Copula Conformal Prediction for Multi-step Time Series Forecasting

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Uncertainty Quantification for Time Series



Covid Forecasts. Patrick McGee / UT Southwestern 2021

Goal: "Cone of uncertainty" valid for all time steps of **y** $\mathbb{P}[\forall h \in \{1, ..., k\}, \mathbf{y}_{t+h} \in \Gamma_h^{1-\alpha}] \ge 1 - \alpha$

Conformal prediction

- Works for *any* underlying prediction model \bullet
- Works for *any* underlying data distribution
- Guaranteed validity in finite samples
- (Under mild assumptions)



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Overview of Conformal Prediction

Training data





Deploy f

Overview of Conformal Prediction

(proper) Training data



Calibration data

Test

Deploy f, Γ

- Run *f*
- Get nonconformity scores e.g. ||y f(x)||
- Calibrate confidence region Γ with scores

Overview of Conformal Prediction

(proper) Training data

Definition 1: Validity Guarantee

Given a test data pair (X, Y) and a desired coverage rate $1 - \alpha \in (0, 1)$, a confidence region $\Gamma^{1-\alpha} : \mathbf{X} \to \{\text{subsets of } \mathbf{Y}\}$ is *valid*.

Calibration data



 $\mathbb{P}(Y \in \Gamma^{1-\alpha}(X)) \ge 1 - \alpha$

UQ for Time Series Setting



Covid Forecasts. Patrick McGee / UT Southwestern 2021

Goal: "Cone of uncertainty" valid for all time steps of y $\mathbb{P}[\forall h \in \{1, \dots, k\}, \mathbf{y}_{t+h} \in \Gamma_h^{1-\alpha}] \ge 1 - \alpha$

UQ for Time Series Setting

Time steps: temporal dependence (non-i.i.d.)



Dataset $\mathcal{D} =$

Dataset \mathcal{D} : independent observations (i.i.d.)

$$\{(\mathbf{X}_{1:t}^{(i)}, \mathbf{y}_{t+1:t+k}^{(i)}\}_{i=1}^{n}$$

Goal: "Cone of uncertainty" valid for all time steps of y $\mathbb{P}[\forall h \in \{1, \dots, k\}, \mathbf{y}_{t+h} \in \Gamma_h^{1-\alpha}] \ge 1 - \alpha$

Producing *k*-step-valid confidence regions

Naive solution: Bonferroni correction (aka. Union bounding) [2]

[2] Stankeviciute, Kamile, Ahmed M Alaa, and Mihaela van der Schaar. "Conformal time-series forecasting." NeurIPS, 2021

- Find the $(1 \alpha/k)$ confidence region for each of the k time steps.
 - Then by Boole's inequality:
 - $\mathbf{P}[\exists h \in \{1, \dots, k\}, \mathbf{y}_{t+h} \notin \Gamma_h^{1-\alpha}] \leq \alpha$
 - Problem: when k is large, this is very inefficient.



Copula Conformal Prediction for Time Series



$$C(u_1, \cdots, u_k) = \mathbb{P}(U_1 \le u_1, \cdots, U_k \le u_k)$$

A copula is a function that synthesizes multiple CDFs to a joint CDF

- $F(x_1, \dots, x_k) = C(F_1(x_1), \dots, F_k(x_k))$ (Sklar's theorem)
- For joint coverage guarantees, we only have to bound the Copula.

Copula Conformal Prediction

(proper) Training data



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We prove that it also has finite-sample **Copula Conformal Prediction**

Theorem 4.1 (Validity of CopulaCPTS). CopulaCPTS (algorithm 1) produces valid confidence regions for the entire forecast horizon. i.e.

 $\mathbb{P}[\forall j \in \{1,\ldots,k\}]$



$$x\}, y_j \in \Gamma_j^{1-\alpha}] \ge 1-\alpha.$$

Use conformal prediction to get each time step's $\mathsf{CDF}\,\hat{F}_h$

Calibrate Copula $C(\hat{F}_1(x_1), \cdots, \hat{F}_k(x_k))$ Use C to find appropriate Γ





Deploy f, Γ

Experiment datasets Synthetic





Drone trajectory following + Gaussian noise

Real-world

Positive COVID-19 cases reported in England (7-day rolling average). Data for the most recent 7 days is incomplete and is not shown.

Up to and including 3 April 2024





Argoverse Motion Forecast Dataset

New Covid cases (380 regions, 500 days)



Results: calibration and sharpness



Results: examples

Argoverse autonomous vehicles dataset

Results: Over different horizon length

Takeaway

- We can use copula functions to better capture uncertainty propagation through time.
- At the cost of having an additional copula calibration dataset, we achieve much sharper confidence intervals.

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 Improvement is more significant for higher dimension data and longer prediction horizons.

Code is open sourced at github.com/Rose-STL-Lab/CopulaCPTS

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Thank you!

If you'd like to chat or collaborate, reach me at <u>shs066@ucsd.edu</u> or on X @huiwensun_ :)