

# Probabilistic Learning To Defer

---

Cuong Nguyen<sup>1</sup>   Thanh-Toan Do<sup>2</sup>   Gustavo Carneiro<sup>1</sup>

April 25th, 2025

<sup>1</sup>Centre for Vision, Speech and Signal Processing, University of Surrey, UK

<sup>2</sup>Department of Data Science and AI, Monash University, Australia



# Introduction

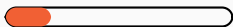
---



Reliability

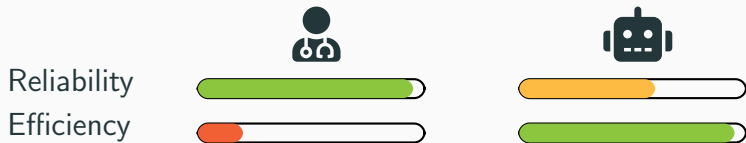


Efficiency

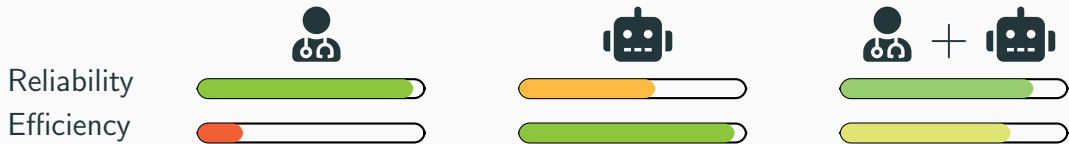


# Introduction

---



# Introduction



**Learning to defer (L2D) aims to leverage:**

- high *reliability* of human, and
- high *efficiency* of machine learning models.

## Background - learning to defer

---

# Background - learning to defer

---

## Modelling

L2D is a mixture of:

# Background - learning to defer

---

## Modelling

L2D is a mixture of:

- $M$  human experts, and

# Background - learning to defer

---

## Modelling

L2D is a mixture of:

- $M$  human experts, and
- a machine learning model (i.e., classifier).



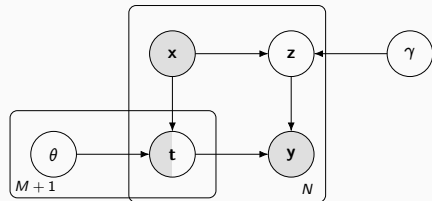
# Background - learning to defer

## Modelling

L2D is a mixture of:

- $M$  human experts, and
- a machine learning model (i.e., classifier).

→ latent variable model: expert selection  $z$



**Figure 1:** Graphical model of L2D.

# Background - learning to defer

## Modelling

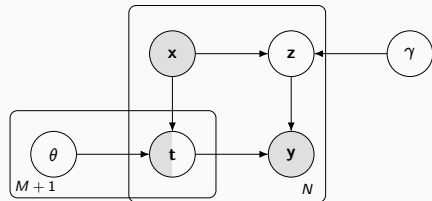
L2D is a mixture of:

- $M$  human experts, and
- a machine learning model (i.e., classifier).

→ latent variable model: expert selection  $z$

## Objective

$$\max_{\gamma, \theta_{M+1}} \frac{1}{N} \sum_{i=1}^N \ln \Pr \left( \mathbf{y}_i, \prod_{m=1}^M \mathbf{t}_i^{(m)} \mid \mathbf{x}_i, \gamma, \prod_{m=1}^{M+1} \theta_m \right). \quad (1)$$



**Figure 1:** Graphical model of L2D.

# Background - learning to defer

## Modelling

L2D is a mixture of:

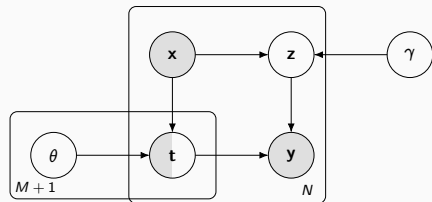
- $M$  human experts, and
- a machine learning model (i.e., classifier).

→ latent variable model: expert selection  $z$

## Objective

$$\max_{\gamma, \theta_{M+1}} \frac{1}{N} \sum_{i=1}^N \ln \Pr \left( \mathbf{y}_i, \prod_{m=1}^M \mathbf{t}_i^{(m)} \mid \mathbf{x}_i, \gamma, \prod_{m=1}^{M+1} \theta_m \right). \quad (1)$$

Learning is performed via the *Expectation - Maximisation* algorithm



**Figure 1:** Graphical model of L2D.

# Motivation

---

**Learning to defer has limitations**

# Motivation

---

## Learning to defer has limitations

⚠ requires *all* human experts must annotate every training sample

# Motivation

---

## Learning to defer has limitations

- ⚠ requires *all* human experts must annotate every training sample
- ⊘ impractical (e.g., each sample is annotated by few human experts), and

# Motivation

---

## Learning to defer has limitations

- ⚠ requires *all* human experts must annotate every training sample
  - 🚫 impractical (e.g., each sample is annotated by few human experts), and
  - 💰 costly, time-consuming, and even infeasible (e.g., radiology),

# Motivation

---

## Learning to defer has limitations

- ⚠ requires *all* human experts must annotate every training sample
  - 🚫 impractical (e.g., each sample is annotated by few human experts), and
  - 💰 costly, time-consuming, and even infeasible (e.g., radiology),
- ⚠ most likely selects the *best human expert* all the time



# Motivation

---

## Learning to defer has limitations

- ⚠ requires *all* human experts must annotate every training sample
  - 🚫 impractical (e.g., each sample is annotated by few human experts), and
  - 💰 costly, time-consuming, and even infeasible (e.g., radiology),
- ⚠ most likely selects the *best human expert* all the time
  - ⚖ unfair workload assignment, and

# Motivation

---

## Learning to defer has limitations

- ⚠ requires *all* human experts must annotate every training sample
  - 🚫 impractical (e.g., each sample is annotated by few human experts), and
  - 💰 costly, time-consuming, and even infeasible (e.g., radiology),
- ⚠ most likely selects the *best human expert* all the time
  - ⚖ unfair workload assignment, and
  - 😞 fatigue, burnout → misdiagnosis.

# Probabilistic L2D - missing annotations

---

# Probabilistic L2D - missing annotations

---

Relax the strong assumption in standard L2D

# Probabilistic L2D - missing annotations

---

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.

# Probabilistic L2D - missing annotations

---

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.



# Probabilistic L2D - missing annotations

---

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.



Each sample is annotated by **some** (or none) human experts.

# Probabilistic L2D - missing annotations

---

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.



Each sample is annotated by **some** (or none) human experts.

→ additional latent variables (i.e., missing annotations  $\mathbf{t}^{(j)}, \forall j \in \mathcal{D}^{\text{unobs.}}$ )



# Probabilistic L2D - missing annotations

---

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.



Each sample is annotated by **some** (or none) human experts.

→ additional latent variables (i.e., missing annotations  $\mathbf{t}^{(j)}, \forall j \in \mathcal{D}^{\text{unobs.}}$ )

**Parameter inference** is performed through the *variational* EM algorithm:

# Probabilistic L2D - missing annotations

---

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.



Each sample is annotated by **some** (or none) human experts.

→ additional latent variables (i.e., missing annotations  $\mathbf{t}^{(j)}, \forall j \in \mathcal{D}^{\text{unobs.}}$ )

**Parameter inference** is performed through the *variational* EM algorithm:

*E-step*: approximate the posterior  $q(\mathbf{z}, \prod_{j \in \mathcal{D}^{\text{unobs.}}} \mathbf{t}^{(j)})$  via variational inference

# Probabilistic L2D - missing annotations

## Relax the strong assumption in standard L2D

Each sample must be annotated by **all** human experts.



Each sample is annotated by **some** (or none) human experts.

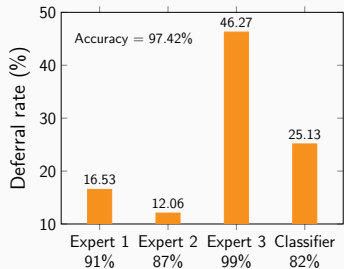
→ additional latent variables (i.e., missing annotations  $\mathbf{t}^{(j)}, \forall j \in \mathcal{D}^{\text{unobs.}}$ )

**Parameter inference** is performed through the *variational* EM algorithm:

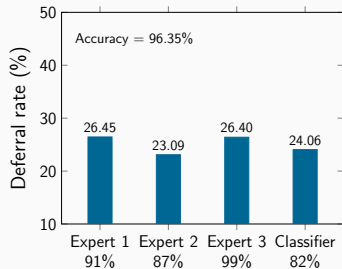
*E-step*: approximate the posterior  $q(\mathbf{z}, \prod_{j \in \mathcal{D}^{\text{unobs.}}} \mathbf{t}^{(j)})$  via variational inference

*M-step*: maximise the “completed”-data log-likelihood w.r.t.  $\gamma$  and  $\{\theta_m\}_{m=1}^{M+1}$ .

# Probabilistic L2D - Control workload assignment



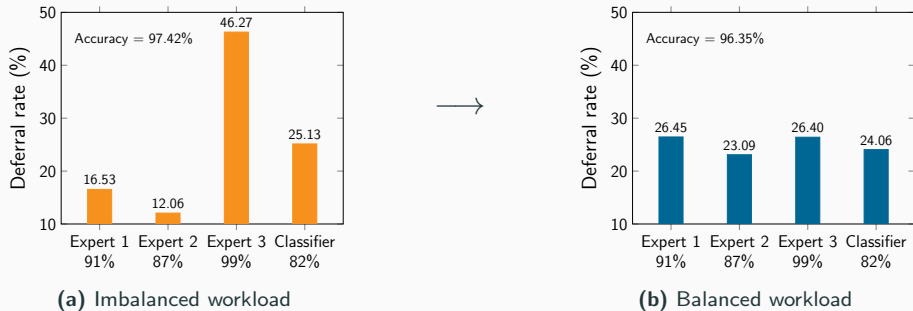
(a) Imbalanced workload



(b) Balanced workload

**Figure 2:** Workload distribution on Chaoyang dataset at coverage of 0.25.

# Probabilistic L2D - Control workload assignment



**Figure 2:** Workload distribution on Chaoyang dataset at coverage of 0.25.

An additional E-step is introduced to calculate the constrained posterior:

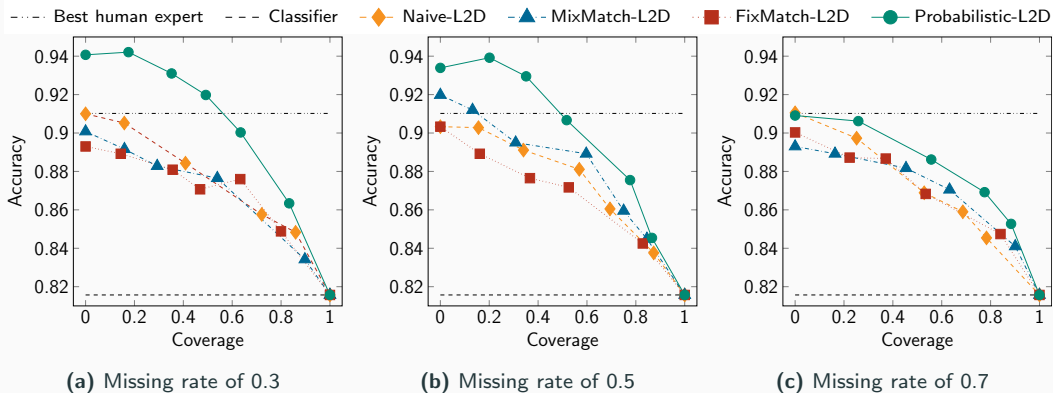
$$\tilde{q}_i^* = \underset{\tilde{q}}{\operatorname{argmin}} \operatorname{KL} [\tilde{q}(\mathbf{z}_i) \| q^*(\mathbf{z}_i)], \forall i \in \{1, \dots, N\} \quad \text{s.t.: } \epsilon_l \preceq \frac{1}{N} \sum_{i=1}^N \tilde{q}(\mathbf{z}_i) \preceq \epsilon_u,$$

where  $q^*$  and  $\tilde{q}$  denote the unconstrained and constrained posteriors of  $\mathbf{z}$ .

## Evaluation - Coverage - accuracy curve on Chaoyang

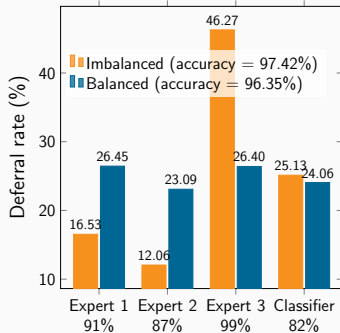
---

# Evaluation - Coverage - accuracy curve on Chaoyang

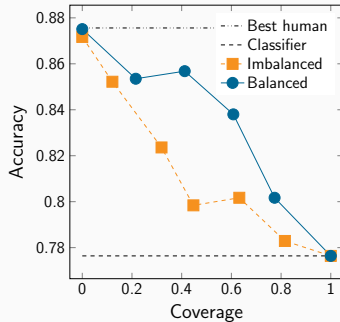


**Figure 3:** Comparison of coverage - accuracy curves between different L2D methods on Chaoyang with 2 human experts, each at a different missing rate.

# Evaluation - Controllable workload



(a) Chaoyang - coverage of 0.25.



(b) MiceBone - missing rate of 0.3

**Figure 4:** ((a)) shows comparisons of two different workload constraints on Chaoyang dataset with 50% missing annotations per expert, where in the *imbalanced* setting,  $\varepsilon_l = 0$  and  $\varepsilon_u = 1$  for each human expert, while in the *balanced* setting,  $\varepsilon_l \approx \varepsilon_u = (1 - \text{coverage})/M$  for each human expert, and ((b)) coverage - accuracy curve on MiceBone at 30% missing rate.



# Conclusion

---

# Conclusion

---

**Propose and develop the probabilistic L2D, which**

# Conclusion

---

**Propose and develop the probabilistic L2D, which**

- addresses the missing annotations, and

# Conclusion

---

**Propose and develop the probabilistic L2D, which**

- addresses the missing annotations, and
- manages the workload between experts.

# Conclusion

---

## **Propose and develop the probabilistic L2D, which**

- addresses the missing annotations, and
- manages the workload between experts.

## **Limitations**

# Conclusion

---

## **Propose and develop the probabilistic L2D, which**

- addresses the missing annotations, and
- manages the workload between experts.

## **Limitations**

- Scalability w.r.t. the number of human experts

# Conclusion

---

## **Propose and develop the probabilistic L2D, which**

- addresses the missing annotations, and
- manages the workload between experts.

## **Limitations**

- Scalability w.r.t. the number of human experts
- Dynamic expert's performance