On Error Propagation of Diffusion Models

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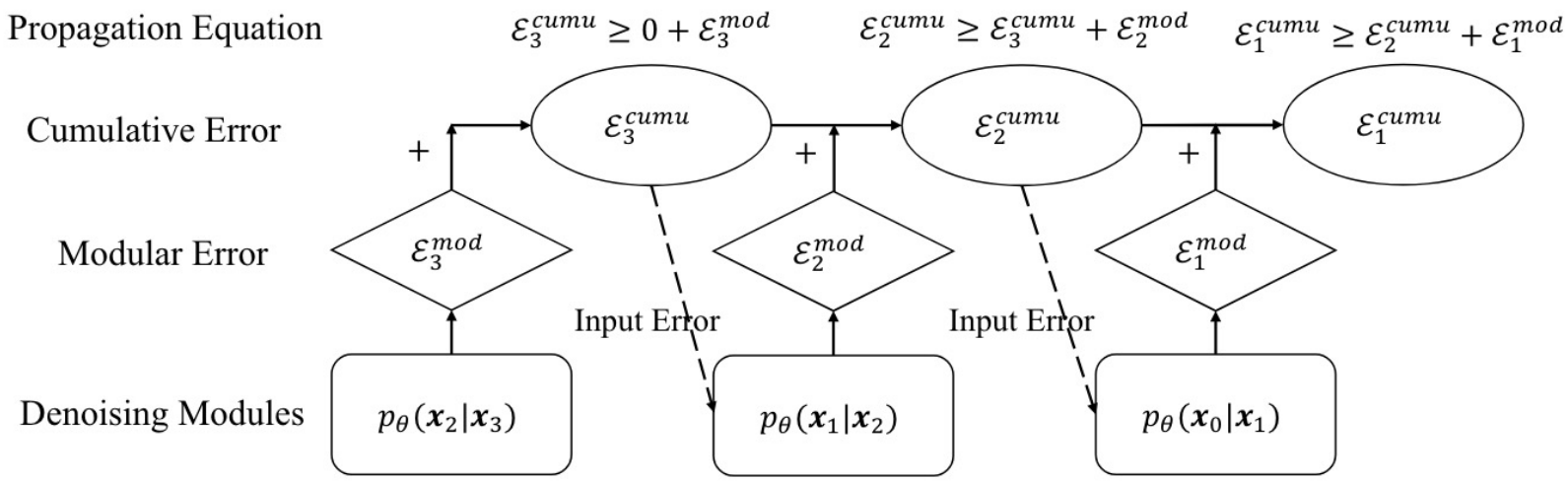
Outline

- Structural Risk of Diffusion Models
- Cumulative Error Estimation
- Method: Regularization with Cumulative Errors
- Experiments on Image Generation

Structural Risk – Part1

- Chain structure might lead to error propagation
  - Why this is not for sure? $\mathcal{E}_t^{\text{cumu}} - \mathcal{E}_t^{\text{mod}} = \mu_t \mathcal{E}_{t+1}^{\text{cumu}}$,



Structural Risk – Part2

- A more solid explanation

Definition of modular errors $$\mathcal{E}_t^{\mathrm{mod}} = \mathbb{E}_{\mathbf{x}_t \sim p_\theta(\mathbf{x}_t)}[D_{\mathrm{KL}}(p_\theta(\mathbf{x}_{t-1} \mid \mathbf{x}_t) \,||\, q(\mathbf{x}_{t-1} \mid \mathbf{x}_t))].$$

Definition of cumulative errors $$\mathcal{E}_t^{\mathrm{cumu}} = D_{\mathrm{KL}}(p_\theta(\mathbf{x}_{t-1}) \,||\, q(\mathbf{x}_{t-1})).$$

Our theorem: propagation equation $$\mathcal{E}_t^{\mathrm{cumu}} \geqslant \mathcal{E}_{t+1}^{\mathrm{cumu}} + \mathcal{E}_t^{\mathrm{mod}},$$

Empirical Evaluation

- Alternative measure with MMD $\frac{1}{4}\mathcal{D}_t^{\text{cumu}} \leqslant \mathcal{E}_t^{\text{cumu}} \leqslant \mathcal{D}_t^{\text{cumu}}.$

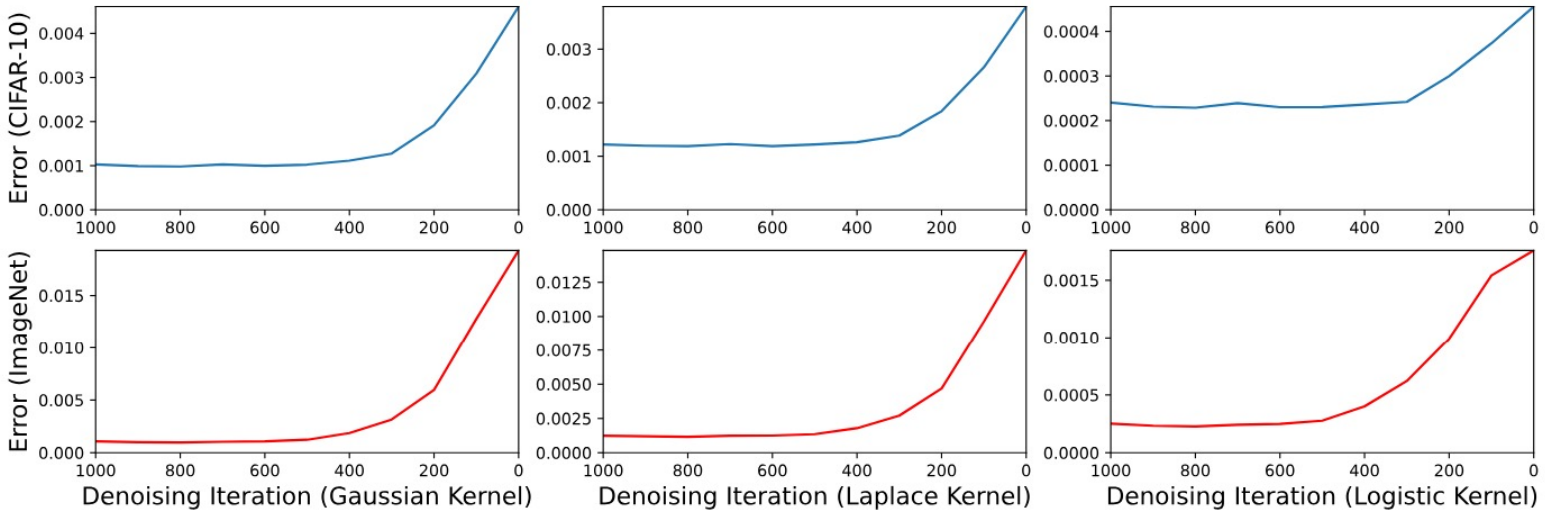


Figure 2: Uptrend dynamics of the MMD error $\mathcal{D}_t^{\text{cumu}}$ w.r.t. decreasing iteration t. The cumulative error $\mathcal{E}_t^{\text{cumu}}$ might show similar behaviors since it is tightly bounded by the MMD error.

Method

- Imposing a regularization $\mathcal{L}_t^{\text{reg}} = \mathcal{D}_t^{\text{cumu}}, \quad \mathcal{L}^{\text{reg}} = \sum_{t=0}^{T-1} w_t \mathcal{L}_t^{\text{reg}},$

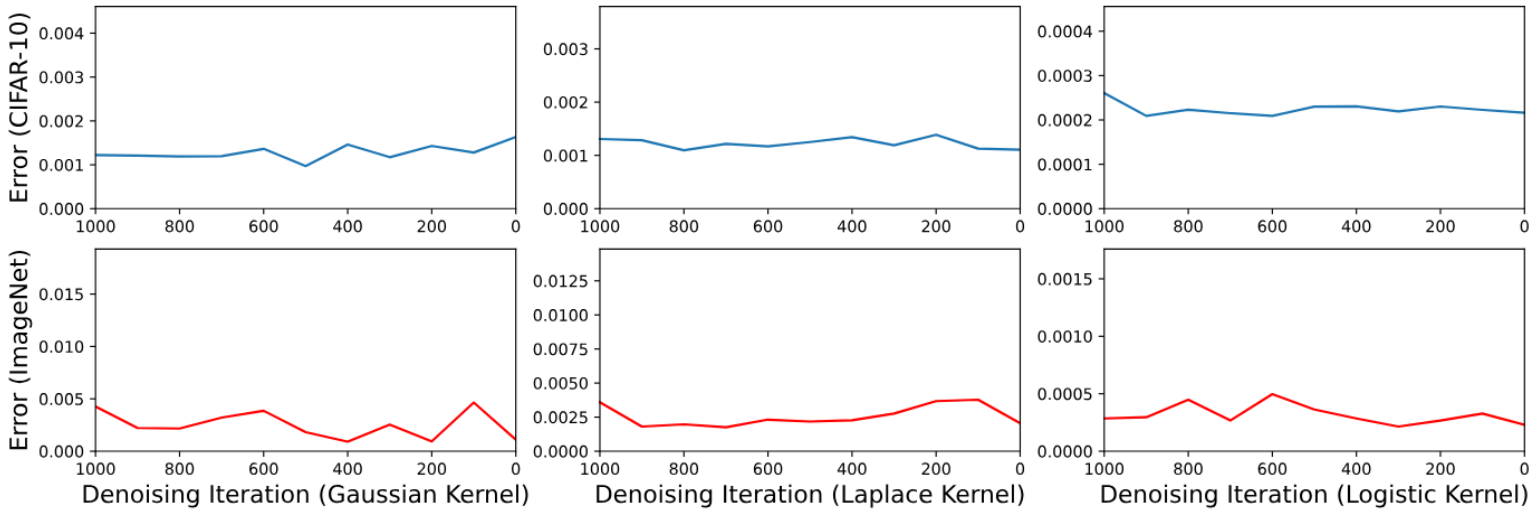


Figure 3: Re-estimated dynamics of the MMD error $\mathcal{D}_t^{\text{cumu}}$ with respect to decreasing iteration t after applying our proposed regularization. These dynamics should be compared with those in Fig. 2, showing that we have well handled error propagation.

Experiments

Approach	CIFAR-10	ImageNet	CelebA
ADM-IP Ning et al. (2023)	3.25	2.72	1.31
DDPM Ho et al. (2020)	3.61	3.62	1.73
DDPM w/ Consistent DM (Daras et al., 2023)	3.31	3.16	1.38
DDPM w/ FP-Diffusion (Lai et al., 2022)	3.47	3.28	1.56
DDPM w/ Our Proposed Regularization	**2.93**	**2.55**	**1.22**

Table 1: FID scores of our model and baselines on different image datasets. The improvements of our approach over baselines are statistically significant with $p < 0.01$ under t-test.