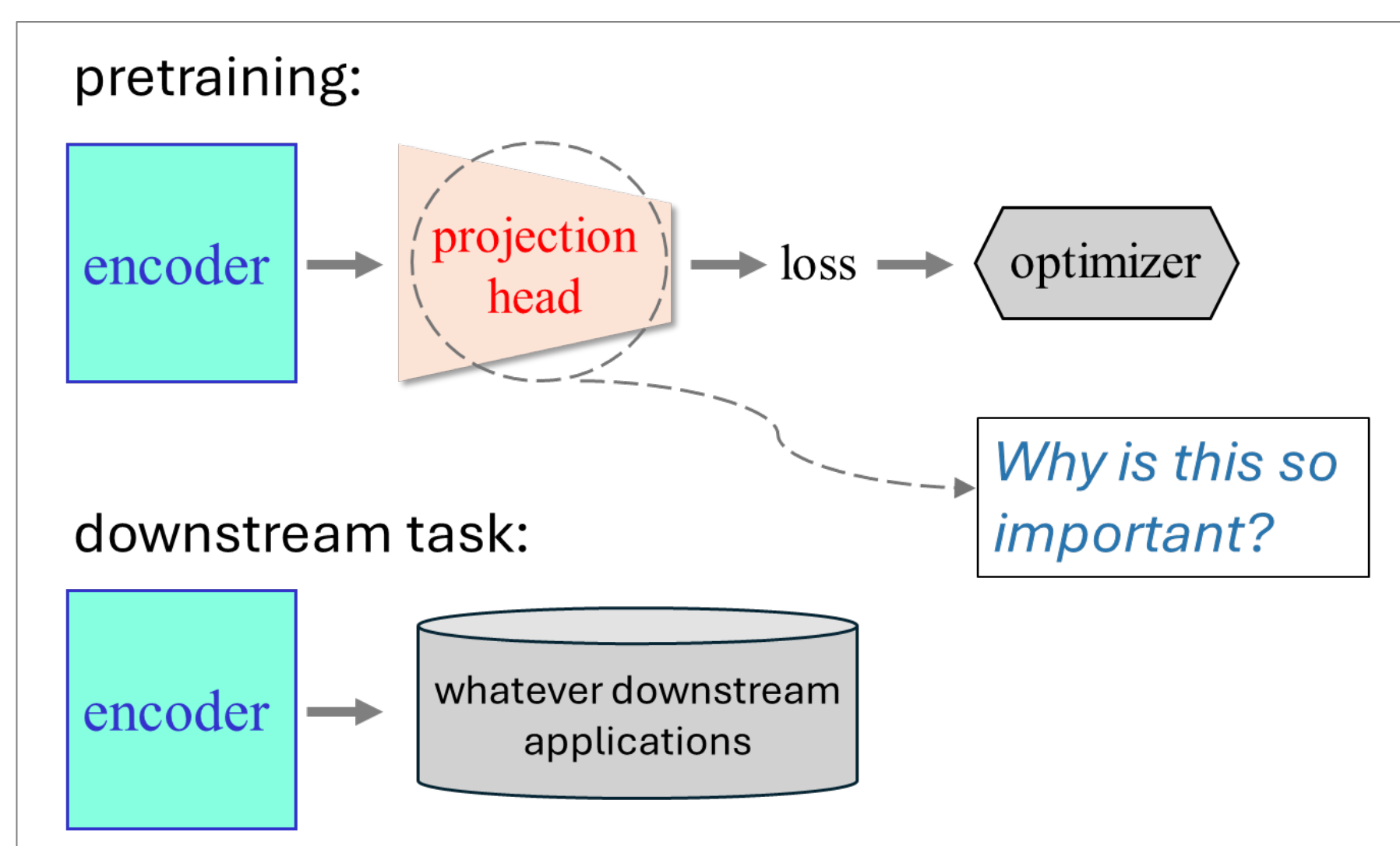


# Investigating the Benefits of Projection Head for Representation Learning

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## Introduction

An effective technique for obtaining high-quality representations is **adding a projection head** on top of the encoder during pretraining, then **discarding** it and using the pre-projection representations for downstream tasks.



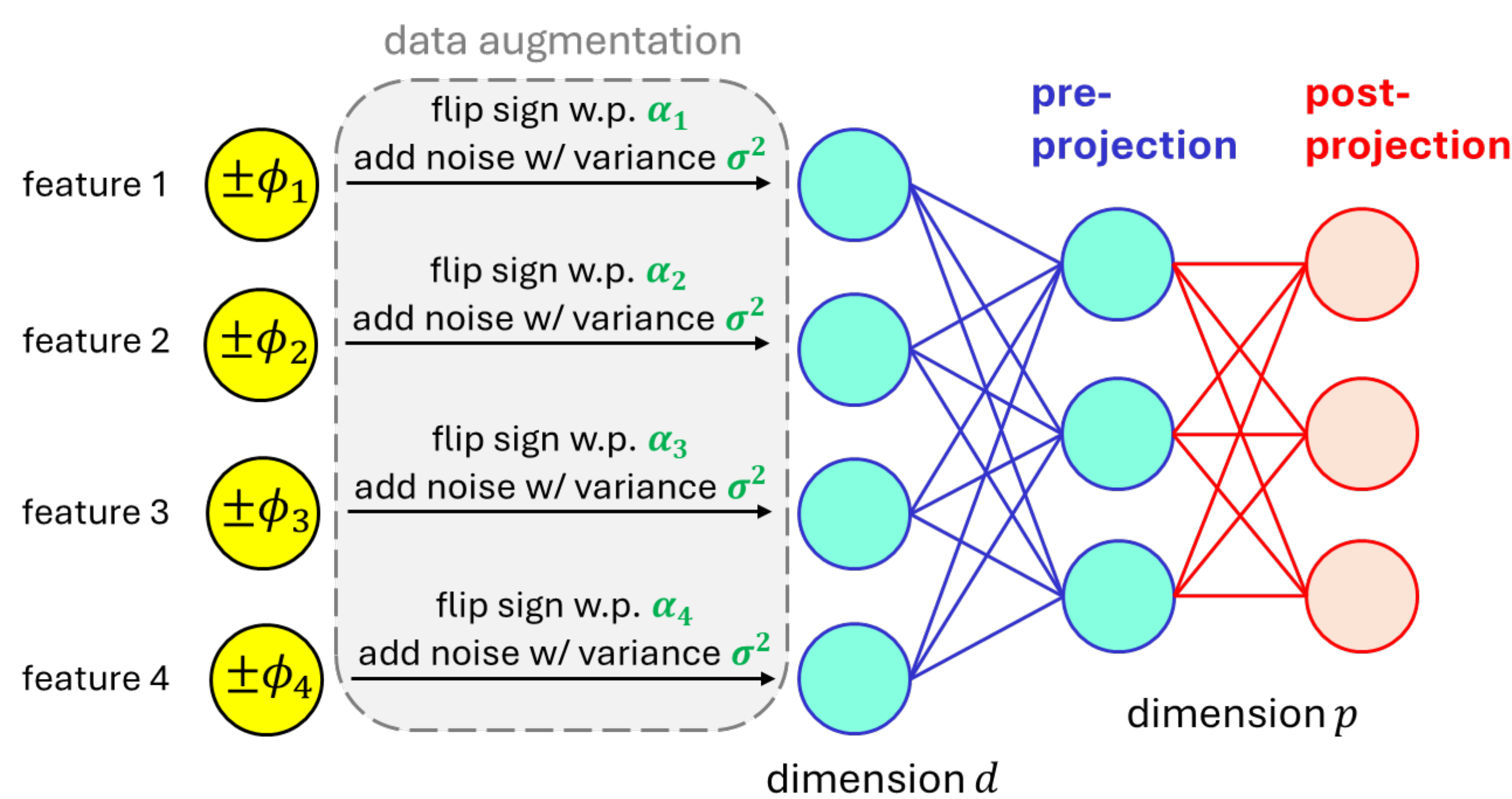
**Our main conclusion:** pre-projection representations represent input features more equally or capture a broader range of features compared to post-projection representations.

## Self-supervised Contrastive Learning

### Pretraining loss

max sim. b/w augmentations of the same examples  
min sim. b/w augmentations of different examples

### Simple data & linear model



## Self-supervised Contrastive Learning (Cont'd)

**Theorem:** Define  $\beta_i = \frac{(1-\alpha_i)^2 \phi_i^2}{\phi_i^2 + \sigma^2}$ ,  $\gamma_i = \sqrt{\frac{(1-\alpha_i)\phi_i}{\phi_i^2 + \sigma^2}}$

feature  $i$  is weighted by  $\begin{cases} 0, & \text{if } \beta_i \text{ is not among the } p \text{ largest } \beta \text{'s} \\ \gamma_i \text{ pre-projection and } \gamma_i^2 \text{ post-projection} \end{cases}$

selection, weighting

### Key insights

- The model selects and weights features based on the interplay between feature strengths ( $\phi$ ), noise ( $\sigma$ ), and data augmentation ( $\alpha$ )
- Features are weighted **more equally pre-projection** than **post-projection**

### When is it beneficial to use pre-projection representations?

Assume that **feature  $i^*$**  is the only one useful for the **downstream task**. Ideally, pretraining should assign a large weight to it relative to other features. If this doesn't occur, i.e., pretraining assigns it a small weight (**small  $\gamma_{i^*}$** ), it would be better to use the pre-projection representations.

Here are some concrete scenarios considering the **interaction** between  $\phi, \sigma, \alpha$

### Corollary (Informal):

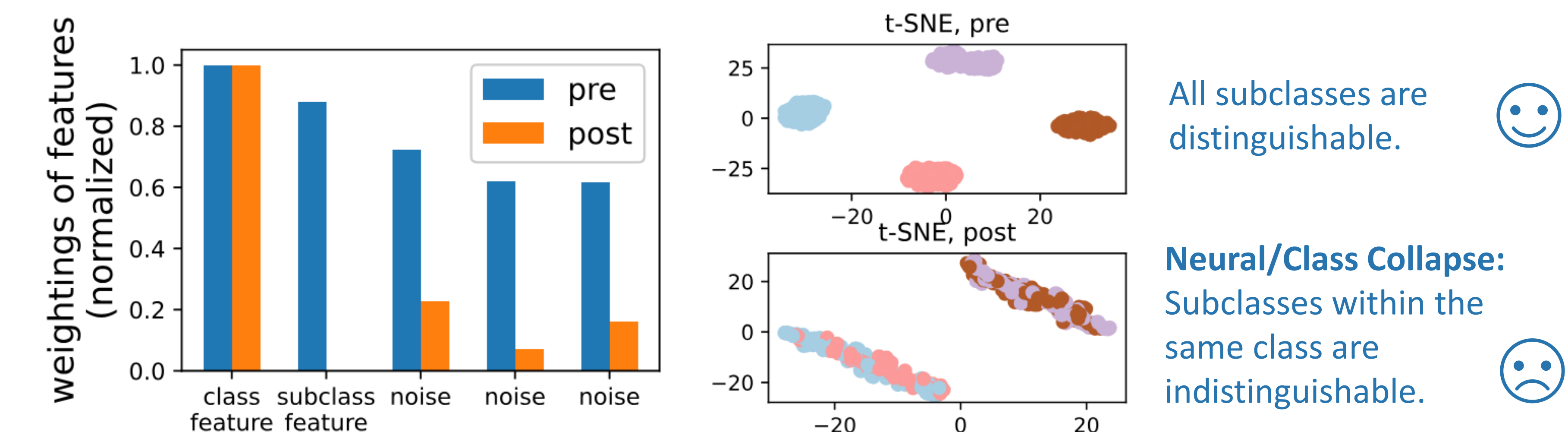
- Data augmentation disrupts **feature  $i^*$**  too much (**large  $\alpha_{i^*} \rightarrow$  small  $\gamma_{i^*}$** )
- Feature  $i^*$**  is too weak in pretraining data (**very small  $\phi_{i^*} \rightarrow$  small  $\gamma_{i^*}$** )
- Feature  $i^*$**  is too strong in pretraining data (**very large  $\phi_{i^*} \rightarrow$  small  $\gamma_{i^*}$** )

### What about non-linear models?

**Theorem (Informal):** Non-linear models allow **pre-projection** representations to capture features that are **entirely absent post-projection**

## Supervised Contrastive Learning and Supervised Learning

Similar conclusions hold for both linear and non-linear models. Notably, the conclusions concerning non-linear models suggest that using pre-projection representations can mitigate **Neural/Class Collapse**, thereby enhancing **coarse-to-fine transferability**.

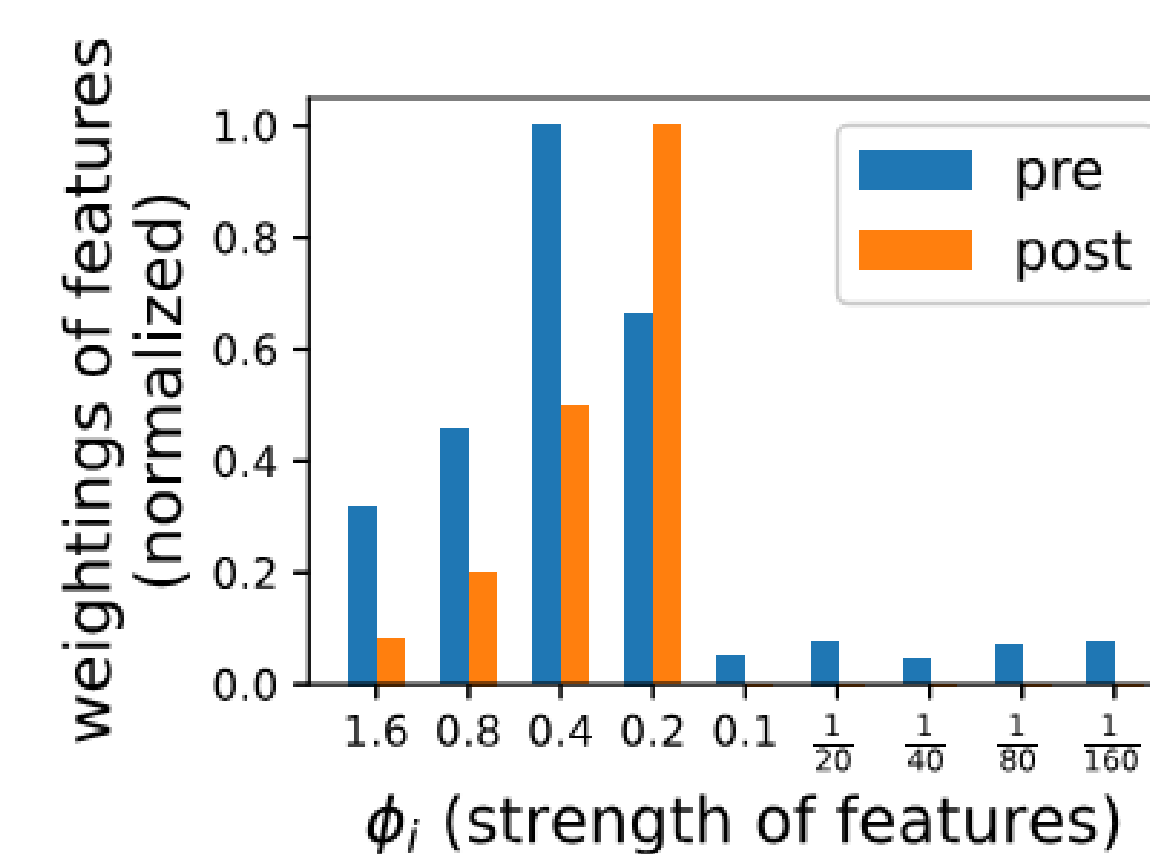
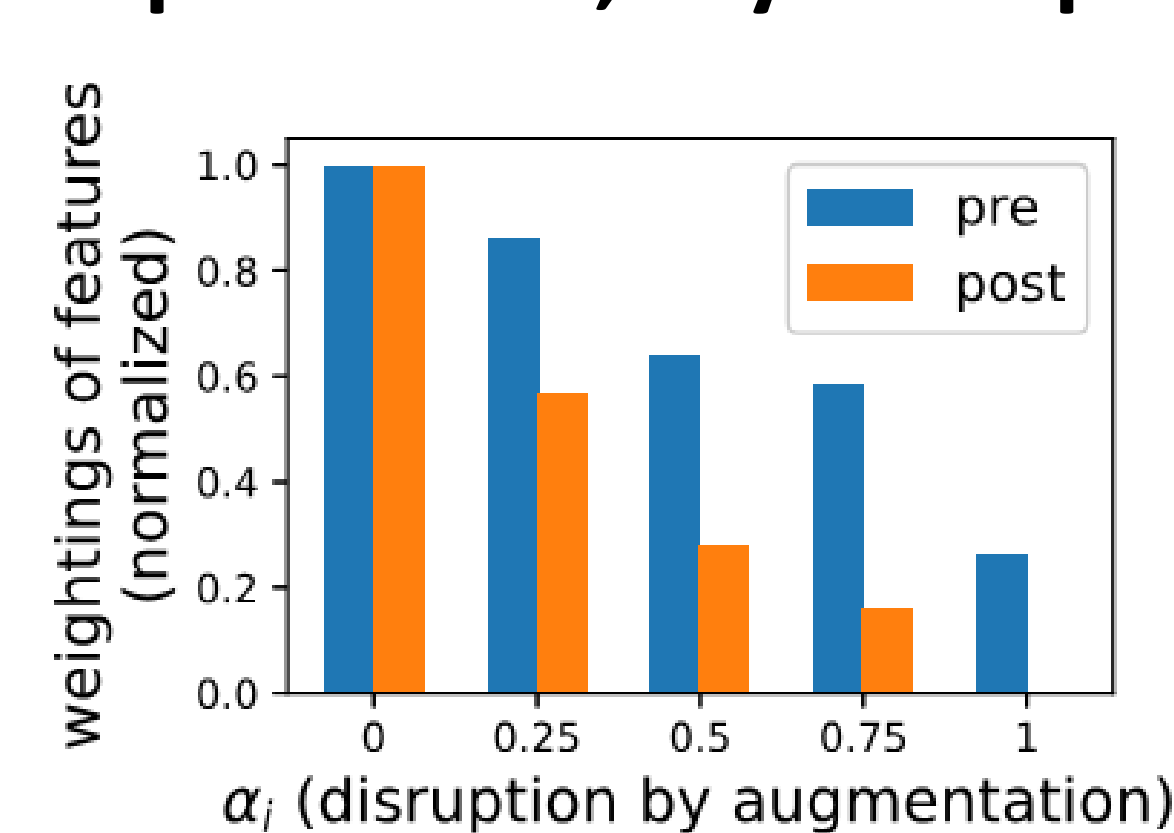


## Replacing the Projection Head with a Fixed Reweighting Head

Scenario	Dataset	Alg.	Performance Measure	Performance		
				vanilla	proj	reweight
synthetic	M-on-C	SSCL	digit clf. acc.	77.0	97.3	97.3
coarse-to-fine	CIFAR100	SCL	fine-grained clf. acc.	21.8	36.0	30.2
coarse-to-fine	CIFAR100	SL	fine-grained clf. acc.	31.44	33.7	32.2
distribution shift	UrbanCars	SL	few-shot adaption acc.	82.2	86.1	87.0

The fixed reweighting head can achieve improvements that are comparable to those of the projection head across many tasks and algorithms.

### Experiments, toy example



### Experiments, semi-synthetic

