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Compressing Latent Space via Least Volume

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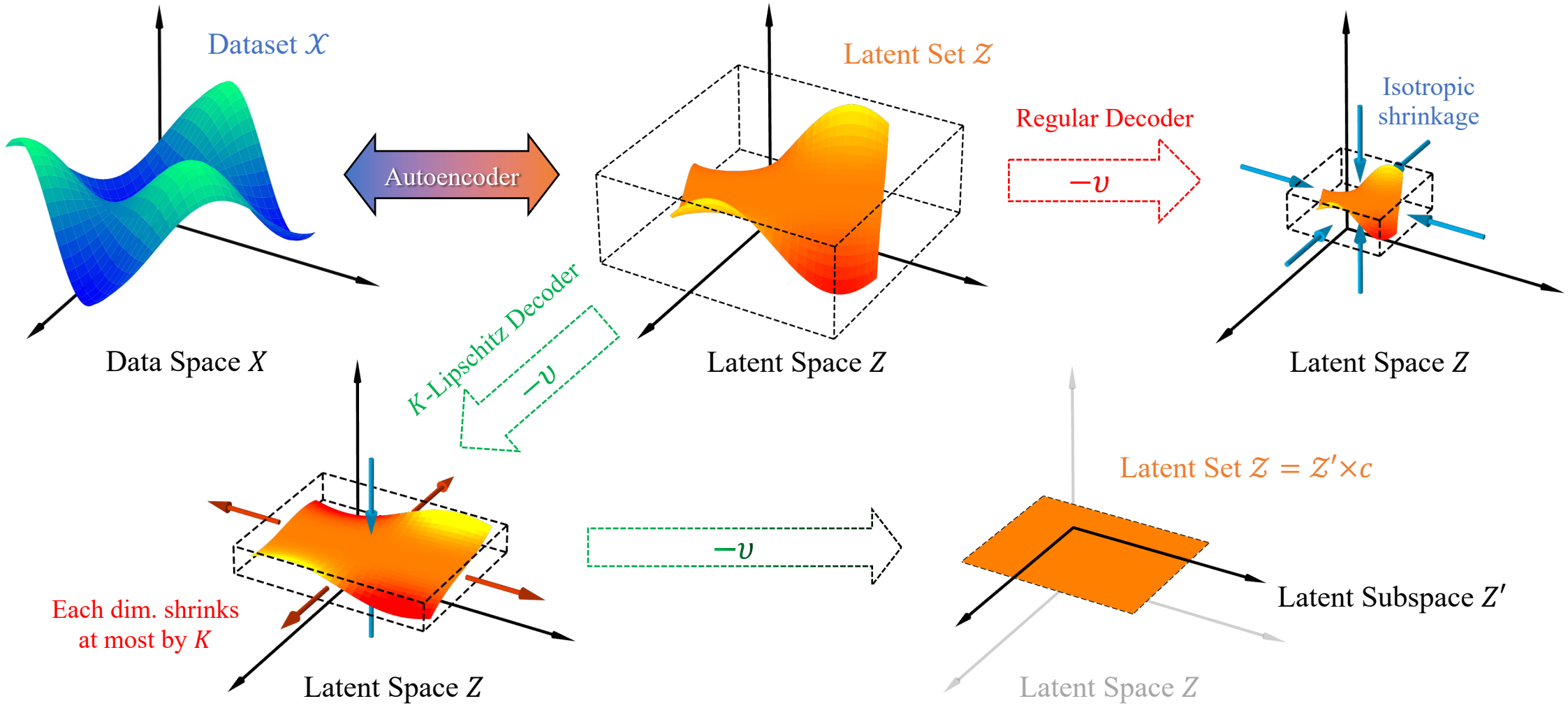
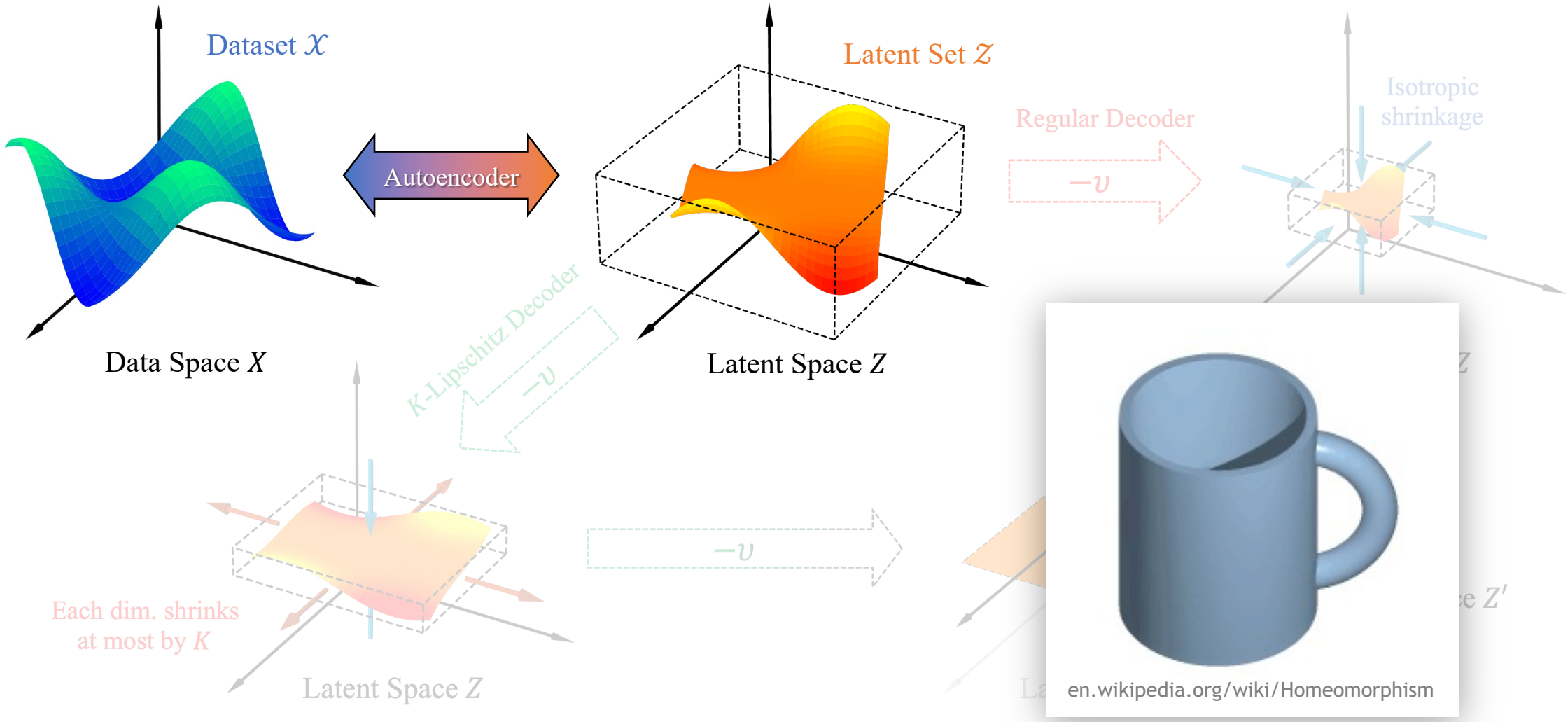
Motivation

- Real-world high dimensional datasets may live on low dimensional nonlinear manifolds.
- Retrieving the least dimensional representations of the datasets could be beneficial:
  - It may alleviate the curse of dimensionality the most.
  - It may improve some downstream tasks, *e.g.*, data generation.
  - It may let us study the dataset's structure more easily.

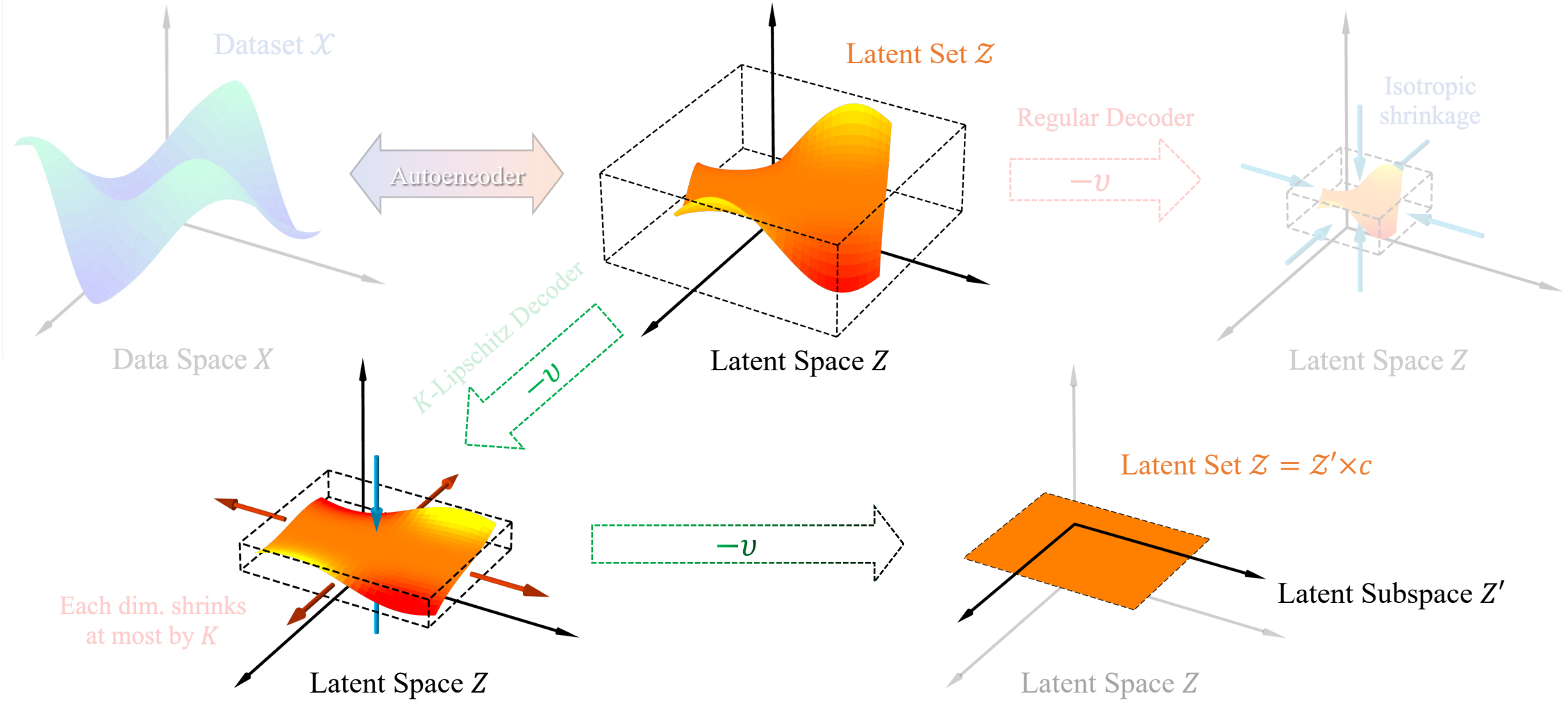
Goal: Obtain the least dimensional representation *automatically*.

Methodology: Least Volume

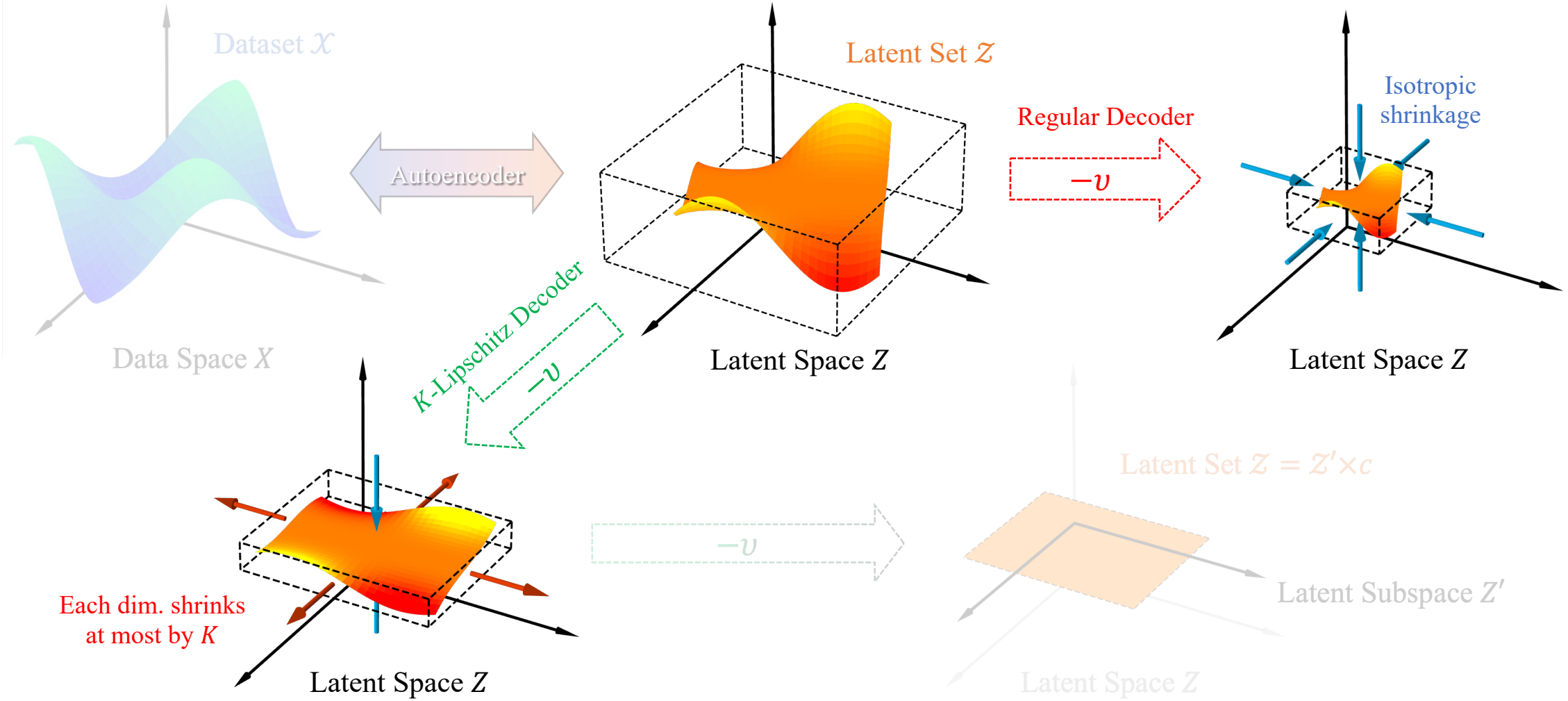




A *continuous* autoencoder's latent set $\mathcal{Z}$ is *homeomorphic* to the dataset $\mathcal{X}$.



A flattened latent set consumes less space $v = \prod_i \sigma_i$ than a curved one.



A K-Lipschitz decoder prevents the latent set's isotropic shrinkage.

Least Volume Formulation

1. Make the decoder g K-Lipschitz with *spectral normalization*.
2. Minimize the loss function with the volume penalty:

$$L = J + \lambda L_{\text{vol}}$$

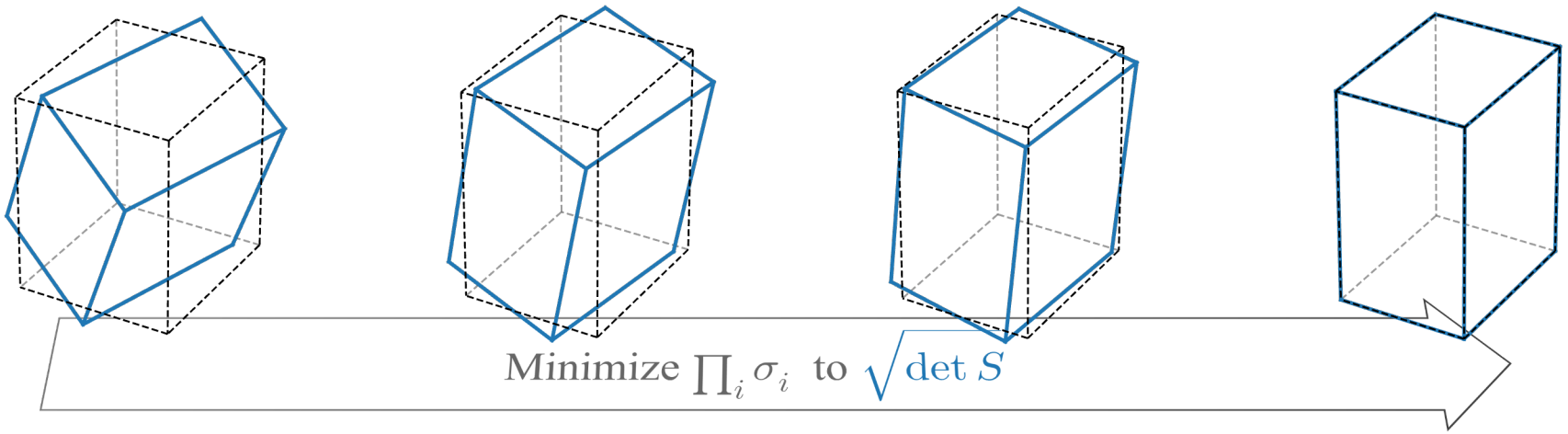
Reconstruction Loss: $J = \mathbb{E}\|g \circ e(x) - x\|$

Volume Penalty: $L_{\text{vol}} = \sqrt[m]{\prod_i (\sigma_i + \eta)}$ ⇠ $v = \prod_i \sigma_i$

We can then reduce the autoencoder's latent dimension *automatically*.

PCA is a Linear Special Case of Least Volume

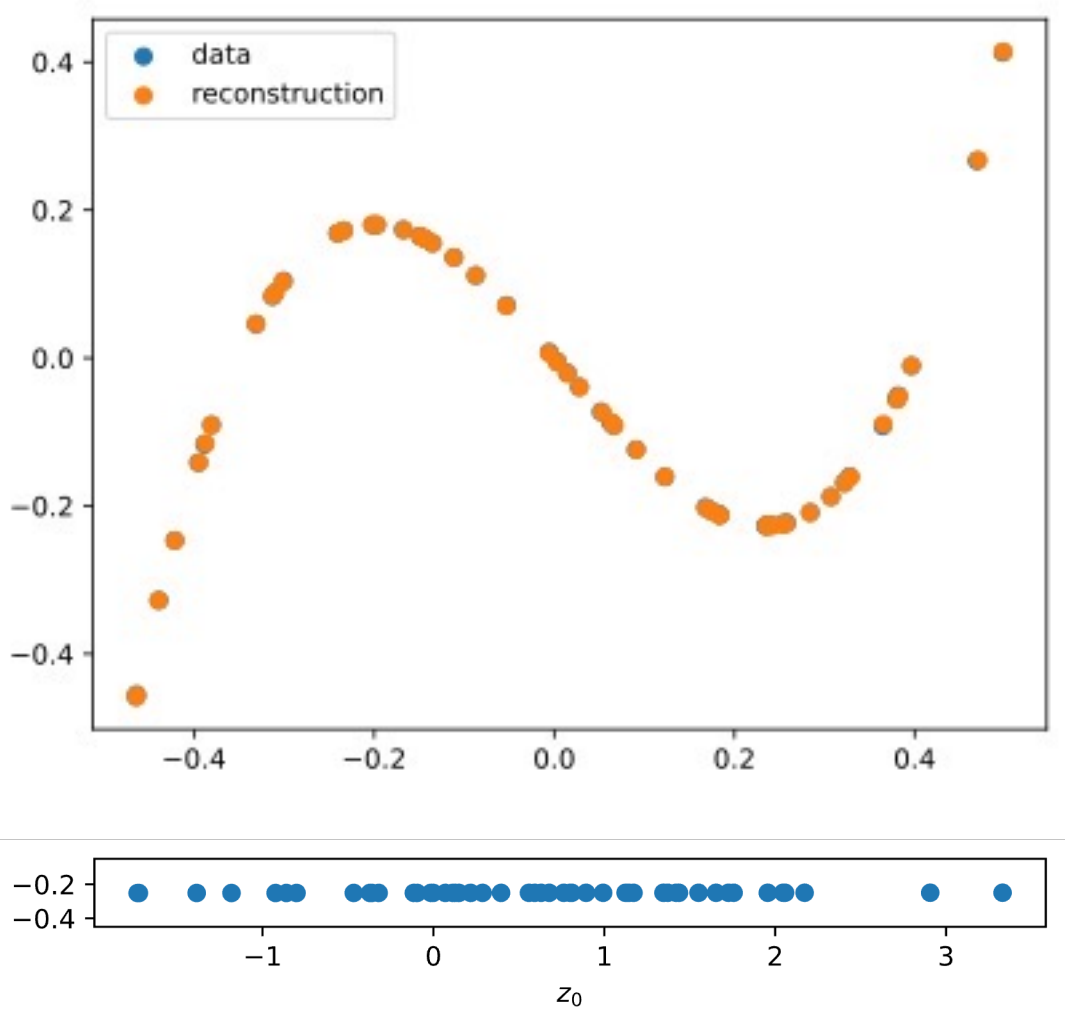
- $v = \prod_i \sigma_i$ is a tight upper-bound of $\sqrt{\det S} = \sqrt{\det \mathrm{Cov}(\mathcal{Z})}$.

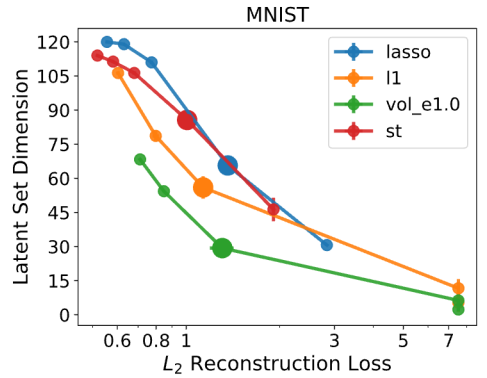


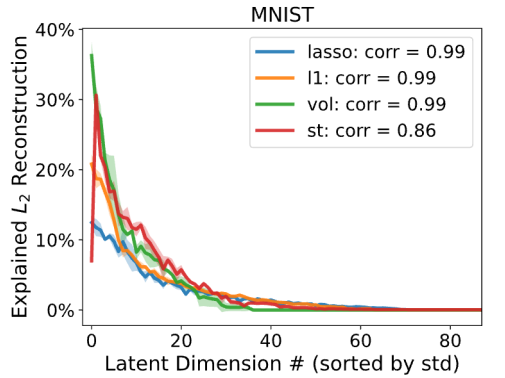
- If the autoencoder is linear, then least volume turns it into PCA.

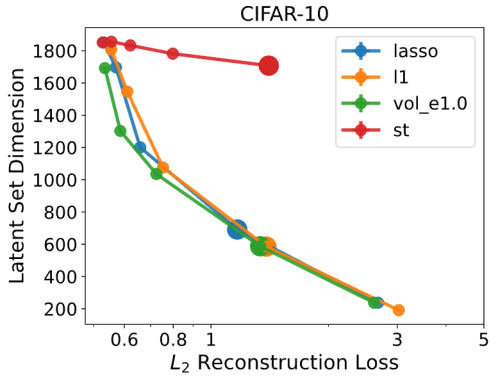
Nonlinear autoencoders may have a similar ordering effect.

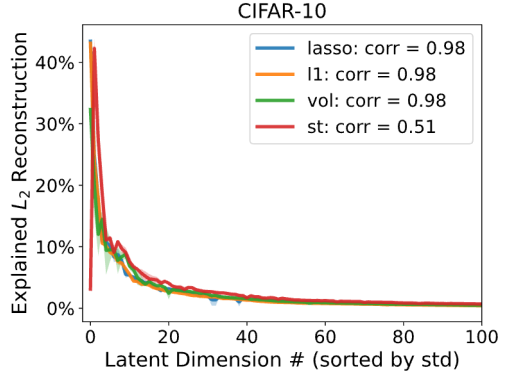
Experiment Results

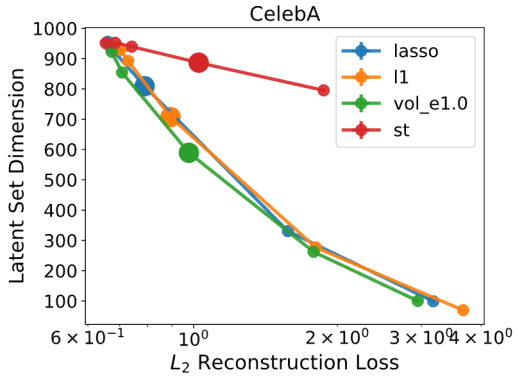


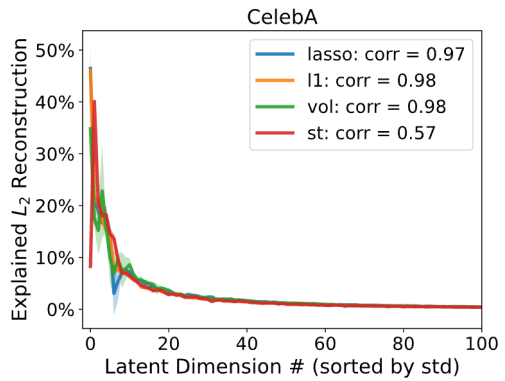




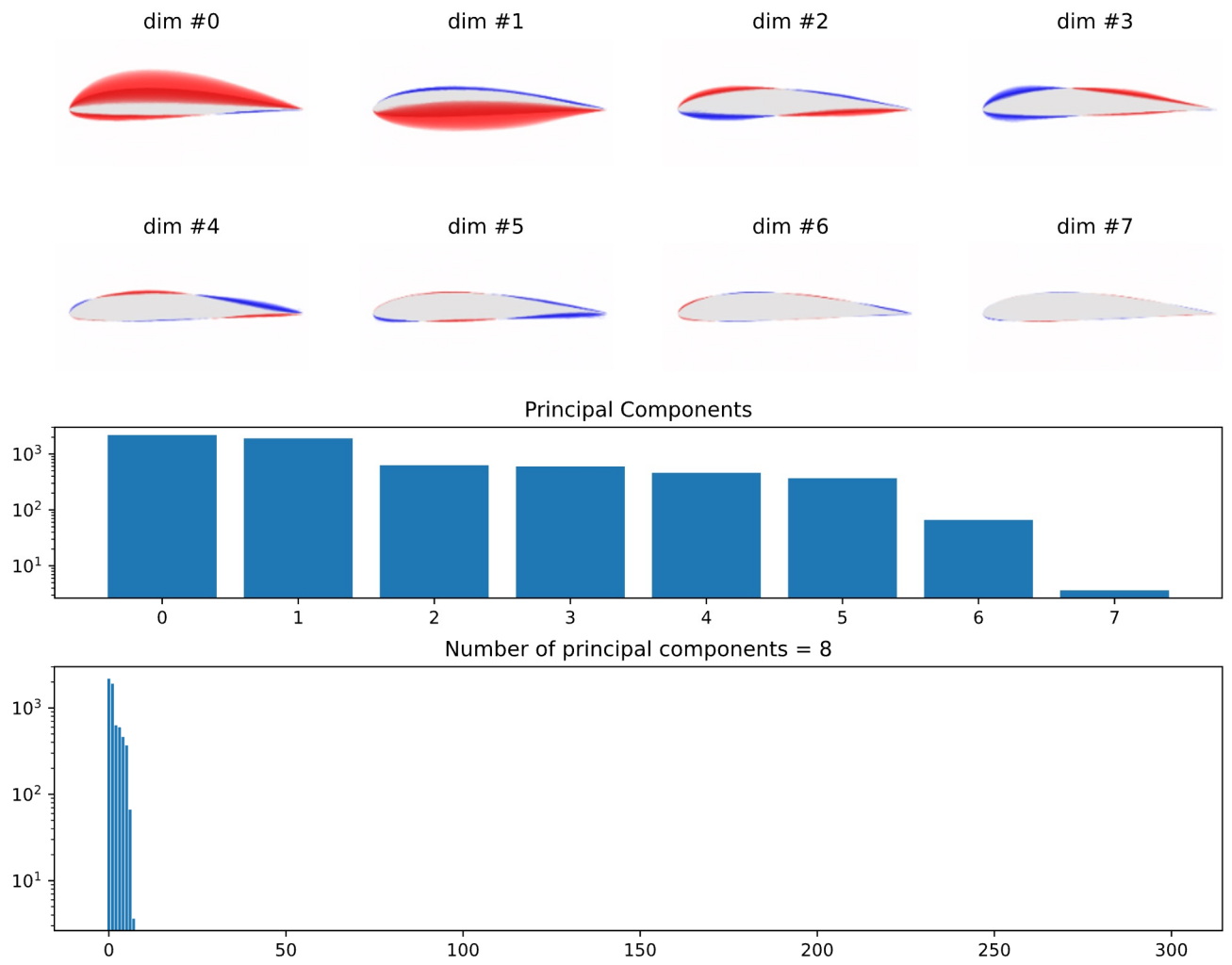


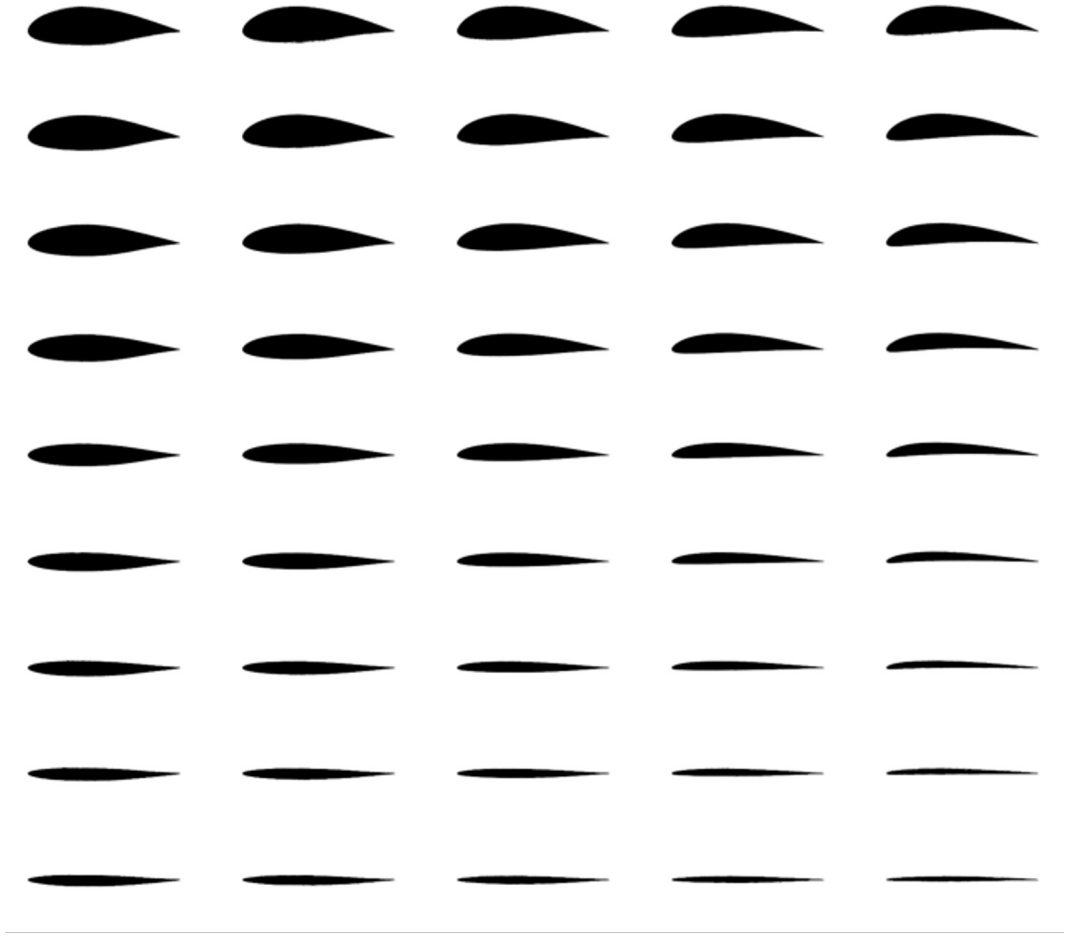




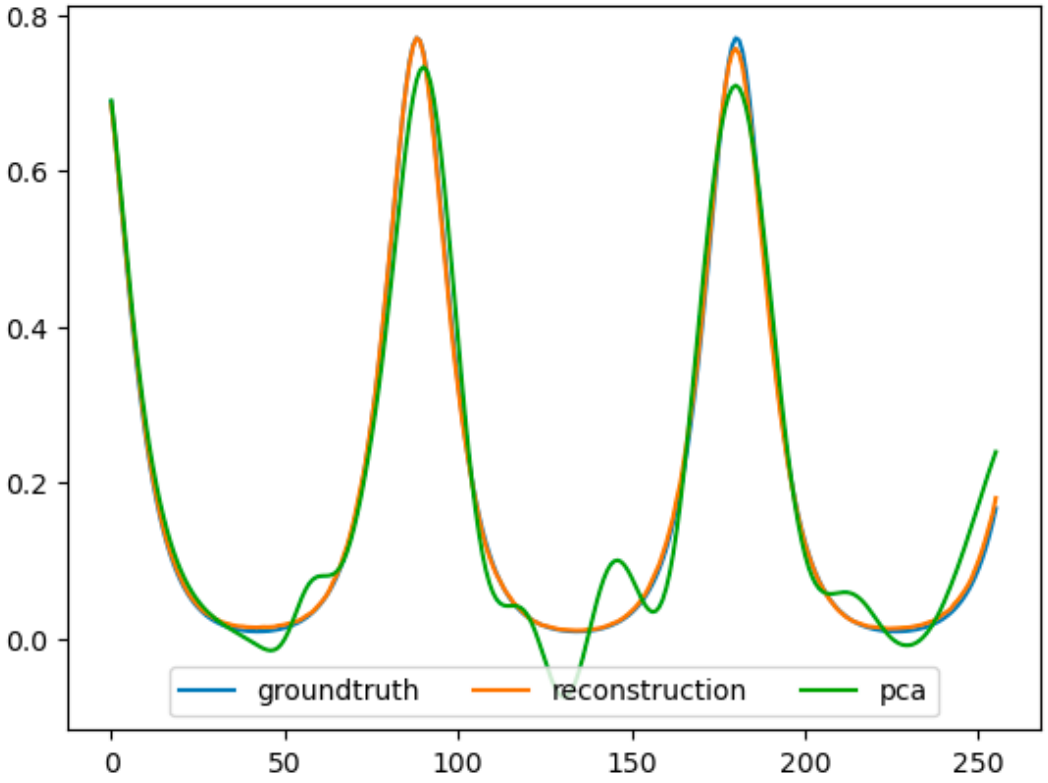


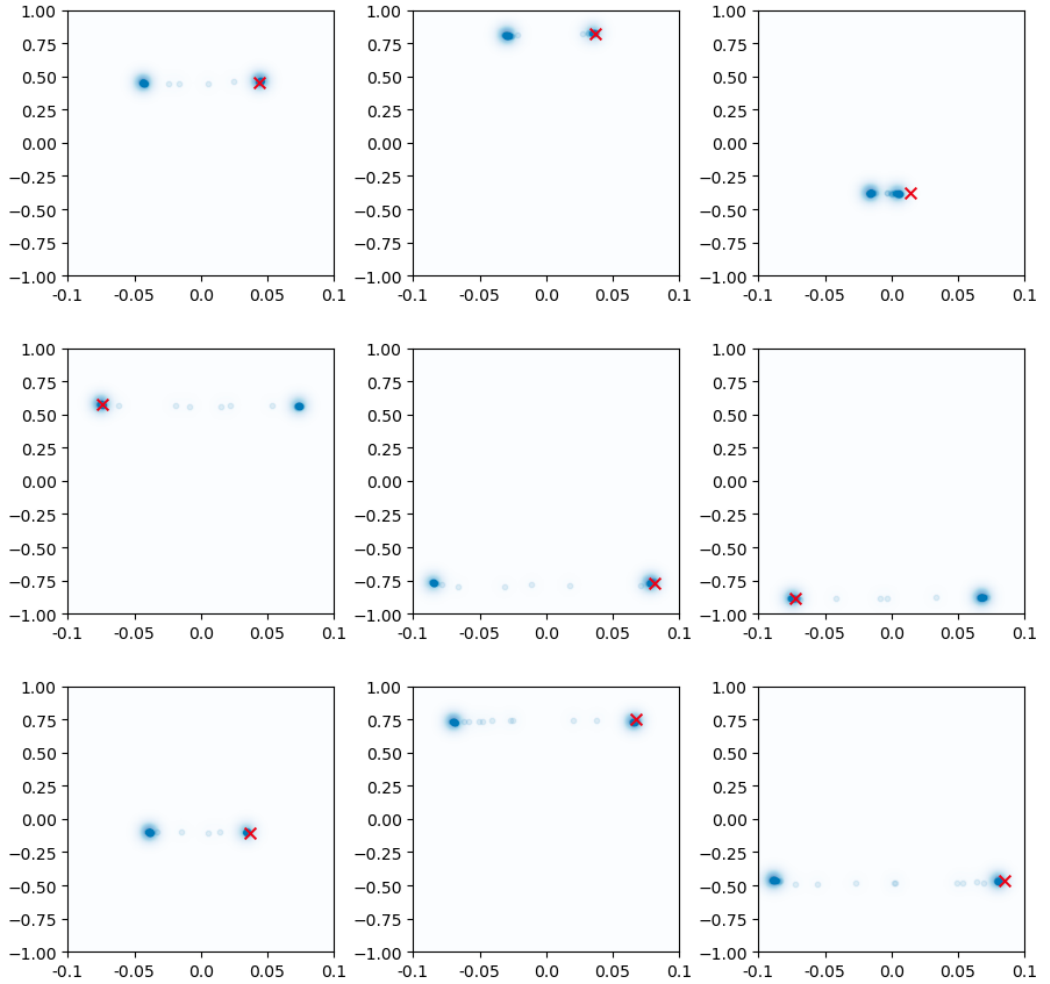
Application: Design Analysis & Parameterization





Application: Time Series Analysis & System Identification





Thanks for watching!

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GitHub: IDEALLab/Least_Volume_ICLR2024

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