IIT Bombay

Sensitivity Analysis for Decision Tree Ensembles

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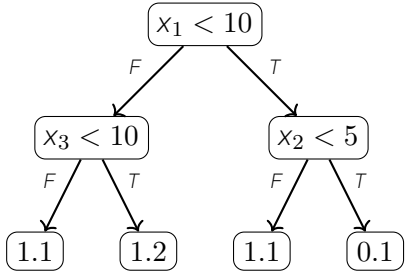
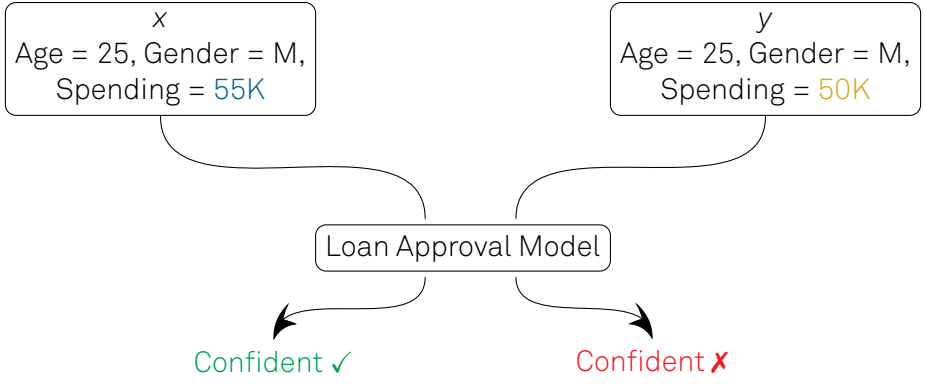
Decision Tree Ensembles

- Tree ensemble models, such as GBDTs and random forests, are widely popular models for a variety of machine learning tasks.
- Given an input $x = (x_1, x_2, x_3)$, a decision tree looks as follows:



- Multiple trees are aggregated by adding their outputs.
  - $c_{raw}(x) = \Sigma_{i=1}^{n} T_i(x)$
  - $c_{label}(x) = \sigma(c_{raw}(x))$
- If $c_{label}(x) > 0.5$ then it is classified as the positive class.

The Sensitivity Problem



Sensitivity Verification Problem

Defn. (p, F)-sensitivity

- Given binary classifier model $c : \mathbb{R}^n \to \{0, 1\}$, a confidence threshold $0 < p < 0.5$ and a subset of features F, do there exist inputs x and $y \in \mathbb{R}^n$ such that :
  - x and y have the same values for features not in F
  - $c_{label}(x) \geq 0.5 + p$
  - $c_{label}(y) \leq 0.5 - p$

Why this matters?

Sensitivity verification is crucial to building **trustworthy AI** systems.

- **Fairness**: Sensitive features (e.g., gender, race) should not unfairly influence decisions.
- **Interpretability**: Understanding which features drive decisions helps build trust in AI systems.
- **Security**: Manipulation of a small sensitive subset of features may enable adversarial attacks.

Contributions

1. We prove that the Sensitivity Verification problem is NP-Hard
   - The Sensitivity Verification problem, that is, checking whether a given tree ensemble classifier is sensitive to a feature set F, is NP-Hard for decision tree ensembles with trees of depth ≥ 3
2. We create an encoding for approximate representations of tree ensembles using Pseudo-Boolean constraints
3. We develop a tool, SensPB, to verify the sensitivity of tree ensembles, beating previous methods by a big margin

Pseudo-Boolean Constraints

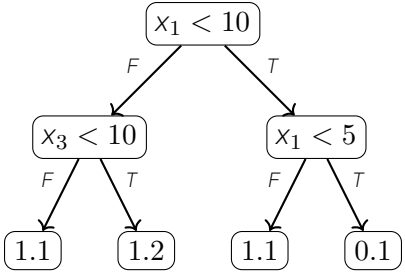
Inequalities involving Boolean variables.

$$\sum a_i x_i \geq A$$

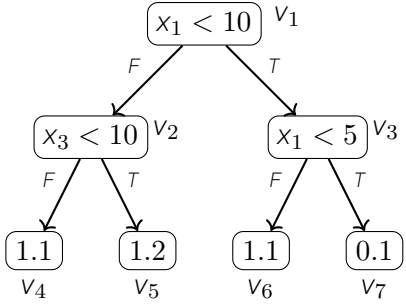
$$x_i \in \{0, 1\}$$

- State of the art solvers are relatively new (5 years or so)
- Stronger than SAT
- Natural for encoding tree ensembles
- Solvers require integer weights

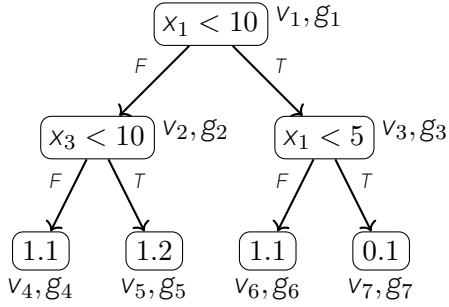
Encoding



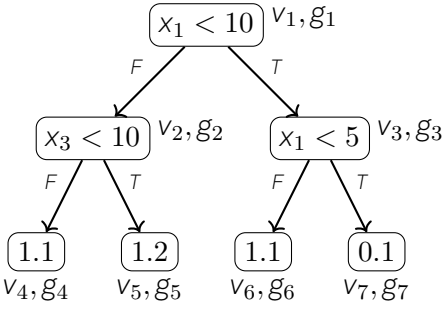
Encoding



Encoding

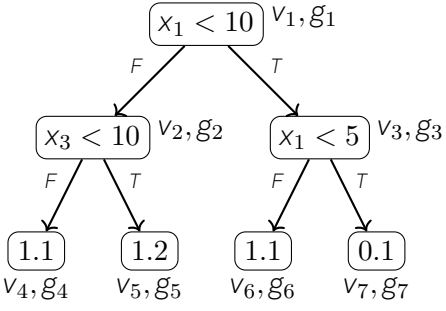

$x_1 < 10$
v_1, g_1
F
T
$x_3 < 10$
v_2, g_2
$x_1 < 5$
v_3, g_3
F
T
F
T
1.1
1.2
1.1
0.1
v_4, g_4
v_5, g_5
v_6, g_6
v_7, g_7

Encode Tree



- $v_1 \wedge g_1 \implies v_3$
- $v_1 \wedge \neg g_1 \implies v_2$
- $v_3 \wedge g_3 \implies v_7$
- ...
- $g_3 \implies g_1$

Encode Classification



SMT Encoding:

$$1.1v_4 + 1.2v_5 + 1.1v_6 + 0.1v_7 \leq -\delta$$

$$1.1v_4 + 1.2v_5 + 1.1v_6 + 0.1v_7 \geq \delta$$

SensPB Encoding: Pick a precision α

$$\lceil \alpha \cdot 1.1 \rceil v_4 + \lceil \alpha \cdot 1.2 \rceil v_5 + \lceil \alpha \cdot 1.1 \rceil v_6 + \lceil \alpha \cdot 0.1 \rceil v_7 \geq \lfloor \alpha \cdot \delta \rfloor$$

$$\lfloor \alpha \cdot 1.1 \rfloor v_4 + \lfloor \alpha \cdot 1.2 \rfloor v_5 + \lfloor \alpha \cdot 1.1 \rfloor v_6 + \lfloor \alpha \cdot 0.1 \rfloor v_7 \leq \lceil -\alpha \cdot \delta \rceil$$

where $\delta = \sigma^{-1}(0.5 + p)$.

Existing Work

- Veritas: Construct a graph using the leaf values and find max-cliques using approximate algorithms
- SMT-based approach: Encode the ensemble tree into an SMT formula and pass it to an SMT Solver

Results

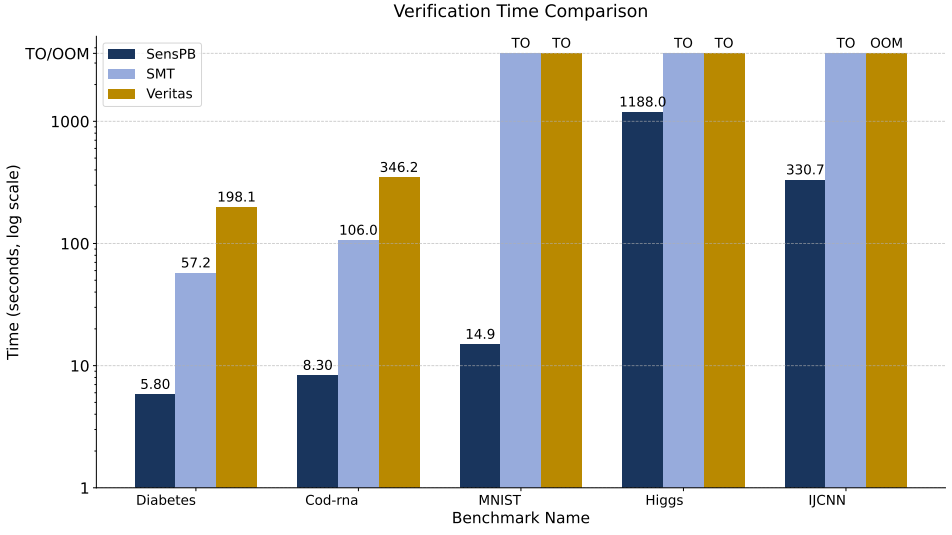


Figure: Time for running the three tools, on a log scale. TO = Time out, set to 3600 seconds. OOM = Out Of Memory

Benchmarks

Table: Tree Ensemble Sizes

Benchmark	#Trees	Depth	#Features
Diabetes	20	5	9
Cod-rna	80	4	8
Binary MNIST	50	6	784
Higgs	100	8	28
IJCNN	60	8	23

Future Work

- Quantifying Sensitivity of models
- Training Insensitive Models
- Further Scalability

Paper and Code

- Paper: https://openreview.net/pdf?id=h0vC0fm1q7
- Code: https://github.com/Arhaan/SensPB

Thanks for attending!

Acknowledgments

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