Competitive Fair Scheduling with Predictions

Presented by
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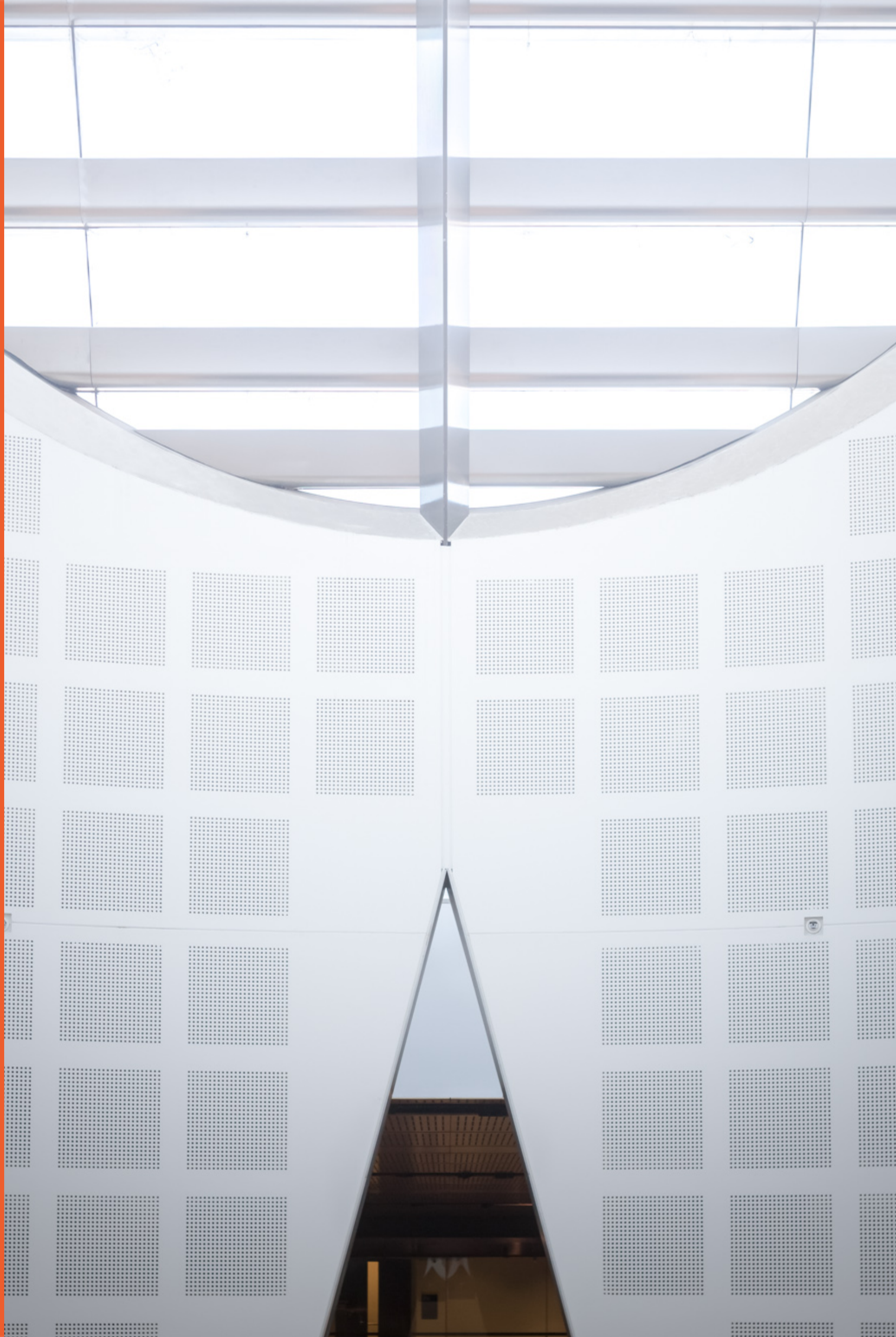

Fair Scheduling with Predictions

Background

Algorithms with predictions (aka learning-augmented algorithms) leverage possibly erroneous predictions to improve decision-making beyond worst-case guarantees.

Problem

- Single machine non-clairvoyant scheduling to minimize max-stretch.
  - $S_j = \frac{C_j - r_j}{p_j^*}$ (stretch of job J_j)
  - C_j (completion time), r_j (release time), p_j^* (job size, i.e., processing time)
  - max-stretch $= \max_{1 \le j \le n} S_j = \max_{1 \le j \le n} \frac{C_j - r_j}{p_j^*}$
- Max-stretch is a fairness measure.
- Job sizes p_j^* are unknown when the job arrives, but a prediction p_j is known. The predictions might not be accurate, and we use $\eta = \max_{1 \le j \le n} \eta_j = \max_{1 \le j \le n} \max\{p_j^*/p_j, p_j/p_j^*\}$ to represent the total prediction error.

Question: How *to perform fair scheduling with imperfect information?*

Key Results

Improved fairness in scheduling

We propose a family of algorithms for different prediction models. In particular, *Relaxed-Greedy (RG)* achieves an $O(\eta^3\sqrt{P})$ competitive ratio, where *P* is the maximum job size ratio.

Consistency-smoothness trade-off

We show how consistency and smoothness trade-off can be achieved by introducing a parametrized algorithm called RG^x.

Enhanced robustness via resource augmentation

The algorithms we initially have suffered poor robustness. We propose to use resource augmentation to address this issue.

Algorithm Overview

Overview

- Key idea: Reduce the problem to two-job-size case (short vs. long jobs) using predictions. If there are only two job sizes, the jobs of the same type should be processed in a FIFO manner.
- Classify jobs based on job size predictions: job J_j short if $p_j \leq \sqrt{p_{\min} p_{\max}}/\mu$, else long.
- Approximate the stretch using $(C_j - r_j)/\sqrt{p_{\min} p_{\max}}$ for a short job and $(C_j - r_j)/p_{\max}$ for a long job; this is called the *relaxed stretch*.
- Greedily schedule the jobs by always running the one with a minimal *relaxed stretch*.

Analysis

- Classifying the jobs into short and long overestimates the job size by $\eta^2\sqrt{P}$, but the problem is reduced to the two-job-size case.
- The greedy heuristic minimizes the relaxed stretch within a constant factor.
- The optimal relaxed stretch is bounded by η times the optimal max-stretch.
- Therefore, the max-stretch bound of our algorithm is $O(\eta^3\sqrt{P})$ times the optimum.

Consistency-Smoothness Trade-offs

- How can we design algorithms that adapt to different levels of prediction confidence?
- Given a user-defined parameter *x*, use $\alpha = p_{\min}^{x} p_{\max}^{1-x} / \sqrt{3}$ to classify long/short jobs.
- The resulting algorithm RG^x has a competitive ratio of $O(\eta^{2+2x} P^{1-x})$.
- At $x = 0$, RG^0 simplifies to *FIFO* (*P*-competitive), achieving the worst consistency $O(P)$ but optimal smoothness $O(1)$. At $x = 1/2$, $\mathsf{RG}^{1/2}$ is the *Relaxed-Greedy* with the best consistency $O(\sqrt{P})$ but the worst smoothness $O(\eta^3)$.
- RG^x interpolates *FIFO* and *Relaxed-Greedy*.

Bounding Robustness via Resource Augmentation

- All algorithms mentioned suffer poor robustness.
- We propose to use resource augmentation to address this issue.
- Suppose you have a $(1+\epsilon)$-speed machine. Run *RG* in the unit-speed and *Round-Robin (RR)* in ϵ-speed. This yields a $(1+\epsilon)$-speed $O(\min\{\eta^3\sqrt{P}, n/\epsilon\})$-competitive algorithm named *RR-augmented RG*.
- *RR*-augmented *RG* achieves the asymptotically optimal robustness.

Thank you

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