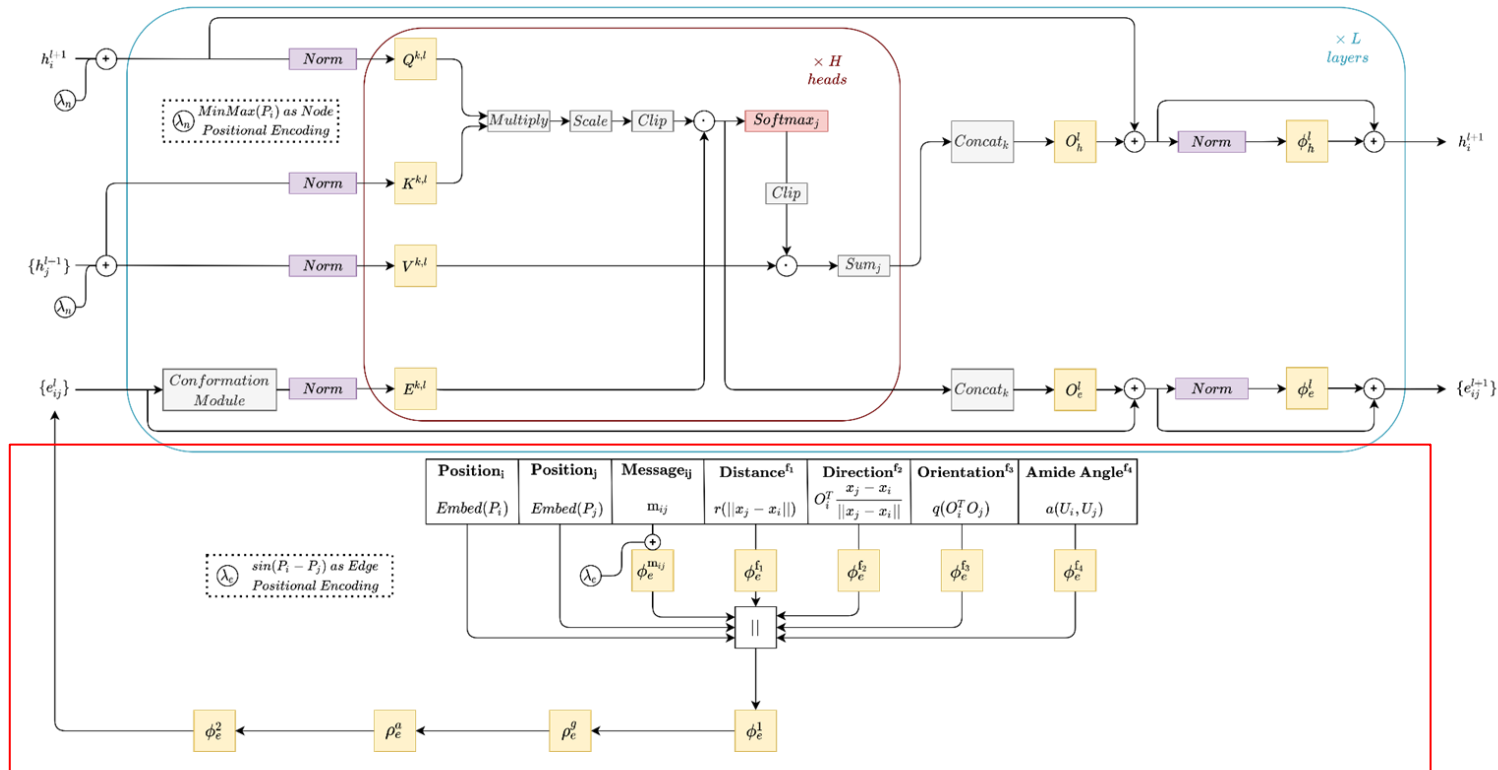
Geometric Transformer



Geometric Transformer



Geometric Transformer



We introduce an Edge Initialization Module to accelerate the Geometric Transformer's training

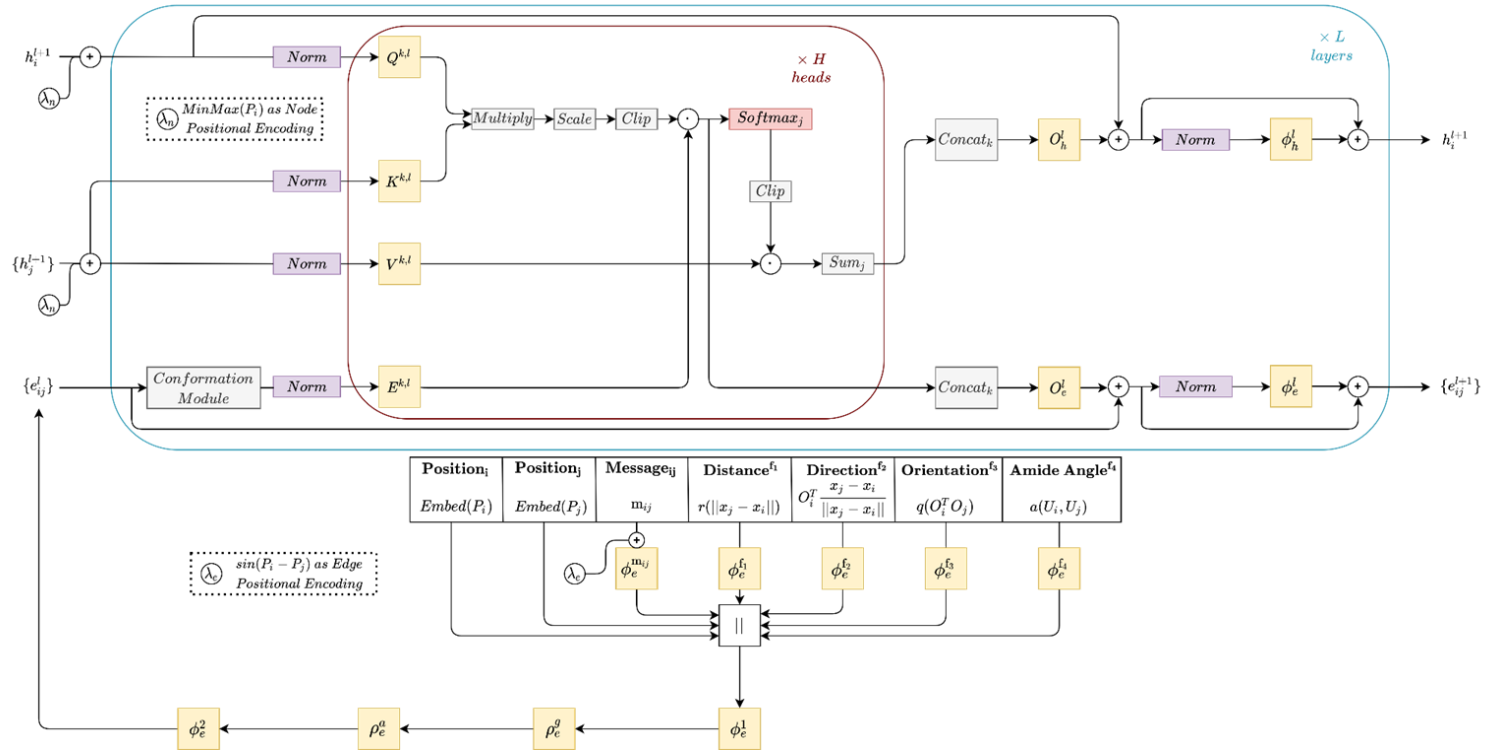
Edge Initialization Module - Definition

$$c_{ij} = \phi_e^1([p_1 \parallel p_2 \parallel \phi_e^{m_{ij}}(m_{ij} \parallel \lambda_e) \parallel \phi_e^{f_1}(f_1) \parallel \phi_e^{f_2}(f_2) \parallel \phi_e^{f_3}(f_3) \parallel \phi_e^{f_4}(f_4)]) \quad (1)$$

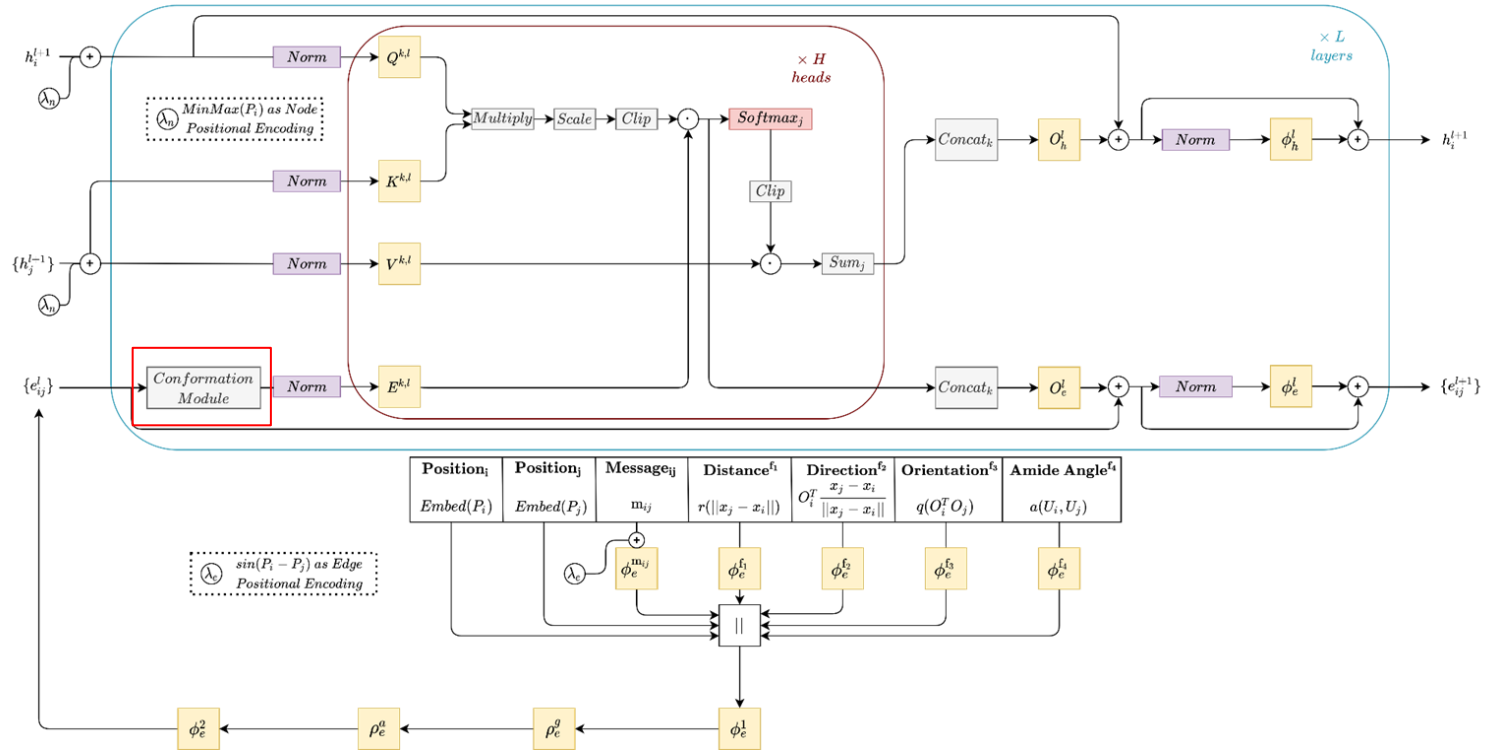
$$e_{ij} = \phi_e^2(\rho_e^a(\rho_e^g(c_{ij}))) \quad (2)$$

1. ϕ_e^i : The i^{th} edge information update function such as a multi-layer perceptron
2. \parallel : Channel-wise concatenation
3. p_1 and p_2 : Trainable one-hot vectors indexed, respectively, by P_i and P_j , the positions of nodes i and nodes j in the chain's underlying amino acid sequence
4. m_{ij} : Any user-predefined features for e (e.g., Euclidean distances between nodes i and nodes j)
5. λ_e : **Edge-wise** sinusoidal positional encodings $\sin(P_i - P_j)$ for e
6. f_1, f_2, f_3 , and f_4 : Our four protein-specific geometric features, in order
7. ρ_e^a and ρ_e^g : Feature addition and channel-wise gating functions, respectively

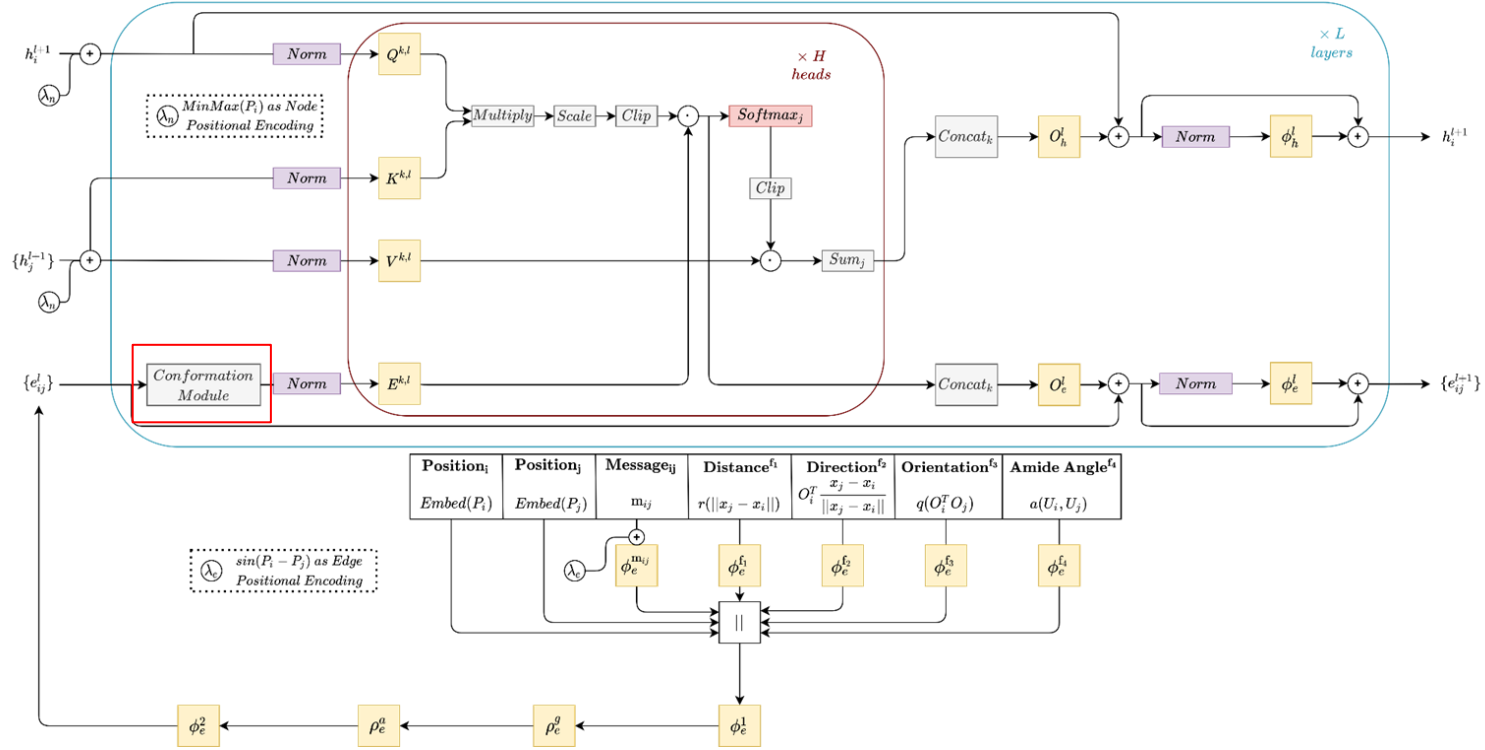
Geometric Transformer



Geometric Transformer

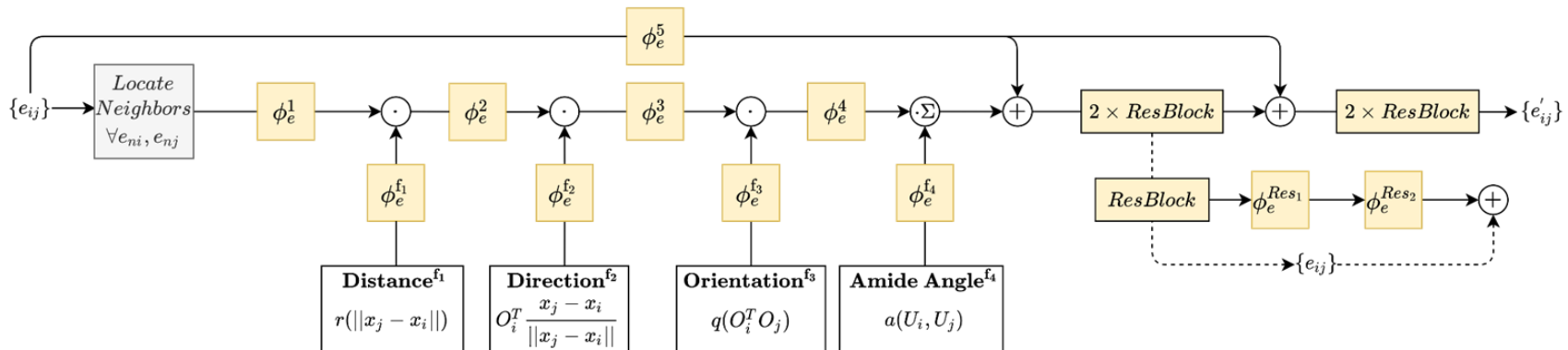


Geometric Transformer

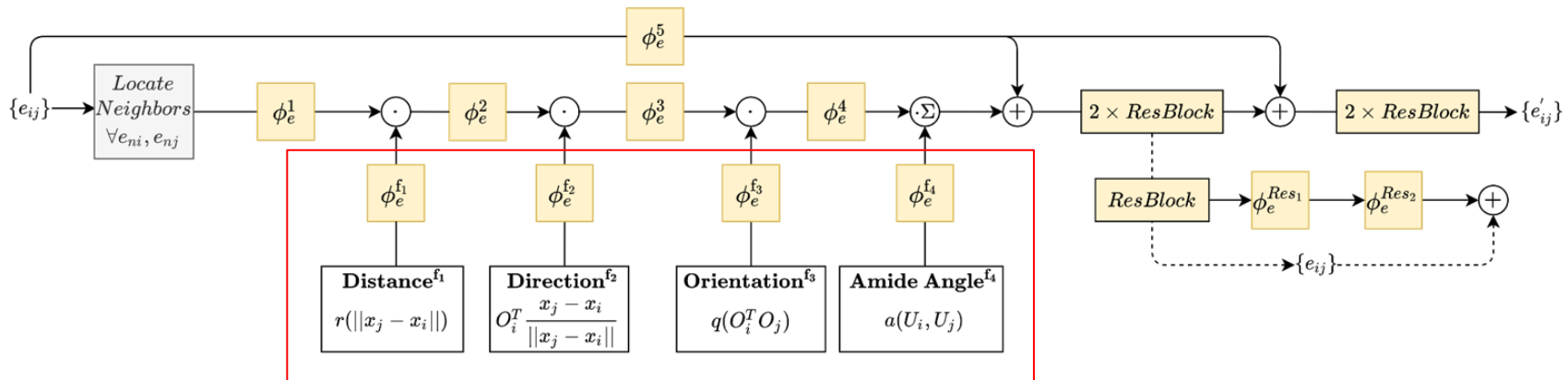


We also introduce a Conformation Module that learns to evolve protein representations using GFG

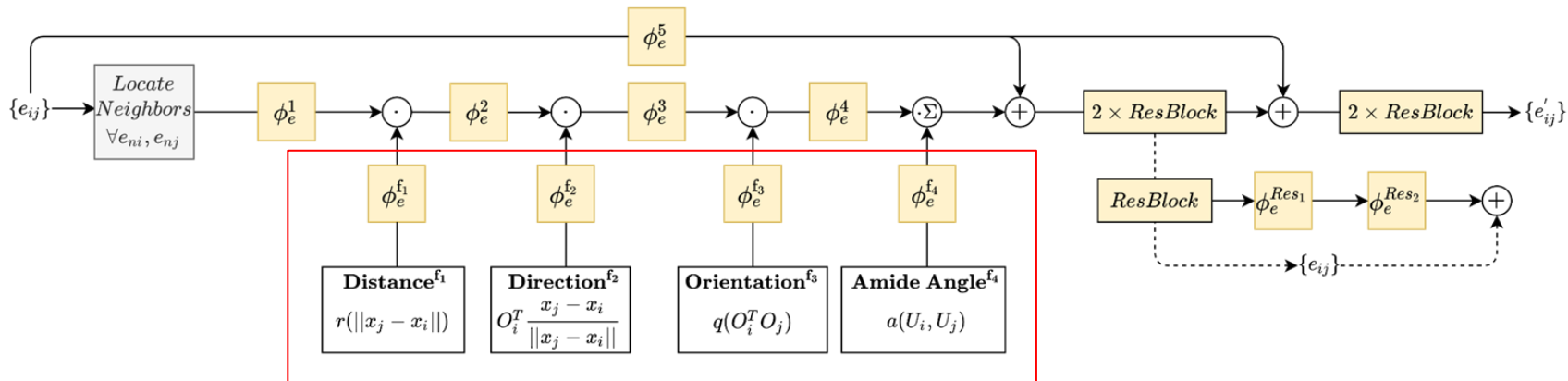
Geometric Transformer *Conformation Module*



Geometric Transformer *Conformation Module*

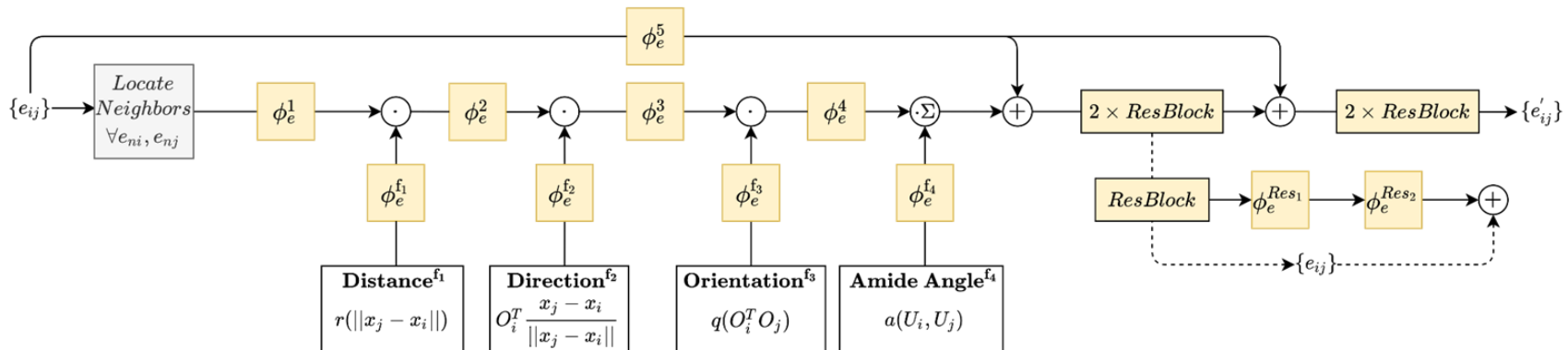


Geometric Transformer Conformation Module

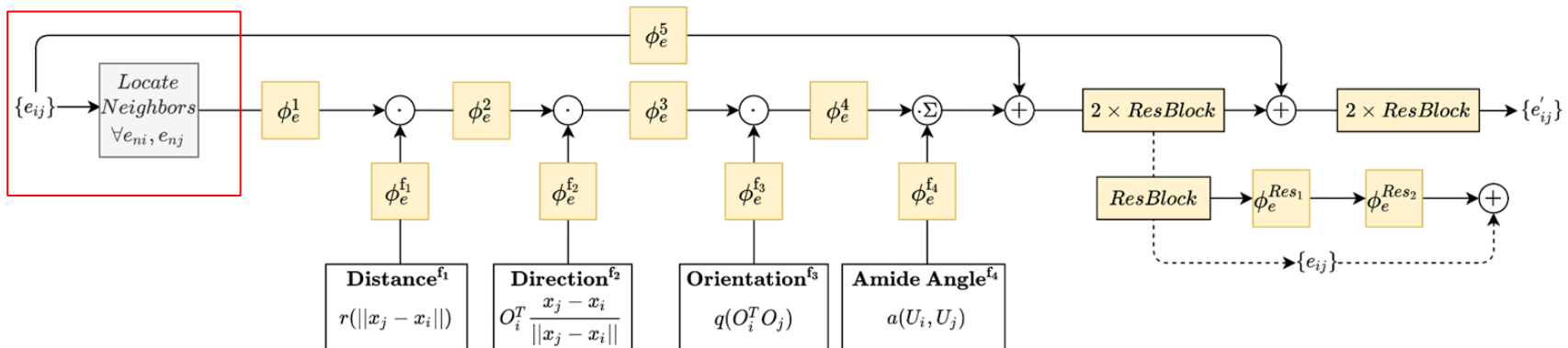


Here, the Conformation Module learns to selectively gate each type of geometric information, similar to work overcoming vanishing gradients with RNNs

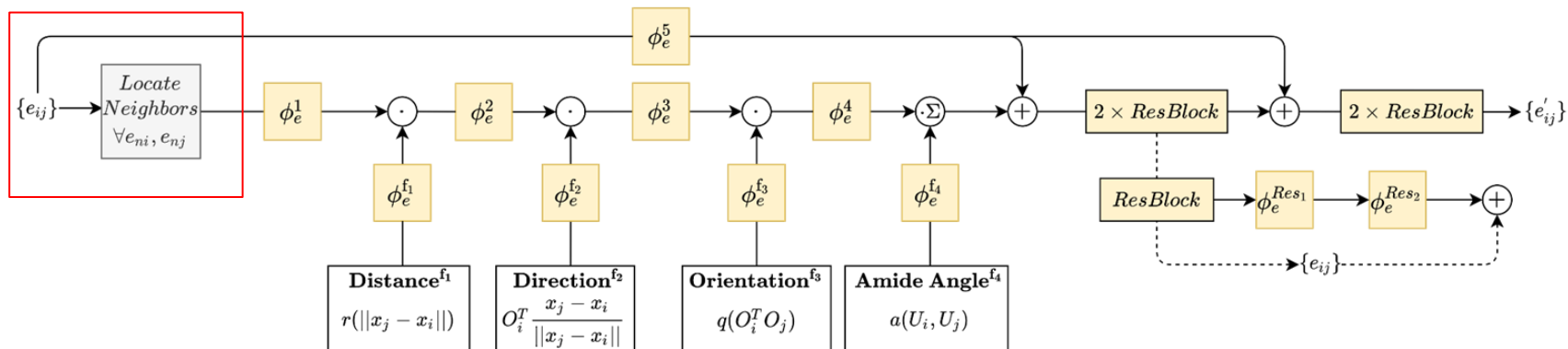
Geometric Transformer *Conformation Module*



Geometric Transformer *Conformation Module*



Geometric Transformer *Conformation Module*



Moreover, it does so by treating an edge's "neighboring" edges as pseudo-nodes, offering the edge a wider receptive field with which to update its representations

Conformation Module - Definition (1)

$$\mathbb{E}_k = \{e_{n_1 i}, e_{n_2 j} \mid (n_1, n_2 \in \mathbb{N}_k) \text{ and } (n_1, n_2 \neq i, j)\} \quad (3)$$

1. \mathbb{E}_k : The edge geometric neighborhood of edge (i.e., pseudo-node) e , defined as the $2n$ edges above in (3)
2. $\mathbb{N}_k \subset \mathbb{N}$: The source nodes for incoming edges on e 's source and destination nodes

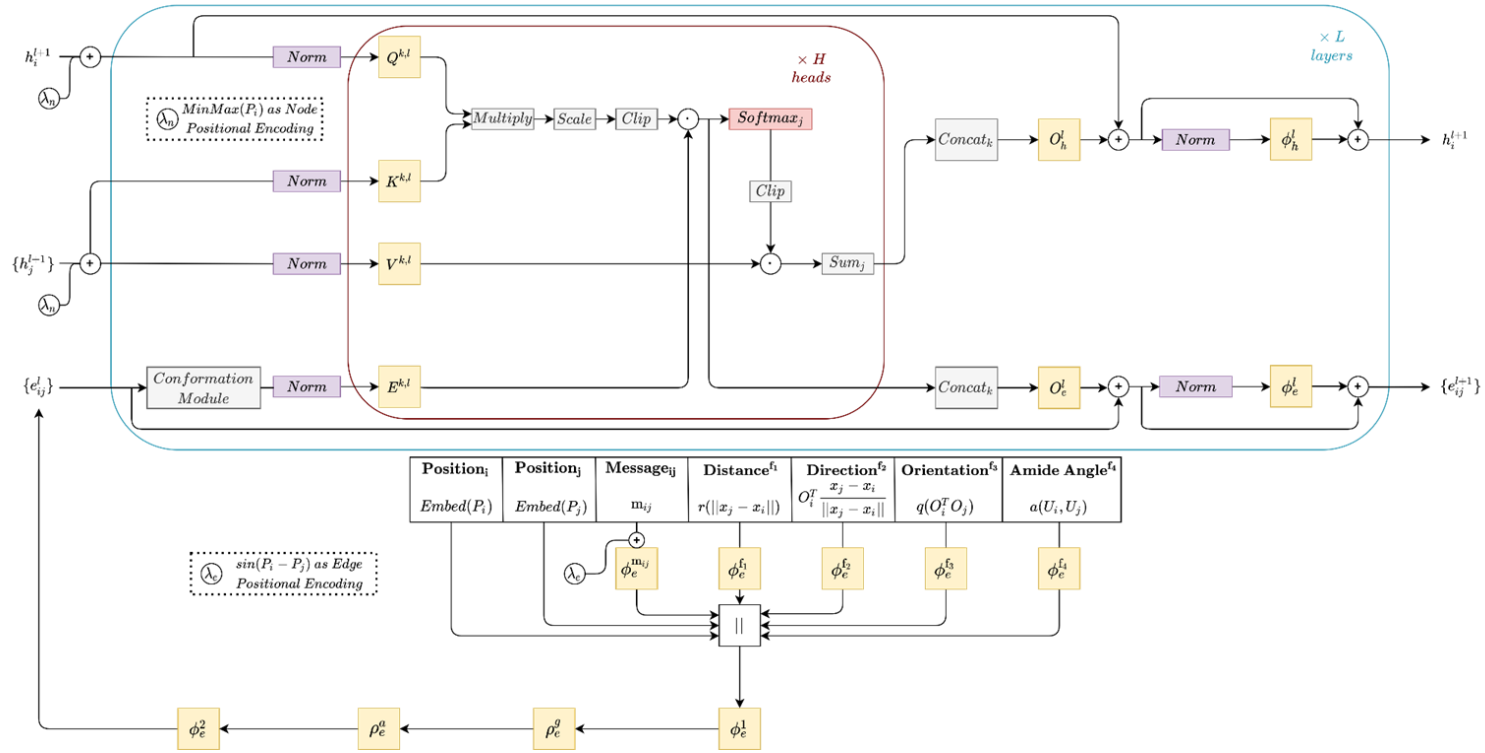
Conformation Module - Definition (2)

$$O_{ij} = \sum_{k \in \mathbb{E}_k} [(\phi_e^n(e_{ij,k}) \odot \phi_e^{f_n}(f_n)), \forall n \in \mathbb{F}] \quad (4)$$

$$e_{ij} = 2 \times ResBlock_2(\phi_e^5(e_{ij}) + 2 \times ResBlock_1(\phi_e^5(e_{ij}) + O_{ij})) \quad (5)$$

1. \mathbb{F} : The set of our protein geometric features
2. \odot : Element-wise multiplication
3. $e_{ij,k}^n$: Neighboring edge e_k 's representation after being gated with f_{n-1}
4. $2 \times ResBlock_i$: The i^{th} application of two unique, successive residual blocks, each defined as $ResBlock(x) = \phi_e^{Res_2}(\phi_e^{Res_1}(x)) + x$

Geometric Transformer



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Test Results on DIPS-Plus Targets

Table 1: The average top- k precision on two types of DIPS-Plus test targets.

Method	16 (Homo)			16 (Hetero)		
	10	$L/10$	$L/5$	10	$L/10$	$L/5$
BI	0	0	0	0.02	0.02	0.02
DH	0.13	0.12	0.09			
CC				0.17	0.16	0.15
DI (GCN)	0.22 (0.06)	0.20 (0.07)	0.18 (0.04)	0.08 (0.01)	0.08 (0.01)	0.07 (0.02)
DI (GT)	0.27 (0.06)	0.24 (0.04)	0.21 (0.04)	0.10 (0.04)	0.09 (0.04)	0.08 (0.04)
DI (GeoT w/o EPE)	0.28 (0.05)	0.24 (0.01)	0.23 (0.03)	0.11 (0.05)	0.10 (0.04)	0.09 (0.03)
DI (GeoT w/o GFG)	0.27 (0.08)	0.24 (0.08)	0.21 (0.08)	0.10 (0.02)	0.09 (0.02)	0.09 (0.01)
DI (GeoT)	0.25 (0.03)	0.25 (0.03)	0.23 (0.02)	0.15 (0.04)	0.14 (0.05)	0.11 (0.04)

Table 2: The average top- k precision and recall on DIPS-Plus test targets of both types.

Method	32 (Both Types)					
	P@10	P@ $L/10$	P@ $L/5$	R@ L	R@ $L/2$	R@ $L/5$
BI	0.01	0.01	0.01	0.01	0.004	0.003
DI (GCN)	0.15 (0.03)	0.16 (0.01)	0.12 (0.02)	0.10 (0.02)	0.06 (0.01)	0.03 (0.003)
DI (GT)	0.18 (0.05)	0.16 (0.04)	0.15 (0.04)	0.13 (0.02)	0.07 (0.01)	0.04 (0.01)
DI (GeoT w/o EPE)	0.19 (0.04)	0.18 (0.03)	0.16 (0.03)	0.14 (0.02)	0.08 (0.02)	0.04 (0.02)
DI (GeoT w/o GFG)	0.18 (0.05)	0.16 (0.04)	0.15 (0.04)	0.14 (0.02)	0.08 (0.02)	0.04 (0.01)
DI (GeoT)	0.20 (0.01)	0.19 (0.01)	0.17 (0.02)	0.15 (0.003)	0.09 (0.004)	0.04 (0.002)

Test Results on CASP-CAPRI 13-14 Targets

Table 3: The average top- k precision on dimers from CASP-CAPRI 13 & 14.

Method	14 (Homo)			5 (Hetero)		
	10	$L/10$	$L/5$	10	$L/10$	$L/5$
BI	0	0	0	0.04	0	0.03
DH	0.02	0.02	0.02			
CC				0.06	0.08	0.05
DI (GCN)	0.12 (0.04)	0.11 (0.03)	0.13 (0.02)	0.10 (0.07)	0.11 (0.08)	0.09 (0.04)
DI (GT)	0.08 (0.03)	0.09 (0.05)	0.08 (0.03)	0.14 (0.02)	0.14 (0.02)	0.12 (0.03)
DI (GeoT w/o EPE)	0.11 (0.01)	0.12 (0.02)	0.11 (0.01)	0.18 (0.07)	0.20 (0.09)	0.18 (0.04)
DI (GeoT w/o GFG)	0.10 (0.02)	0.10 (0.02)	0.09 (0.02)	0.14 (0.03)	0.17 (0.03)	0.14 (0.02)
DI (GeoT)	0.18 (0.05)	0.13 (0.03)	0.11 (0.02)	0.30 (0.09)	0.31 (0.07)	0.24 (0.04)

Table 4: The average top- k precision and recall across all targets from CASP-CAPRI 13 & 14.

Method	19 (Both Types)					
	P@10	P@ $L/10$	P@ $L/5$	R@ L	R@ $L/2$	R@ $L/5$
BI	0.01	0	0.01	0.02	0.01	0.001
DI (GCN)	0.12 (0.04)	0.10 (0.05)	0.09 (0.04)	0.11 (0.001)	0.06 (0.01)	0.02 (0.01)
DI (GT)	0.10 (0.03)	0.09 (0.03)	0.08 (0.02)	0.11 (0.02)	0.06 (0.01)	0.02 (0.01)
DI (GeoT w/o EPE)	0.13 (0.02)	0.14 (0.03)	0.13 (0.02)	0.12 (0.01)	0.07 (0.01)	0.03 (0.01)
DI (GeoT w/o GFG)	0.11 (0.01)	0.12 (0.02)	0.10 (0.02)	0.11 (0.01)	0.06 (0.01)	0.03 (0.01)
DI (GeoT)	0.21 (0.01)	0.19 (0.01)	0.14 (0.01)	0.13 (0.02)	0.08 (0.01)	0.04 (0.003)

Test Results on Docking Benchmark 5 Targets

Table 5: The average top- k precision and recall on DB5 test targets.

55 (Hetero)						
Method	P@10	P@ L /10	P@ L /5	R@ L	R@ L /2	R@ L /5
BI	0	0.002	0.001	0.003	0.001	0.0004
CC	0.002	0.003	0.003	0.007	0.003	0.001
DI (GCN)	0.005 (0.002)	0.006 (0.001)	0.007 (0.001)	0.013 (0.002)	0.008 (0.001)	0.003 (0.001)
DI (GT)	0.008 (0.004)	0.008 (0.005)	0.008 (0.004)	0.010 (0.005)	0.006 (0.003)	0.003 (0.002)
DI (GeoT w/o EPE)	0.011 (0.004)	0.009 (0.004)	0.011 (0.002)	0.018 (0.01)	0.010 (0.004)	0.0034 (0.002)
DI (GeoT w/o GFG)	0.008 (0.001)	0.008 (0.001)	0.009 (0.002)	0.014 (0.01)	0.006 (0.002)	0.003 (0.001)
DI (GeoT)	0.013 (0.001)	0.009 (0.003)	0.011 (0.001)	0.018 (0.001)	0.010 (0.001)	0.0034 (0.001)

Disclaimer

- New **standardized** means of evaluating interface contact prediction methods are needed
- One way to do this may be to use structures generated by AlphaFold to curate a **new** benchmark dataset of homomeric and heteromeric targets

Conclusion

- We introduce the **Geometric Transformer** for predicting contact points in protein interfaces
- The Geometric Transformer lays the foundation for conducting **alternative** forms of message-passing on 3D graphs (e.g., treating edges as pseudo-nodes) and exploiting geometric feature gating to **enhance the expressiveness** of graph-like Transformers architectures

Extensions (1)

- We have already extended some ideas introduced with the **Geometric Transformer** (e.g., edge message-passing) to other problems in protein bioinformatics (i.e., **EnQA** for protein model quality assessment)

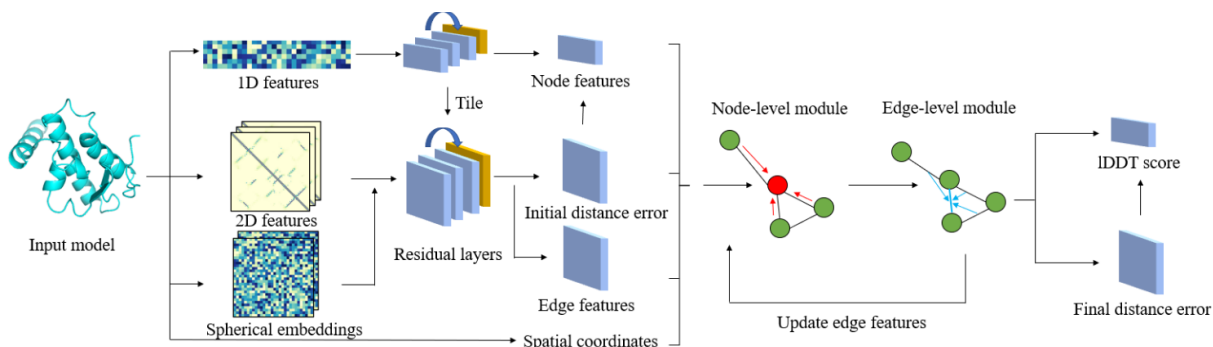


Figure 2. The illustration of the overall architecture of EnQA. The 1D/2D features from the input model are first converted into hidden node and edge features for the 3D-equivariant graph module. The spatial coordinates of Ca atoms of the residues are also used as an extra feature. The node and edge network modules update the graph features iteratively. In the end, the final per-residue IDDT score and distance errors of residue pairs are predicted from the updated node/edge features and spatial coordinates by the 3D-equivariant network.

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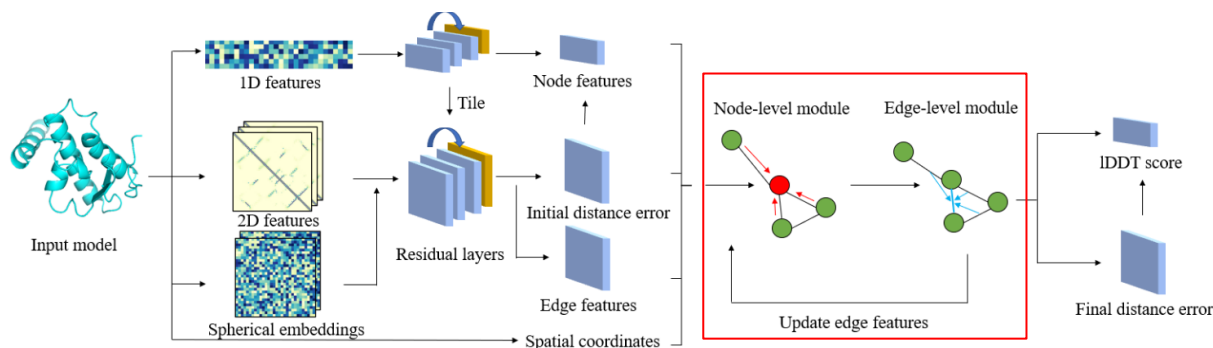


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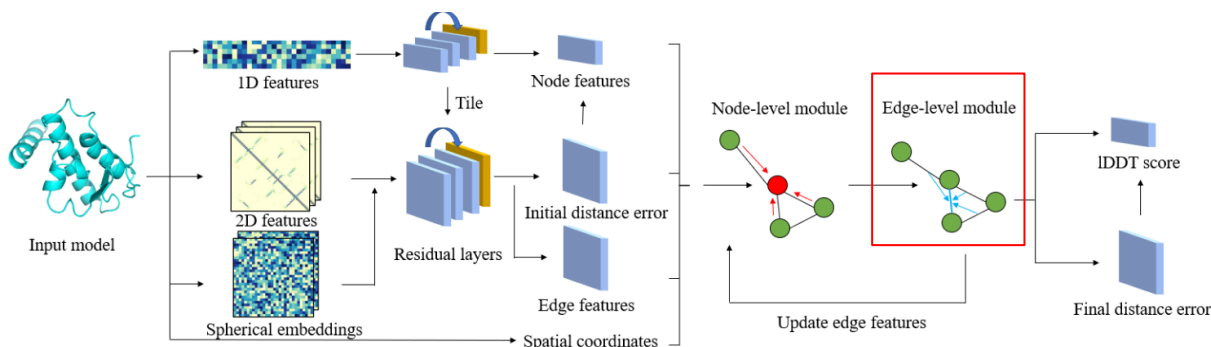
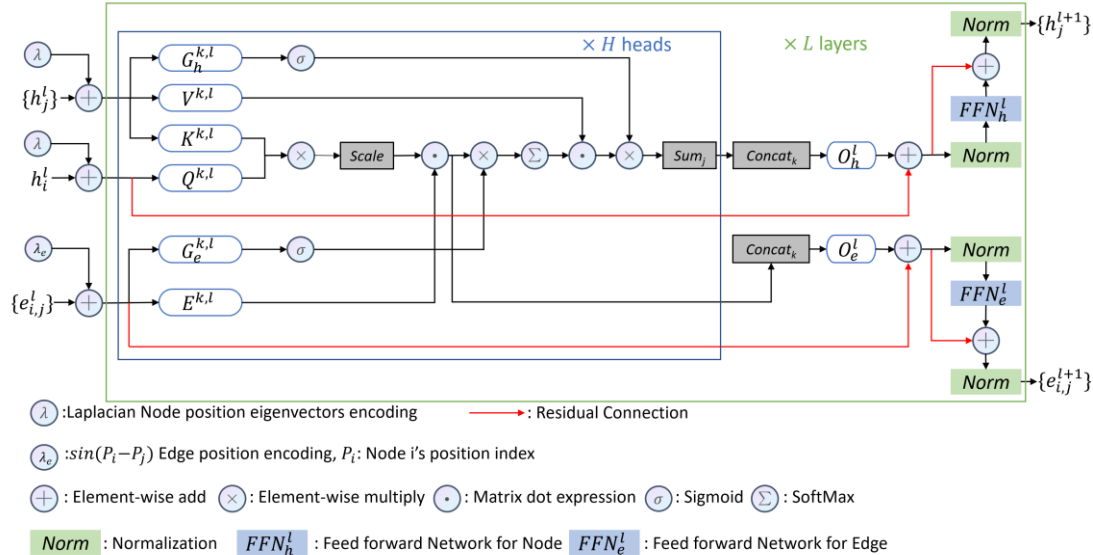


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Extensions (2)

- Likewise, we have also adapted the idea of graph-based feature gating in the **Geometric Transformer** to other problems in protein bioinformatics (i.e., **DPROQ** for quaternary protein model quality assessment)



Future Directions

- Ablating the CNN component of our DeepInteract pipeline and instead treating all chains as a **single** heterogeneous geometric graph
- Replacing the node-local self-attention mechanism in the Geometric Transformer with a node-global multi-head attention module - **long-range** interactions may play an important role in determining interface contact points, especially if modeling the complex as a single graph
- **Investigating the impact** of protein-protein interface interaction information on models designed for protein-protein, protein-ligand, or protein-DNA docking, just to name a few examples

Reproducibility

- Source code, data, and pre-trained models are **available** on GitHub at <https://github.com/BioinfoMachineLearning/DeepInteract>

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