ICLR

Analyzing and Improving Optimal-Transport-based Adversarial Networks

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Overview

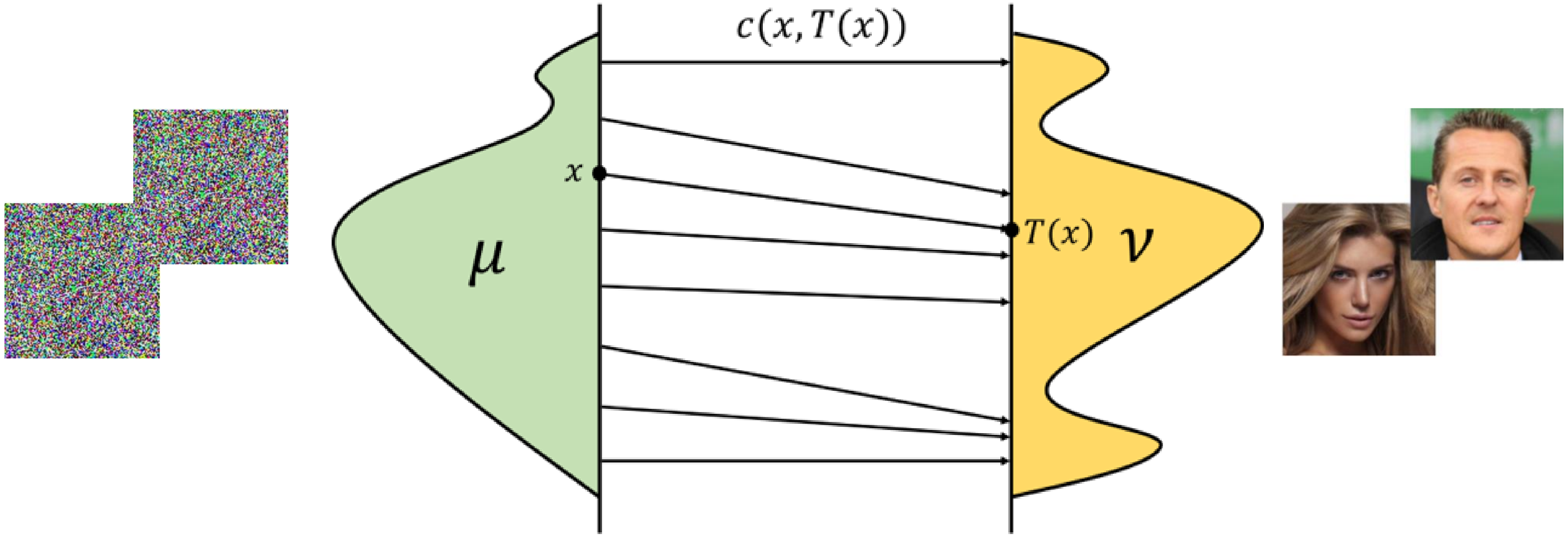
- Unify various adversarial algorithms through **Unbalanced Optimal Transport Model (UOTM)**

- Compare and analyze properties of adversarial algorithms through the unified perspective

- Improve adversarial algorithms based on our analysis

Notations

- Throughout the presentation, μ and ν is source (prior) and target (data) distribution, respectively.
- c is a quadratic (transport) cost functional, i.e. $c(x, y) = \tau \parallel x - y \parallel^2$.



Preliminaries

- **UOT problem relaxes the hard constraint on marginal distributions into soft penalization.**

OT :

$$C(\mu,\nu) := \inf_{\pi \in \Pi(\mu,\nu)} \left[\int_{\mathcal{X}\times\mathcal{Y}} c(x,y)\, d\pi(x,y) \right].$$

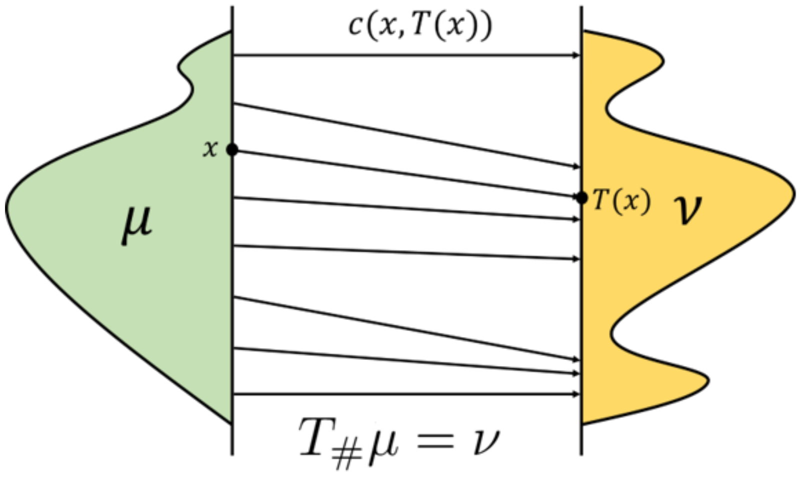
Hard Constraint

UOT :

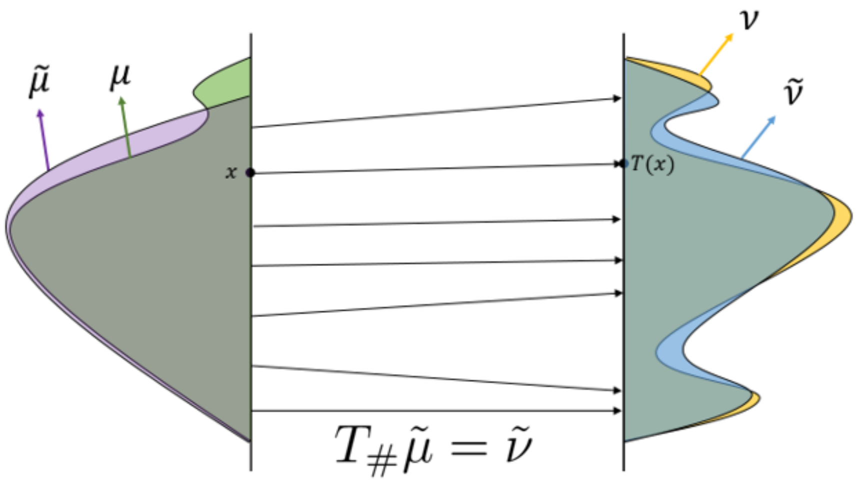
$$C_{ub}(\mu,\nu) := \inf_{\pi \in \mathcal{M}_+(\mathcal{X}\times\mathcal{Y})} \left[\int_{\mathcal{X}\times\mathcal{Y}} c(x,y)\, d\pi(x,y) + D_{\Psi_1}(\pi_0|\mu) + D_{\Psi_2}(\pi_1|\nu) \right].$$

Positive Measure

with Soft Penalization



OTM



UOTM

Preliminaries

Let $g_i(x) = -\Psi_i^*(-x)$ for simplicity. Note that $\boldsymbol{c(x, y) = \tau\|x - y\|^2}$.

Primal :

$$\inf_{\pi \in \mathcal{M}_+(\mathcal{X}\times\mathcal{Y})} \left[\int_{\mathcal{X}\times\mathcal{Y}} c(x,y)\, d\pi(x,y) + D_{\Psi_1}(\pi_0|\mu) + D_{\Psi_2}(\pi_1|\nu) \right]$$

Semi-dual :

$$\sup_{v_\phi} \left[\int_{\mathcal{X}} g_1 \left(\inf_{T_\theta} [c\,(x, T_\theta(x)) - v_\phi\,(T_\theta)] \right) d\mu(x) + \int_{\mathcal{Y}} g_2(v_\phi(y))\, d\nu(y) \right]$$

[1] J. Choi, J. Choi, M. Kang. “Generative Modeling through the Semi-dual Formulation of Unbalanced Optimal Transport." *NeurIPS*, 2023.

Unified View of OT-based Adversarial Networks

- Let $g_i(x) = -\Psi_i^*(-x)$ for simplicity.

Primal : $$\inf_{\pi \in \mathcal{M}_+(\mathcal{X}\times\mathcal{Y})} \left[\int_{\mathcal{X}\times\mathcal{Y}} c(x,y)\, d\pi(x,y) + D_{\Psi_1}(\pi_0|\mu) + D_{\Psi_2}(\pi_1|\nu) \right]$$

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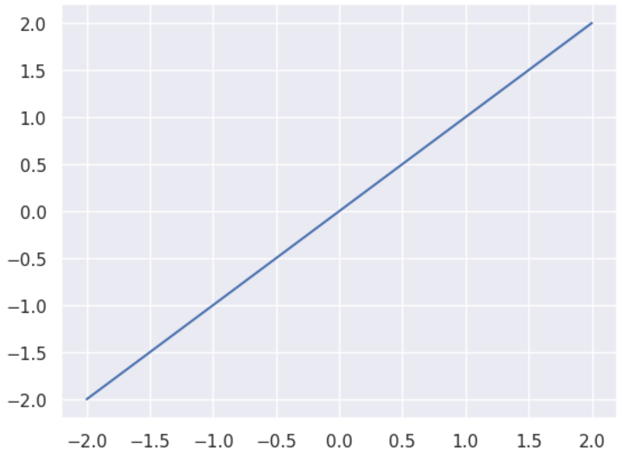
D_Ψ

$$D_\Psi(P|Q) = \begin{cases} 0, & \text{if } P = Q \ a.e. \\ \infty, & \text{else} \end{cases}$$

Convex Indicator

→ **OTM**

g



Unified View of OT-based Adversarial Networks

- Let $g_i(x) = -\Psi_i^*(-x)$ for simplicity.

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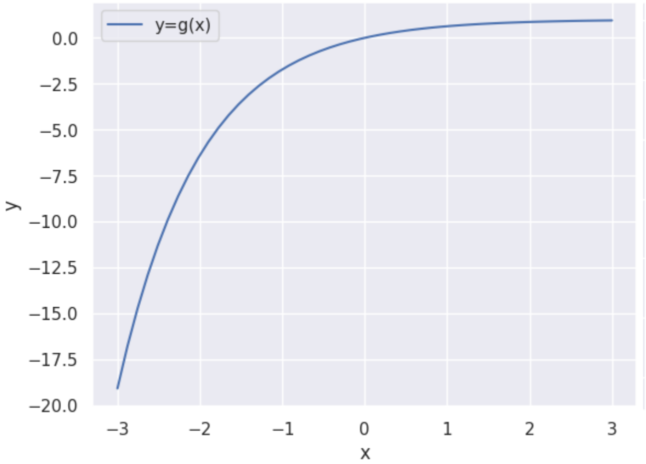
D_Ψ | g

KL divergence



χ^2 **divergence**

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	$g_1 = g_2 = \mathrm{Id}$	$g_1 = \mathrm{Id},\ g_2 = \mathrm{Ccv}$	$g_1 = g_2 = \mathrm{Ccv}$
$c \equiv 0$	WGAN [3]	f-GAN [4]	UOTM w/o cost
$c \neq 0$	OTM [2]	Source-fixed-UOTM	UOTM [1]

[1] J. Choi, J. Choi, M. Kang. “Generative Modeling through the Semi-dual Formulation of Unbalanced Optimal Transport." *NeurIPS*, 2023.
[2] L. Rout, A. Korotin, E. Burnaev. “Generative Modeling with Optimal Transport Maps”, *ICLR*, 2022.
[3] M. Arjovsky, S. Chintala, L. Bottou, “Wasserstein generative adversarial networks”, *ICML*, 2017.
[4] S. Nowozin, B. Cseke, R. Tomioka, “f-GAN: Training Generative Neural Samplers using Variational Divergence Minimization”, *NeurIPS*, 2016.

Properties of OT-based Adversarial Networks

Note that $c(x, y) = \tau \|x - y\|^2$.

$$\sup_{v_\phi} \left[\int_{\mathcal{X}} g_1 \left(\inf_{T_\theta} \left[c\left(x, T_\theta(x)\right) - v_\phi\left(T_\theta\right) \right] \right) d\mu(x) + \int_{\mathcal{Y}} g_2(v_\phi(y)) d\nu(y) \right]$$

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- **The presence of cost function $c(\cdot,\cdot)$ mitigates mode collapse. [2]**
- **The strictly concave $g_1 \& g_2$ helps stabilizing training. [2]**
- **UOT-based algorithms offers more outlier robustness. [1]**

[1] J. Choi, J. Choi, M. Kang. “Generative Modeling through the Semi-dual Formulation of Unbalanced Optimal Transport." *NeurIPS*, 2023.
[2] J. Choi, J. Choi, M. Kang. “Analyzing and Improving Optimal-Transport-based Adversarial Netoworks." *ICLR*, 2024.

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$$\sup_{v_\phi} \left[\int_{\mathcal{X}} g_1 \left(\inf_{T_\theta} [c(x, T_\theta(x)) - v_\phi(T_\theta)] \right) d\mu(x) + \int_{\mathcal{Y}} g_2(v_\phi(y)) d\nu(y) \right]$$

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- **For UOTM, under some regularity condition, there exists unique Lipschitz continuous optimal potential $v^\star$. Moreover, the collection of c-convex potential of UOTM which has a negative loss satisfies equi-Lipschitzness. [2]**

[1] J. Choi, J. Choi, M. Kang. "Generative Modeling through the Semi-dual Formulation of Unbalanced Optimal Transport." *NeurIPS*, 2023.
[2] J. Choi, J. Choi, M. Kang. "Analyzing and Improving Optimal-Transport-based Adversarial Netoworks." *ICLR*, 2024.

Properties of OT-based Adversarial Networks

- **Desirable properties of UOTMs**
  - Stabilize training (∵ concave g_1 and g_2)
  - Prevent mode collapse (∵ cost function c)
  - Robust to outliers (∵ soft marginal penalization)
  - Lipschitzness of potentials

- **Limitation of UOTMs**
  - Distributional matching error (∵ soft marginal penalization)

$$\inf_{\pi \in \mathcal{M}_+(\mathcal{X}\times\mathcal{Y})} \left[\int_{\mathcal{X}\times\mathcal{Y}} c(x,y)\, d\pi(x,y) + D_{\Psi_1}(\pi_0|\mu) + D_{\Psi_2}(\pi_1|\nu) \right]$$

Soft Penalization

Improving OT-based Adversarial Networks

We introduce new hyperparameter α_1 and α_2. We gradually increase these hyperparameters while training.

Start training with UOTM algorithm (Small α_1&α_2) → Gradually increase α_1&α_2 for better distribution matching

$$\inf_{\pi \in \mathcal{M}_+} \left[\int c(x,y) d\pi(x,y) + \alpha_1 D_{\Psi_1}(\pi_0|\mu) + \alpha_2 D_{\Psi_2}(\pi_1|\nu) \right]$$

Thank you!