PIG: Physics-Informed Gaussians as Adaptive Parametric Mesh Representations

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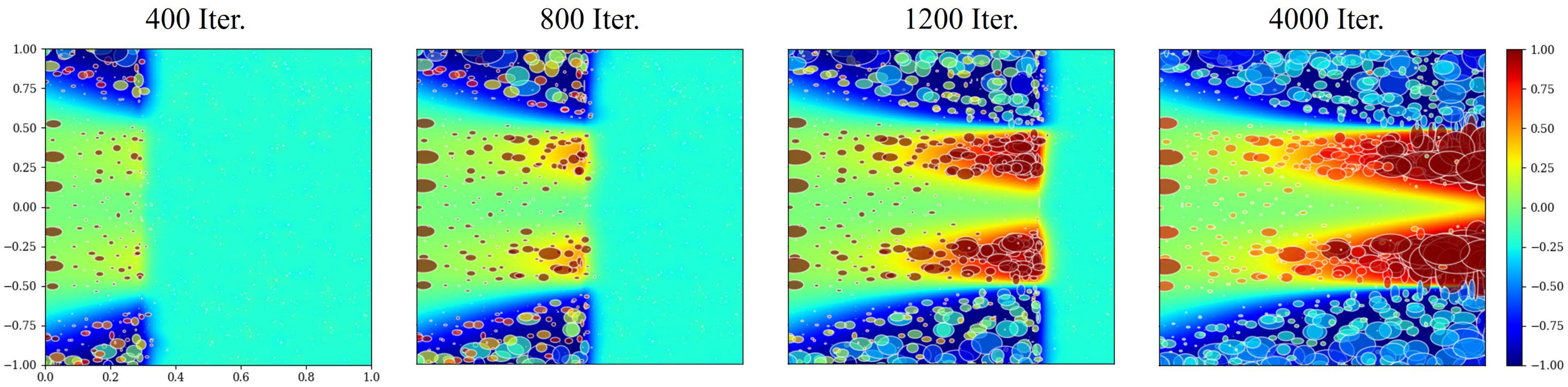
https://namgyukang.github.io/Physics-Informed-Gaussians/

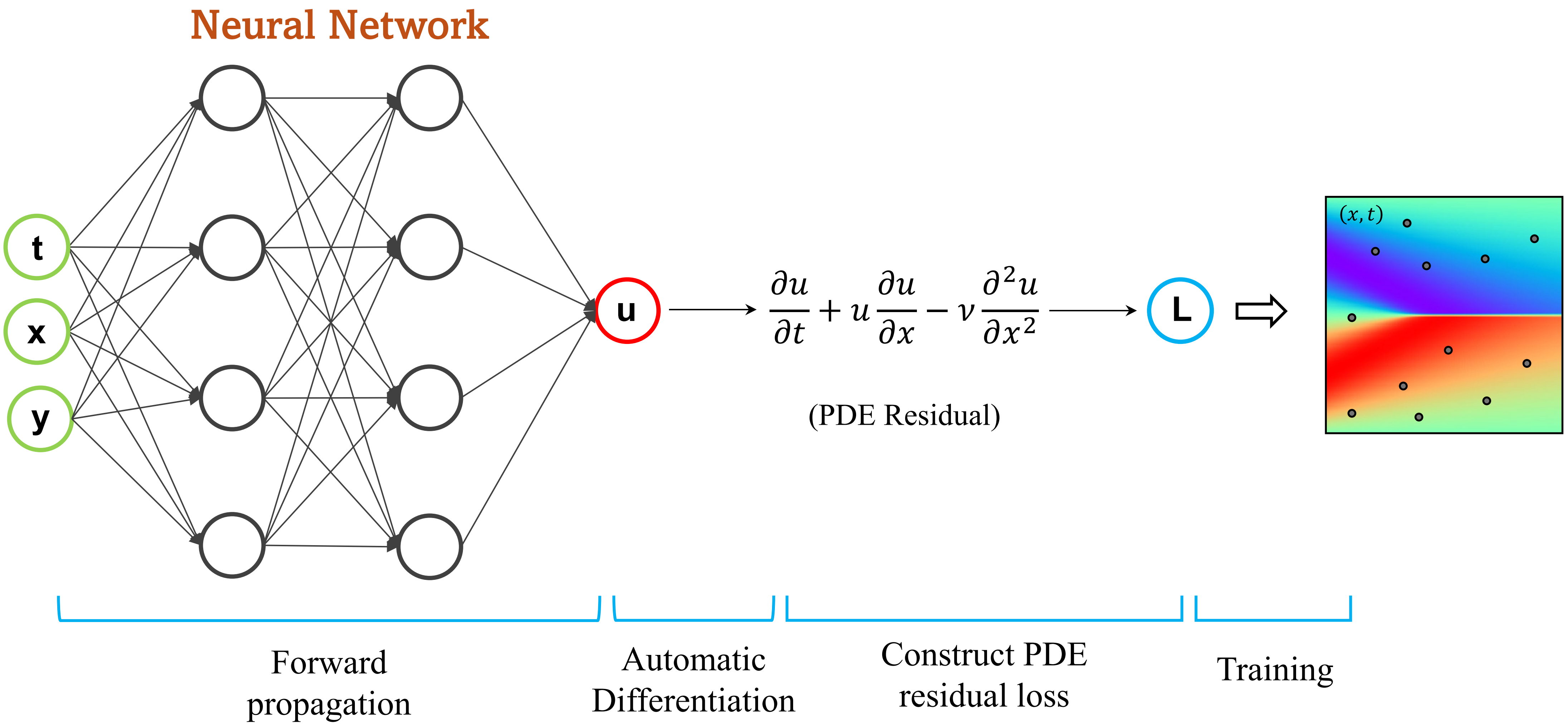
Physics-Informed Gaussians

Neural network + Parametric Learnable Gaussains!



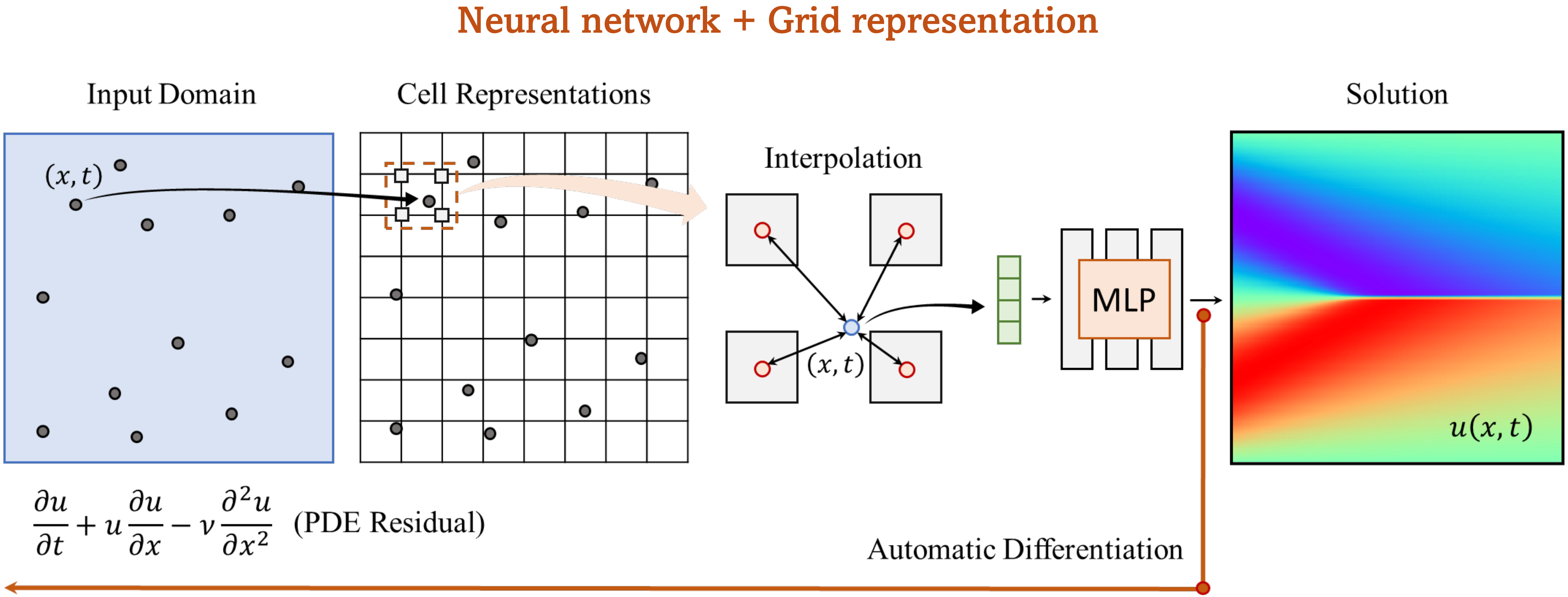
- Proposed Physics-Informed Gaussians (PIGs) combine feature embeddings using Gaussian functions with a lightweight MLP.
- Our approach uses trainable parameters for the coefficient value, mean and variance of each Gaussian, allowing for dynamic adjustment of their positions and shape during training

Physics-Informed Neural Network



Physics-Informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations, M.Raissi et al., Journal of Computational Physics, 2019

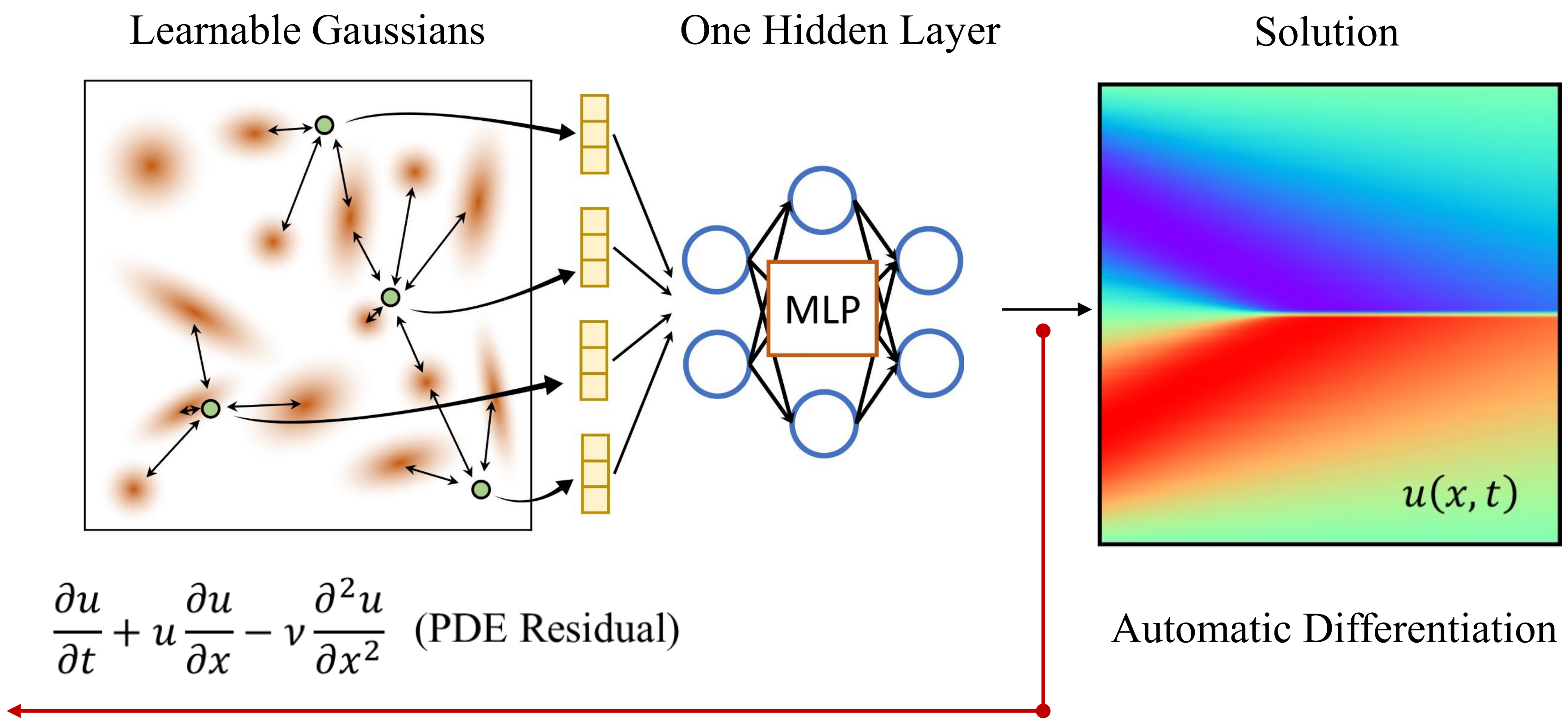
Physics-Informed Parametric Grid Representations



- Disentangle the trainable parameters with respect to the input coordinates
- Minimize conventional spectral bias problem

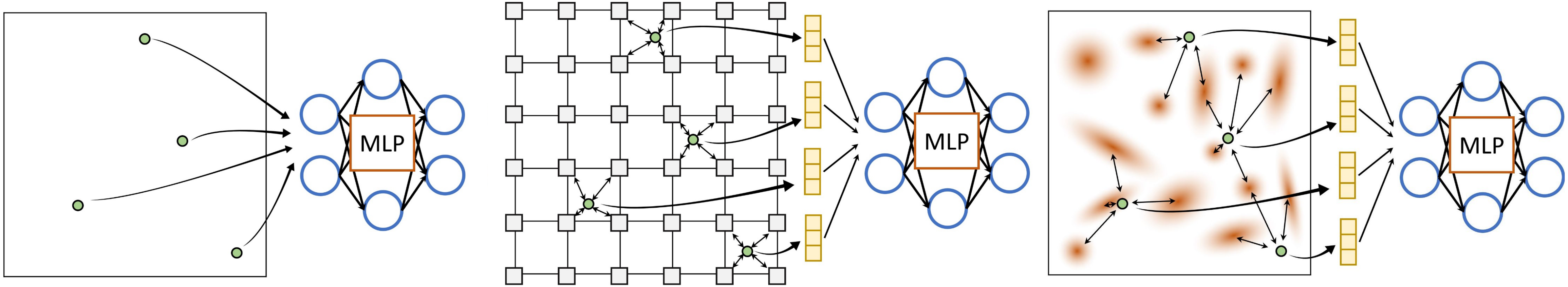
Physics-Informed Gaussians

Neural network + Parametric Learnable Gaussains!



- An adaptive mesh representation in which Gaussian parameters are trained to dynamically adjust their positions and shapes.
- This adaptability enables our model to optimally approximate PDE solutions, unlike models with fixed parameter positions.

Physics-Informed Gaussians



(a) PINN

(b) Parametric Grid

(c) PIG (ours)

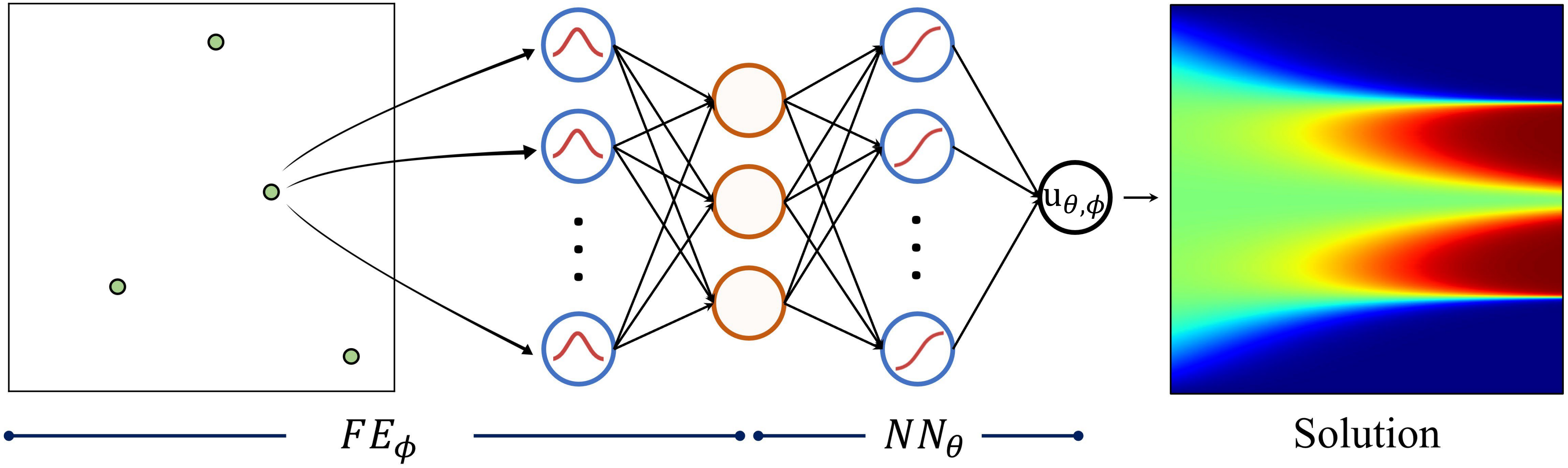
- Spectral bias (neural net bias) (-)
- Autograd + NN training (+)
- Mesh-free (+)

- No spectral bias (+)
- Autograd + NN training (+)
- Fixed Grid points (-)

- No spectral bias (+)
- Autograd + NN training (+)
- Adaptive Grid points (+)

Physics-Informed Gaussians as a Neural Network

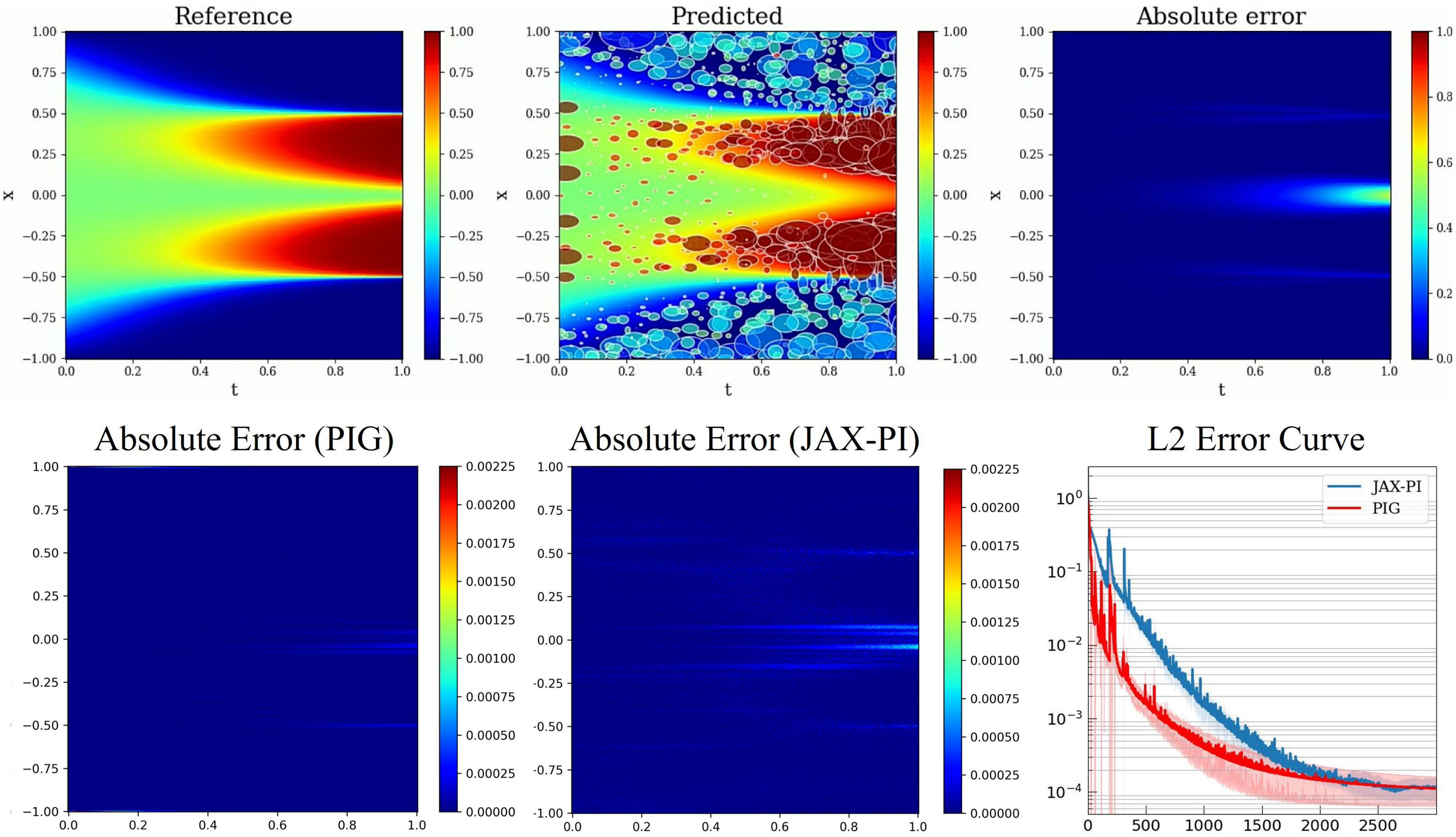
Neural network + Parametric Learnable Gaussains!



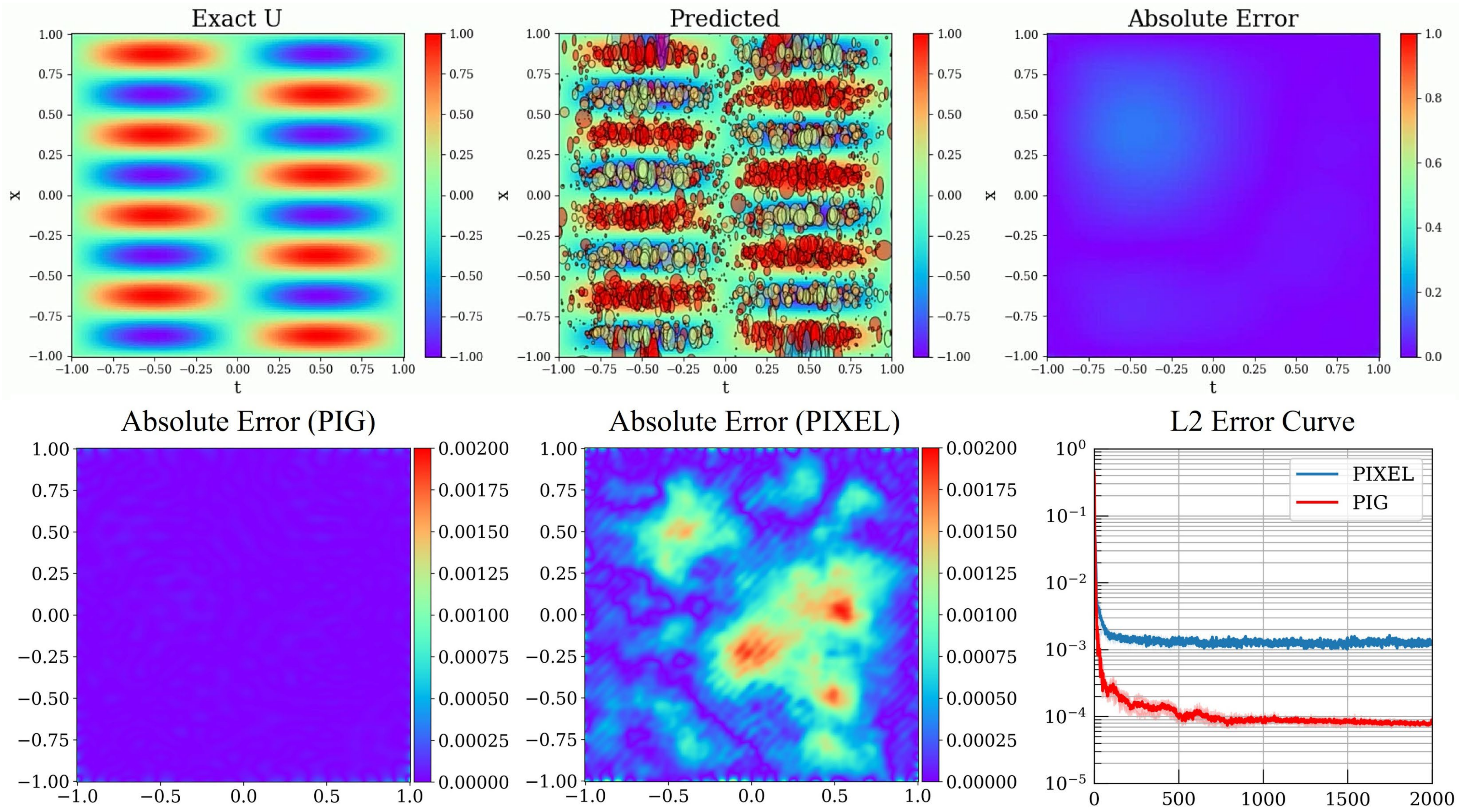
$$\mathrm{FE}_{\phi} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{f_i G_i}, \quad \mathrm{G_i} = e^{-\frac{1}{2}(\mathrm{x}-\mu_{\mathrm{i}})^{\mathrm{T}} \Sigma_{\mathrm{i}}^{-1} (\mathrm{x}-\mu_{\mathrm{i}})}$$

$$\mathrm{u}_{\theta,\phi}(x) = \mathrm{NN}_{\theta}(FE_{\phi}(x))$$

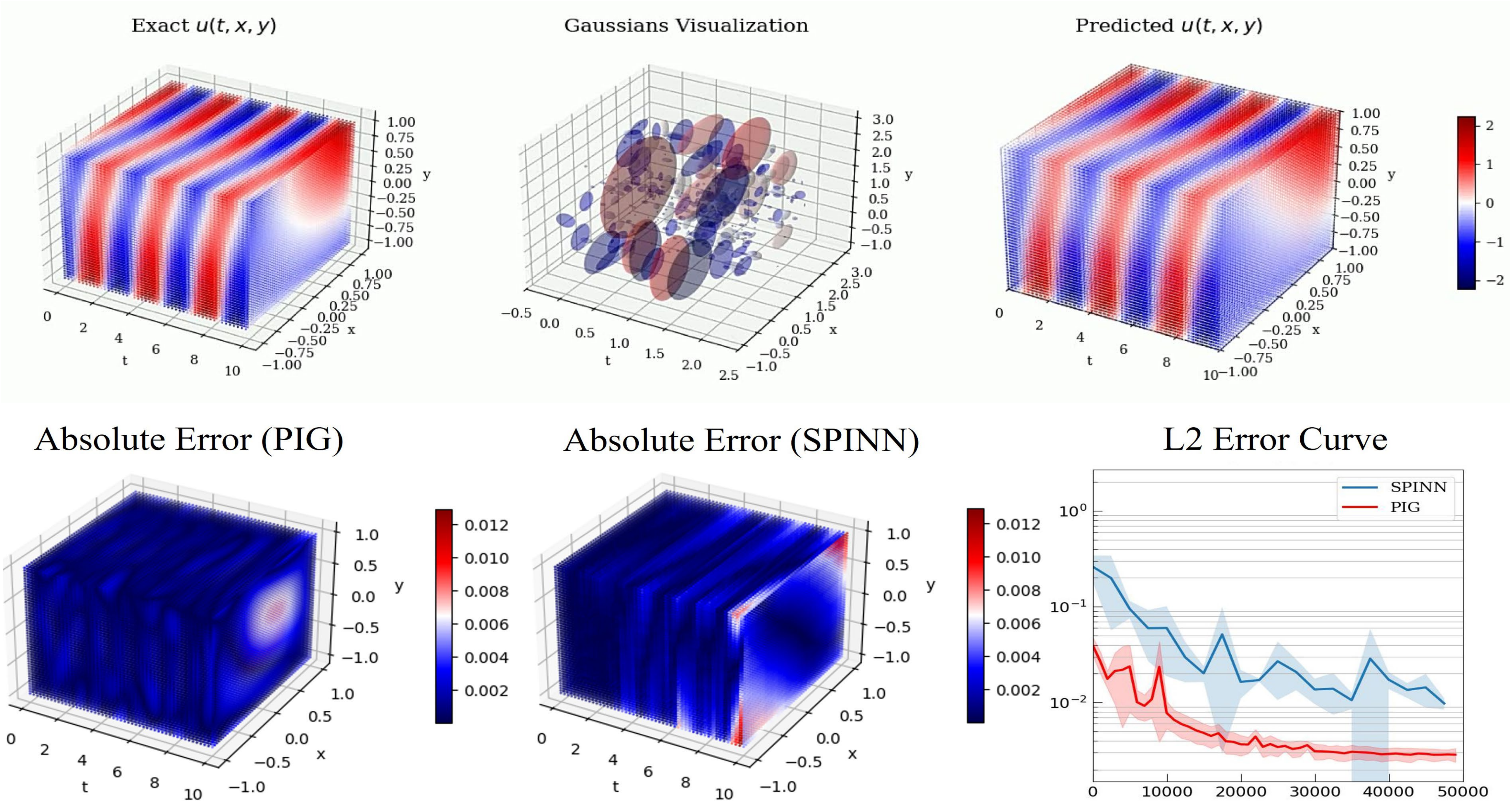
Experiments (Allen Cahn Eq.) Best L2 rel. Error: 5.93e-05



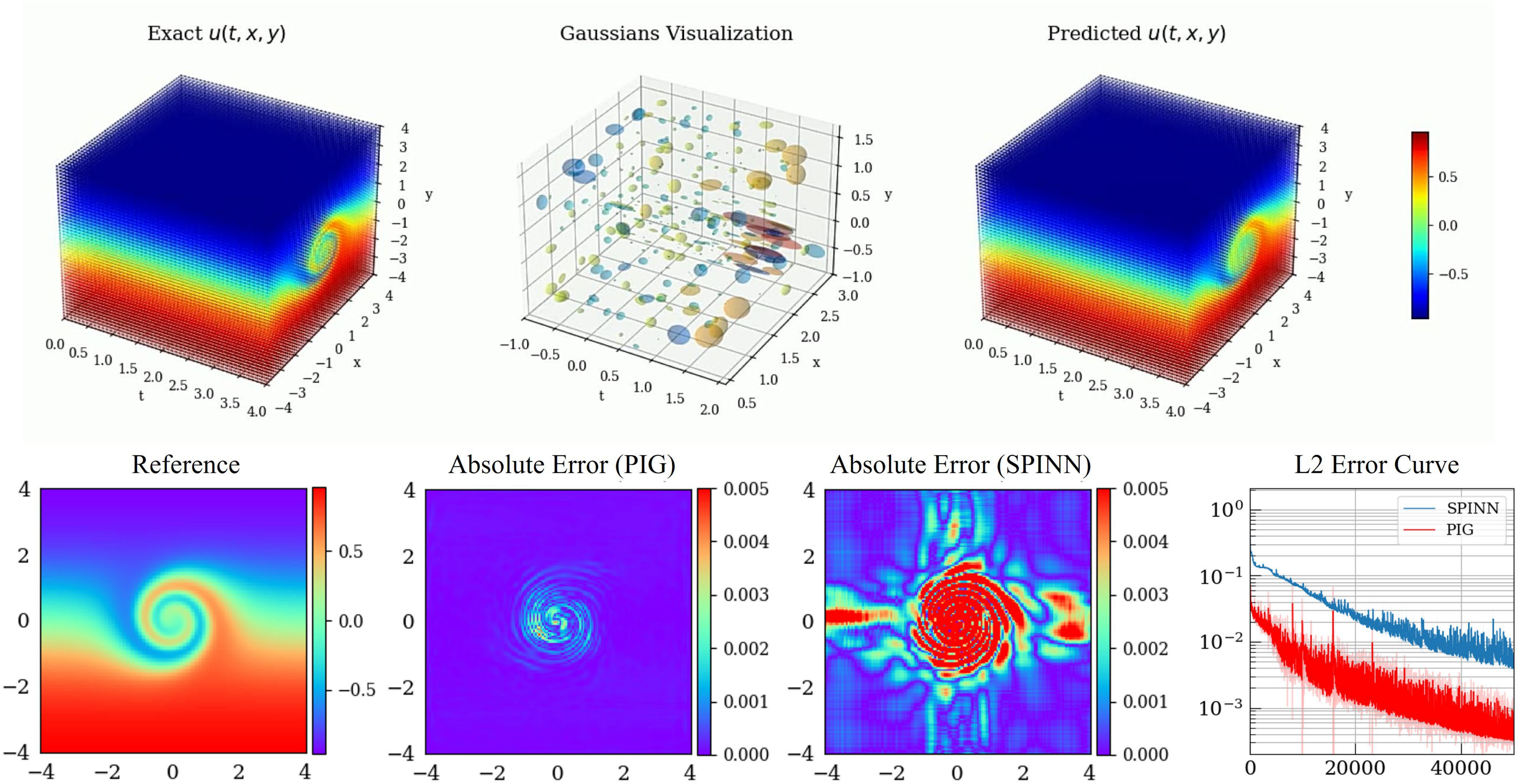
Experiments (Helmholtz Eq.) Best L2 rel. Error: 2.12e-05



Experiments (Klein-Gordon Eq.) Best L2 rel. Error: 2.36e-03

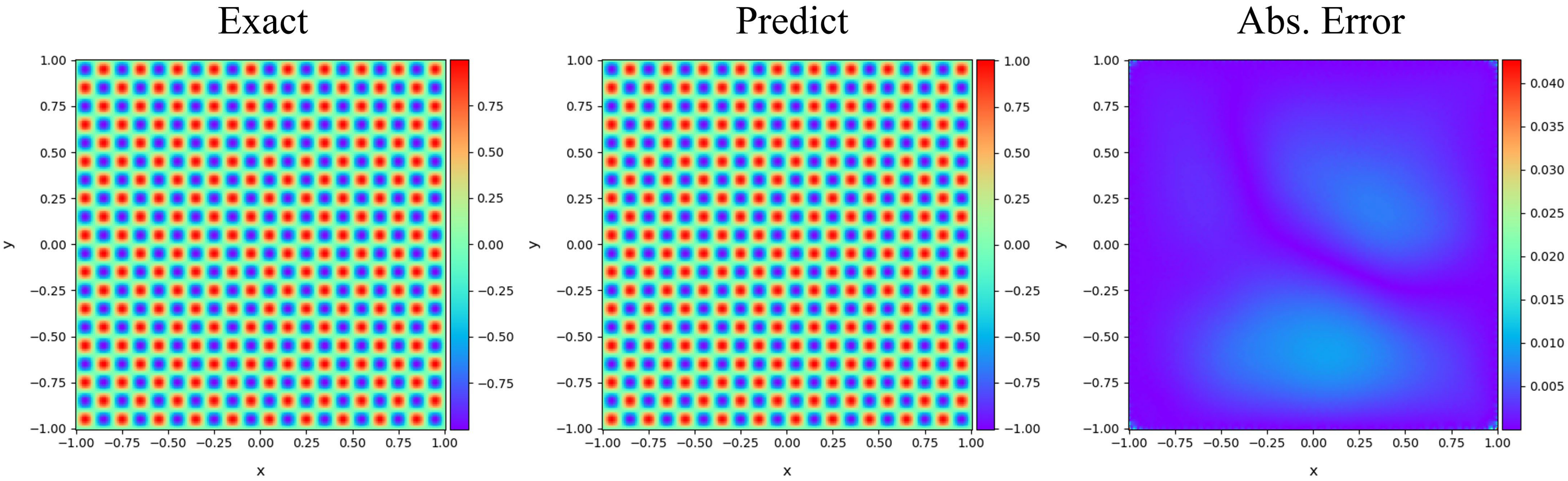


Experiments (Flow-Mixing Eq.) Best L2 rel. Error: 2.67e-04



Experiments

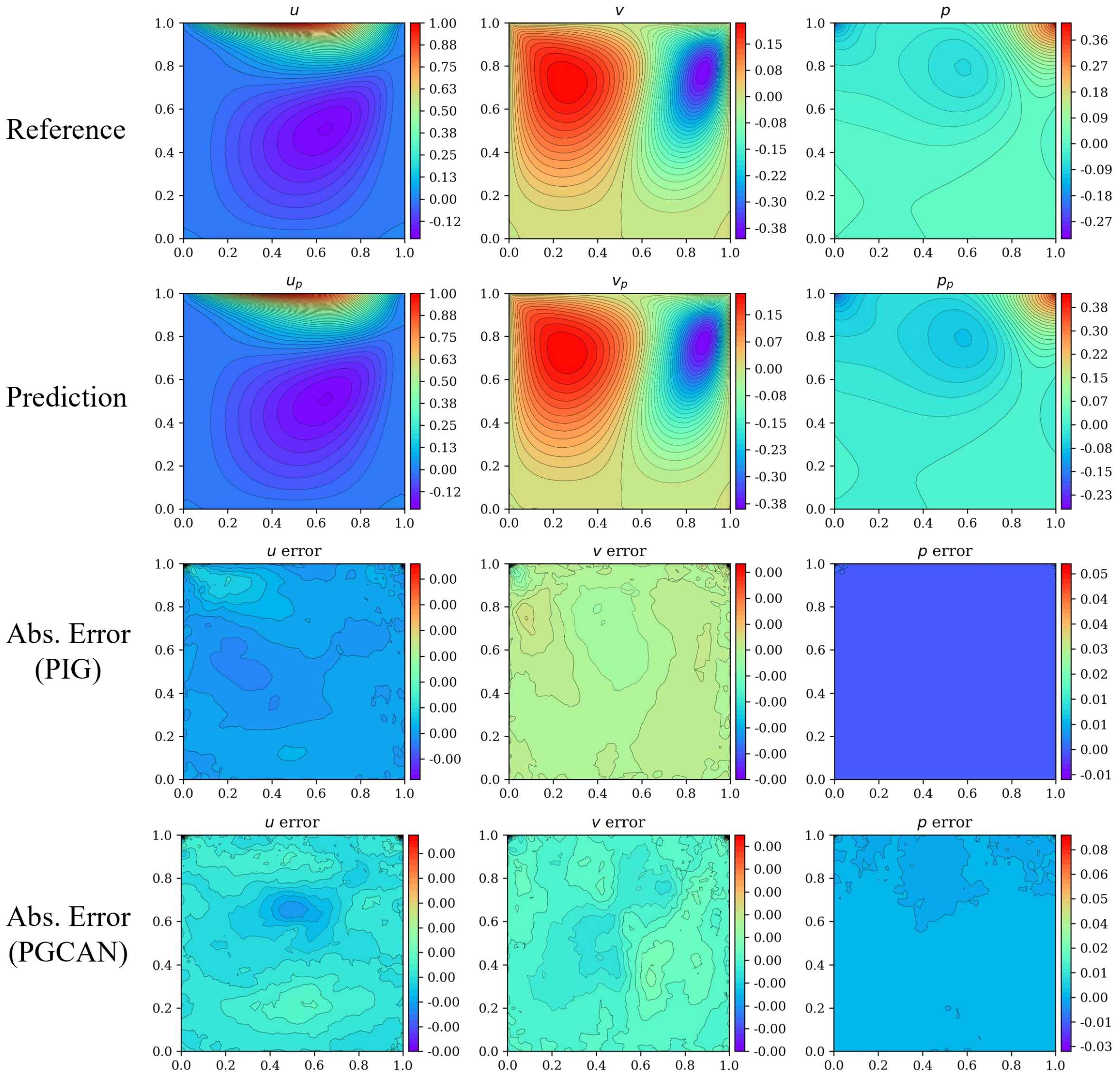
Helmholtz Eq. with high frequency components $(a_1, a_2) = (10, 10)$
Our L2 rel. Error: 7.09e-03
Baseline L2 rel. Error : 7.47e-02

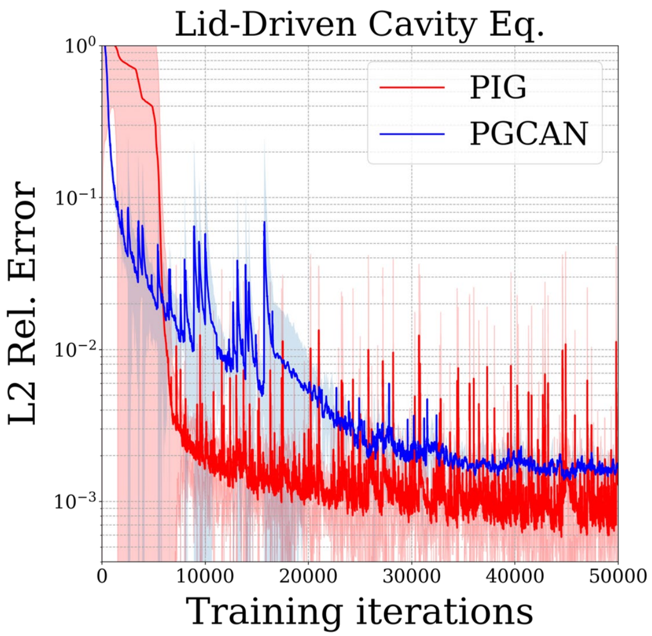


Experiments (Lid-Driven Cavity Eq.)

Our L2 rel. Error: 4.04e-04

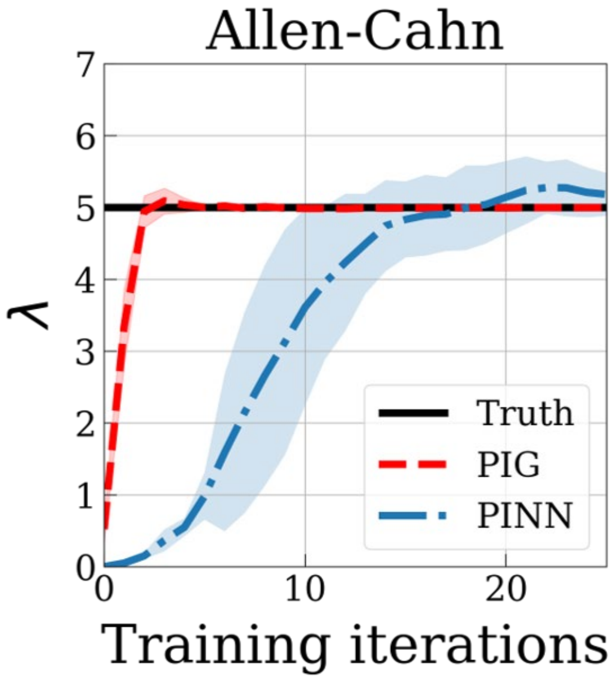
Baseline L2 Rel. Error: 1.22e-03



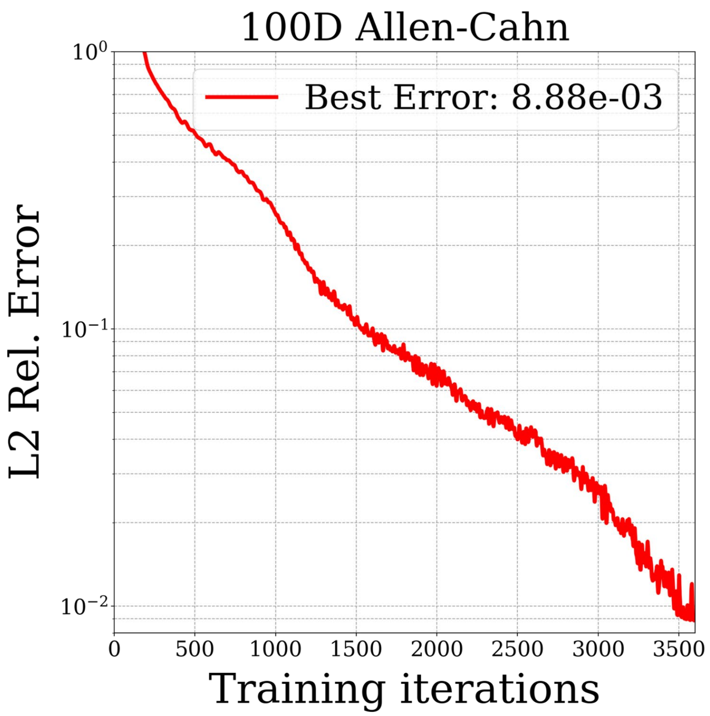


Experiments

Allen Cahn Eq.
Inverse Problem



Allen Cahn Eq.
100 Dim. Forward Problem



Poisson Eq.
100 Dim. Forward Problem