Università di Pisa

MultiSTOP: Solving Functional Equations with Reinforcement Learning

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CONTRIBUTIONS

- MultiSTOP: an extension of the BootSTOP algorithm to solve functional equations with additional constraints.
- Analysis on a 1D physical model with no analytical solution.
- Discussion on the degeneracy problem for similar terms.

Problem

- **Parametric functional equation**

$$h(x) + \sum_{n\geq 1} C_n^2 F_{\Delta_n}(x) = 0$$

- **Unknowns:** $C_n^2 \geq 0$ and function parameters $\Delta_n \geq 0$.

$$F_{\Delta_n}(x) = x^2 f_{\Delta_n}(1-x) + (1-x)^2 f_{\Delta_n}(x)$$

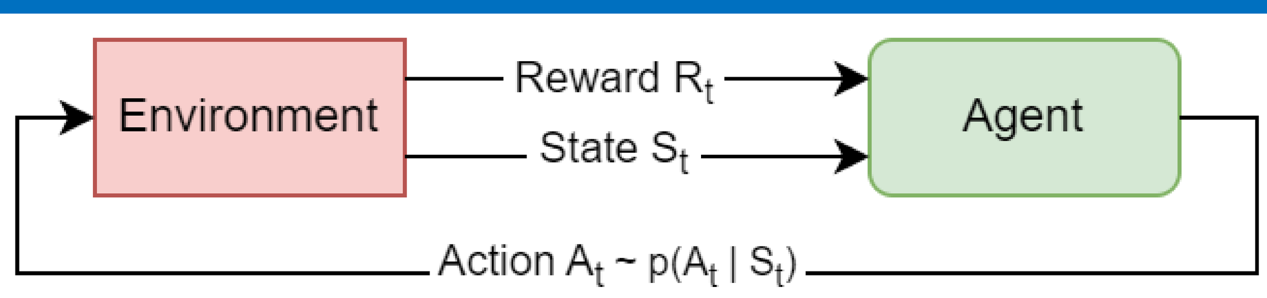
- $\boldsymbol{f_{\Delta_n}(x)}$ **is analytic** in x and Δ_n.
- **Two integral constraints**[1] with the same unknowns

$$\sum_{n\geq 1} C_n^2 \int_0^{\frac{1}{2}} (x^2 - x + 1)\frac{f_{\Delta_n}(x)}{x^2} \partial_x \log\big(x(1-x)\big)\mathrm{d}x = A_1$$

$$\sum_{n\geq 1} C_n^2 \int_0^{\frac{1}{2}} \frac{f_{\Delta_n}(x)}{x^2}(2x-1)\,\mathrm{d}x = A_2$$

- These constraints **improve precision on unknowns**[1].

Reinforcement Learning Framework



BootSTOP[2] algorithm

- Approximate with a **finite number** of terms ($n \leq 10$).
- **Evaluate** equation on a **fixed set** of N points.

- **State:** current guess of solution

$$(\boldsymbol{C^2}, \Delta) = (C_1^2, C_2^2, \dots, C_{10}^2, \Delta_1, \Delta_2, \dots, \Delta_{10})$$

- **Action:** cyclically from 1 to 10, change (C_n^2, Δ_n).
- **Reward:** evaluate the equation on the current guess and the N points and take the squared norm $\|\boldsymbol{E}(\boldsymbol{C^2}, \Delta)\|_2$.

$$R = \frac{1}{\|\boldsymbol{E}(\boldsymbol{C^2}, \Delta)\|_2}$$

- **RL algorithm:** Soft Actor Critic[3].

MultiSTOP

- Include **additional constraints** into the same framework.
- **Evaluate** integral equations on the current guess:

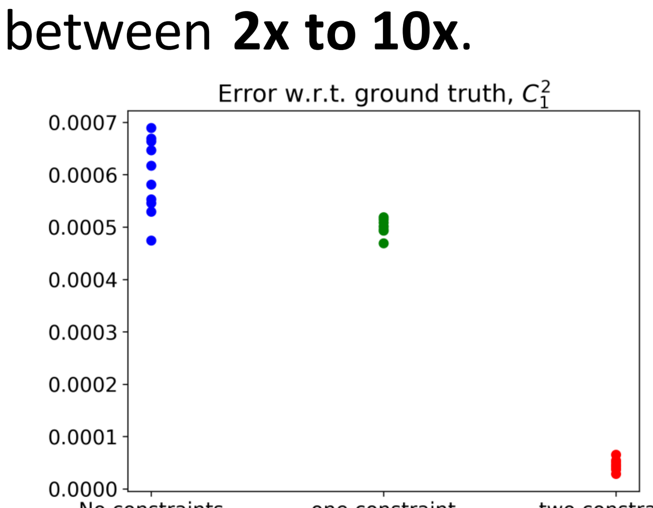
$$\boldsymbol{I_1}(\boldsymbol{C^2}, \Delta) = \sum_{1\leq n\leq 10} C_n^2\, \mathrm{Int}_1\big[f_{\Delta_n}(x)\big] - A_1$$

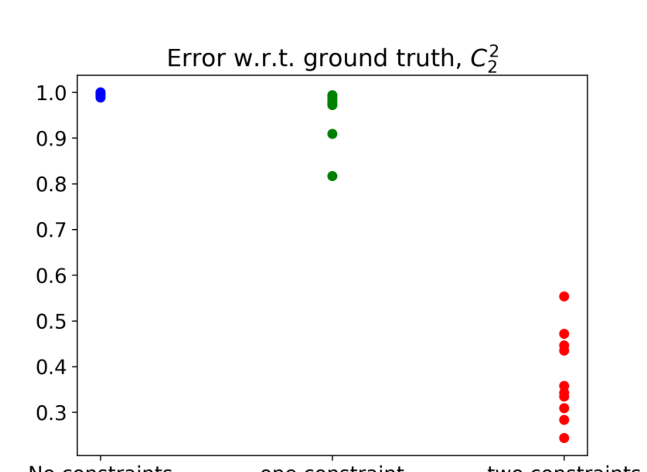
$$\boldsymbol{I_2}(\boldsymbol{C^2}, \Delta) = \sum_{1\leq n\leq 10} C_n^2\, \mathrm{Int}_2\big[f_{\Delta_n}(x)\big] - A_2$$

- Take **absolute value** and include in the reward:

$$\boldsymbol{R_1} = \frac{1}{\|\boldsymbol{E}(\boldsymbol{C^2}, \Delta)\|_2 + \boldsymbol{w_1}|\boldsymbol{I_1}(\boldsymbol{C^2}, \Delta)| + \boldsymbol{w_2}|\boldsymbol{I_2}(\boldsymbol{C^2}, \Delta)|}$$

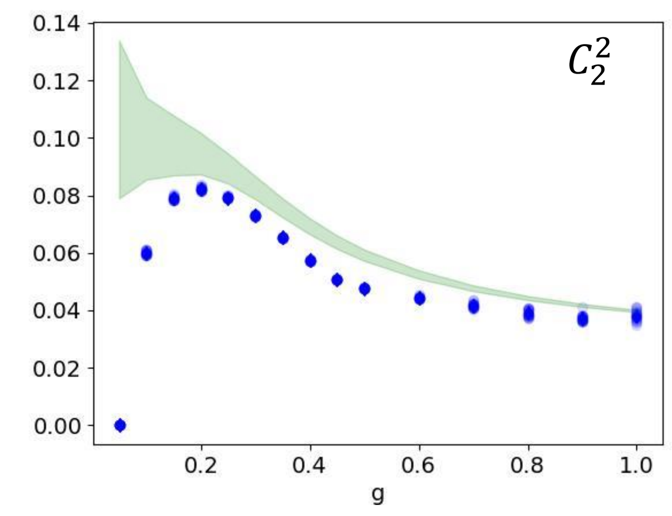
- $\boldsymbol{R_1}$ forces **together** the constraints.
- Using **MultiSTOP** the relative error on C_1^2, C_2^2, C_3^2 is reduced between **2x to 10x**.

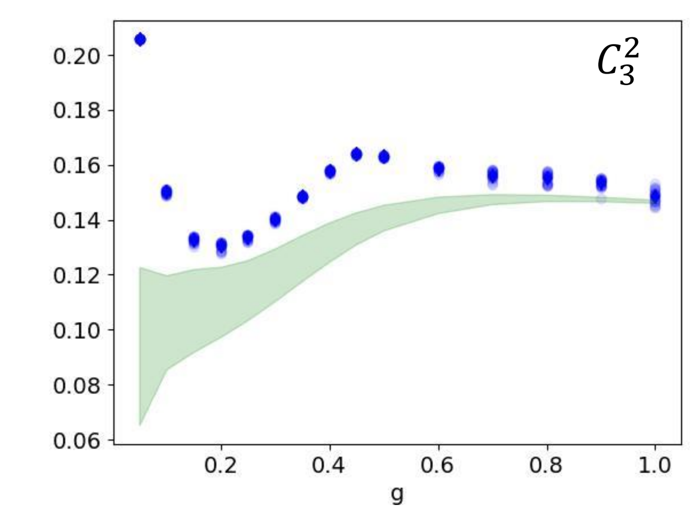




RESULTS

- **1D defect CFT** in a 4D supersymmetric Yang-Mills theory**.** Model and results depend on parameter $\boldsymbol{g}$.
- **Weak coupling $\boldsymbol{g} \leq \boldsymbol{1}$**: hardest case.

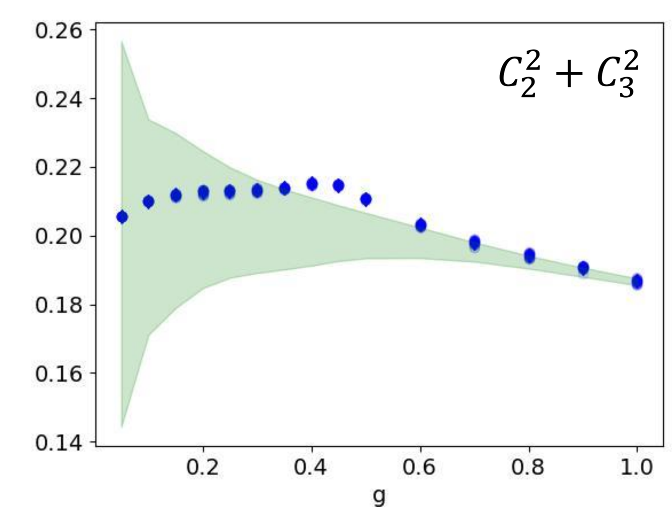




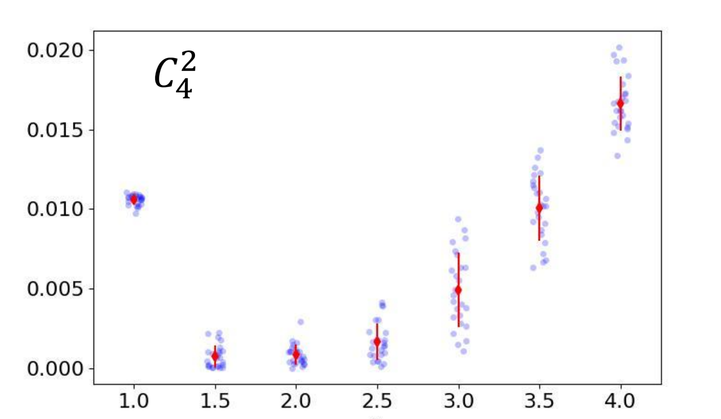
- **Degeneracy:** because $\Delta_2 \approx \Delta_3 \approx 2$, we have

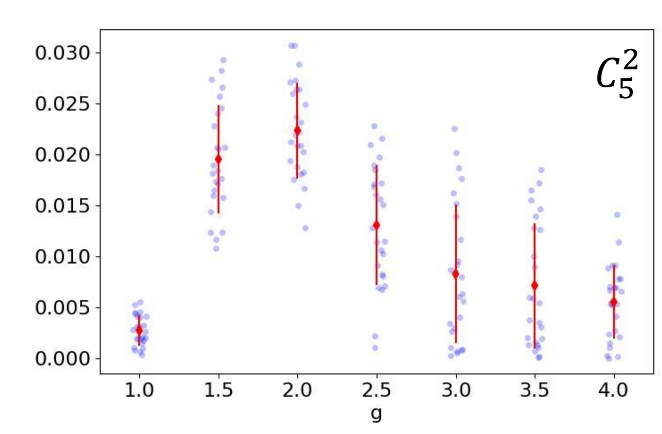
$$C_2^2 F_{\Delta_2} + C_3^2 F_{\Delta_3} \approx (C_2^2 + C_3^2) F_2$$

- Results on the sum are much better.

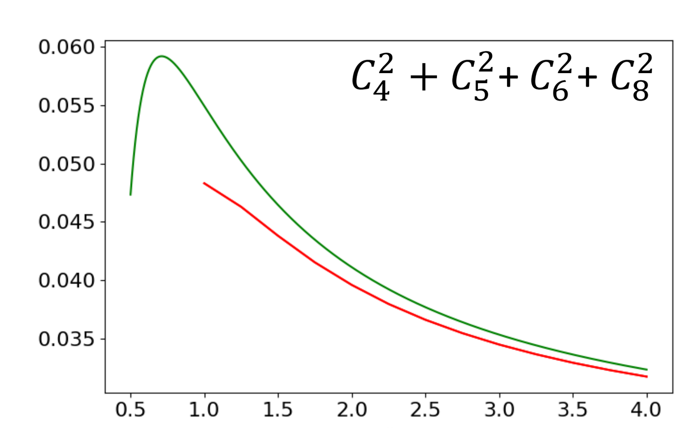


- **Strong coupling $\boldsymbol{g} \geq \boldsymbol{1}$**: C_1^2, C_2^2, C_3^2 are known.





- $C_7^2 \xrightarrow[g\to 4]{} 0$ as expected[4].
- Results follow theoretical expectations[4] (green line).



CONCLUSION

- Capacity to solve **multiple parametric equations.**
- Great improvements with multiple physical constraints.
- In case of degeneracy, other functional equations can be included (future work).
- If no degeneracy, **precision is very high** (red line).
- Applicable to functional equations and possibly **PDEs using parametric basis functions.**

References

[1]Andrea Cavaglià, Nikolay Gromov, Julius Julius, and Michelangelo Preti. Bootstrability in defect CFT: integrated correlators and sharper bounds. Journal of High Energy Physics, 2022(5), may 2022. doi: 10.1007/jhep05(2022)164.

[2]Gergely Kàntor, Constantinos Papageorgakis, and Vasilis Niarchos. Solving conformal field theories with artificial intelligence. Physical Review Letters, 128(4), jan 2022

[3]Tuomas Haarnoja, Aurick Zhou, Pieter Abbeel, and Sergey Levine. Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor. PMLR, 10–15 Jul 2018.

[4]Pietro Ferrero and Carlo Meneghelli. Unmixing the Wilson line defect CFT. Part I and part II, December 2023.