## Critical Points in Quantum Generative Models

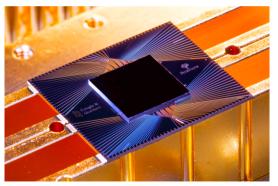
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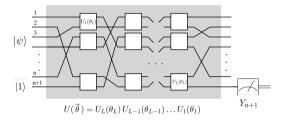
## Quantum Computing

- Quantum computers believed to be superpolynomially more efficient than "classical" computers in solving certain tasks
- Currently in an era of noisy, intermediate-scale quantum devices
- ► What can they do that's useful?



## Quantum Generative Models

▶ Quantum neural network¹—generalization of "classical" neural networks



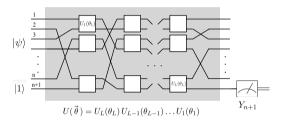
▶ Provably more expressive than classical counterparts<sup>2</sup>

<sup>&</sup>lt;sup>1</sup>Farhi and Neven 2018.

<sup>&</sup>lt;sup>2</sup>Gao et al. 2021.

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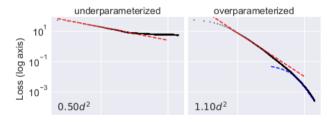
- ▶ Provably more expressive than classical counterparts<sup>2</sup>
- ► Are these models trainable?

<sup>&</sup>lt;sup>1</sup>Farhi and Neven 2018.

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# Untrainability Results

- Analytic results: gradient vanishes exponentially in model size for deep models<sup>3</sup>
- Numerical results: poor local minima quality in shallow models<sup>4</sup>, in contrast with typical behavior of local minima in neural networks<sup>5</sup>



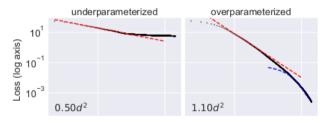
<sup>&</sup>lt;sup>3</sup>McClean et al. 2018; Cerezo et al. 2021.

<sup>&</sup>lt;sup>4</sup>Kiani, Lloyd, and Maity 2020.

<sup>&</sup>lt;sup>5</sup>Choromanska et al. 2015.

# Untrainability Results

- ► Analytic results: gradient vanishes exponentially in model size for deep models<sup>3</sup>
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Can these numerical results be proven?

<sup>&</sup>lt;sup>3</sup>McClean et al. 2018; Cerezo et al. 2021.

<sup>&</sup>lt;sup>4</sup>Kiani, Lloyd, and Maity 2020.

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#### Overview

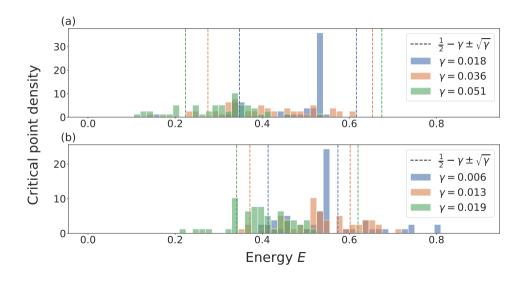
#### We show analytically:

The existence of this trainability phase transition at  $\gamma=1$ , governed by "order parameter"

$$\gamma \sim \frac{p}{2^{n+1}}$$

- ▶ The asymptotic distribution of local minima in these models
- Heuristic reasons why certain classes of models may not experience this poor quality of local minima

### **Numerical Confirmation**



#### Conclusion

- Even shallow quantum models can be difficult to train!
- ▶ Is there a way to show similar results for more general quantum models?

# Thank You!

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