Robust Angular Synchronization via Directed Graph Neural Networks

Yixuan He [1] Gesine Reinert [1,2]
David Wipf [3] Mihai Cucuringu [1,2]

[1]Department of Statistics, University of Oxford, United Kingdom
[2]The Alan Turing Institute, United Kingdom
[3]Amazon Web Services AI Shanghai Lablet, China

Goal: estimate a collection of group elements, given a small subset of potentially noisy measurements of their pairwise ratios $g^*_{i,j} = g^*_i g^{*-1}_j$.

- Over the group $SO(2)$: *angular synchronization* aims at obtaining an accurate estimation (up to a constant additive phase) for a set of unknown angles $\theta_1, \ldots, \theta_n \in [0, 2\pi)$ from m noisy measurements of their offsets $\theta_i - \theta_j \mod 2\pi$.

X\Y	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5
Θ_1	\	1.4		3.38	5.88
Θ_2		\	4.28	1.11	
Θ_3			\		
Θ_4				\	4.3
Θ_5					\

Θ_1: ?	Θ_2: ?	Θ_3: ?	Θ_4: ?	Θ_5: ?

- *k-synchronization* extends to the heterogeneous setting: given only the graph union of the same set of nodes and k disjoint sets of edges, estimate the k sets of angles simultaneously.

Why GNNSync?

- A key limitation of existing methods for angular synchronization is their poor performance in the presence of **considerable noise** (large measurement errors).
- Neural networks (NNs)?
- The angular synchronization problem is not directly amenable to a standard NN architecture $\rightarrow$ a customized GNN architecture and loss functions are needed

GNNSync Upset Loss

X\Y	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5
Θ_1	\	1.4		3.38	5.88
Θ_2		\	4.28	1.11	
Θ_3			\		
Θ_4				\	4.3
Θ_5					\

Observed (⬆) vs Predicted (⬇)

X\Y	Θ_1	Θ_2	Θ_3	Θ_4	Θ_5
Θ_1	\	$r_1 - r_2$		$r_1 - r_4$	
Θ_2		\	$r_2 - r_3$	$r_2 - r_4$	
Θ_3			\		
Θ_4				\	$r_4 - r_5$
Θ_5					\

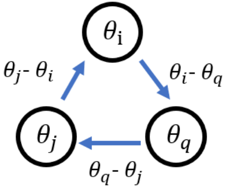
Upset loss function

$\{1.4 - [(r_1 - r_2) \bmod 2\pi]\}^2 +$

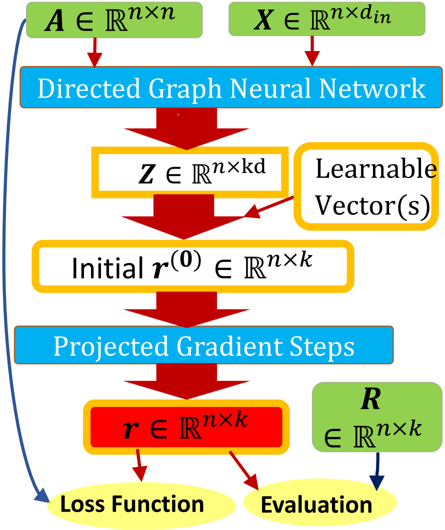
$\{3.38 - [(r_1 - r_4) \bmod 2\pi]\}^2$

$+ \ldots$

GNNSync Cycle Loss

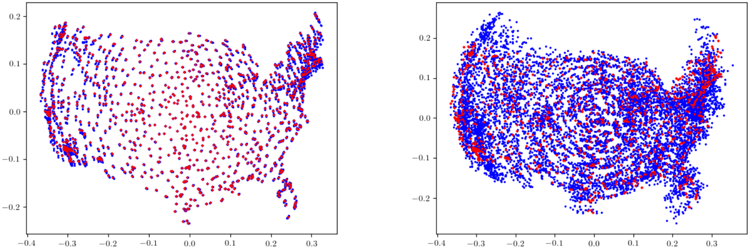


When there is no noise in the data, the angles of any cycle exhibit *cycle consistency*: they add up to 0 $\rightarrow$ we devise a novel loss function to account for deviations from cycle consistency in noisy data.


$A \in \mathbb{R}^{n\times n}$
$X \in \mathbb{R}^{n\times d_{in}}$
Directed Graph Neural Network
$Z \in \mathbb{R}^{n\times \mathrm{kd}}$
Learnable Vector(s)
Initial $r^{(0)} \in \mathbb{R}^{n\times k}$
Projected Gradient Steps
$r \in \mathbb{R}^{n\times k}$
$R \in \mathbb{R}^{n\times k}$
Loss Function
Evaluation

GNNSync Experiments

We have satisfactory performance on synthetic data sets in terms of MSE values, and our GNNSync can also work on Sensor Network Localization (SNL); the SNL problem seeks to reconstruct the 2D coordinates of a cloud of points from a sparse set of pairwise noisy Euclidean distances.



(a) Low noise. (b) High noise.

Figure: Sensor network localization on the U.S. map.

Conclusion

We proposed a general neural network framework for angular synchronization and a heterogeneous extension. Future directions:

- extending the framework to more general group synchronization problems
- optimizing the loss functions under constraints
- training with some supervision of ground-truth angles
- exploring the interplay with low-rank matrix completion
- exploring the graph realization problem, of recovering point clouds from a sparse noisy set of pairwise distances

Paper: `https://arxiv.org/abs/2310.05842`

Code: `https://github.com/SherylHYX/GNN_Sync`

More about me: `https://sherylhyx.github.io/`