



**ICLR**

# Offline RL with Smooth OOD Generalization in Convex Hull and its Neighborhood

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ICLR 2025



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# Outline

Introduction

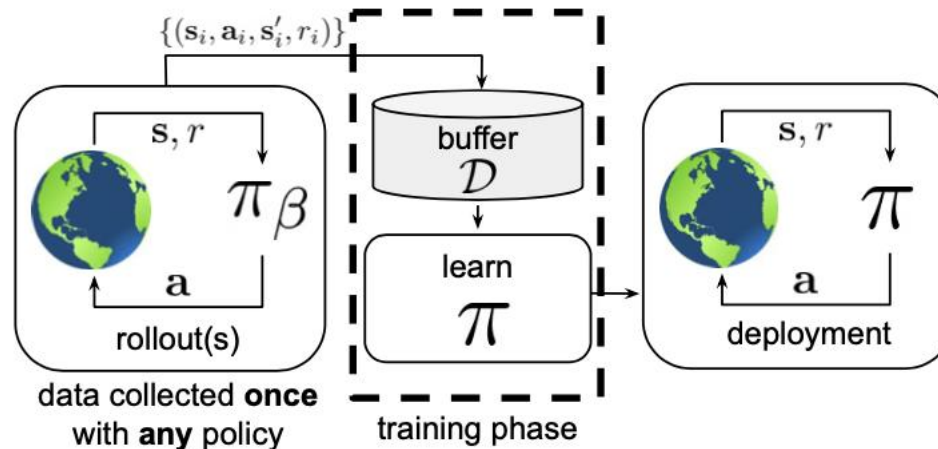
Methods

Experiments

# Introduction

## Background: Offline Reinforcement Learning

- Offline Reinforcement Learning (RL) learns the optimal policy solely from offline datasets  $D = \{(s_i, a_i, r_i, s'_i, d_i)\}_{i=1}^N$ ,  $d_i \in \{0, 1\}$
- Key challenges
  - The **distribution shift** between behavior policy  $\mu$  (dataset policy) and learned policy  $\pi$
  - The **overestimation** issue of **out-of-distribution (OOD)** actions, leading to suboptimal policy

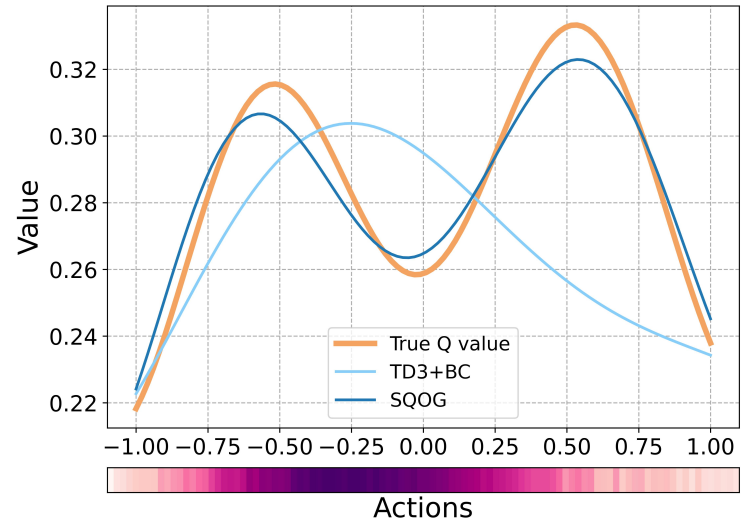
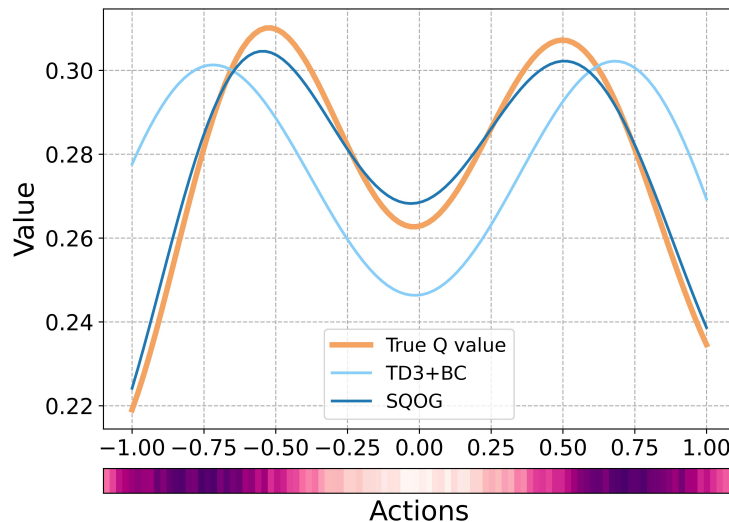


# Introduction

## Background: Over-constraint issue in Offline RL

- Recent solutions are too conservative, introducing an over-constraint issue.

- Over-constraint issue (on Q-value)



- TD3+BC: the learned policy is **overly close** to the behavior policy
- SQOG (our method): alleviates the over-constraint issue

- Goal: improve Q-value estimation by enhancing Q-function generalization in dataset OOD regions.

# Introduction

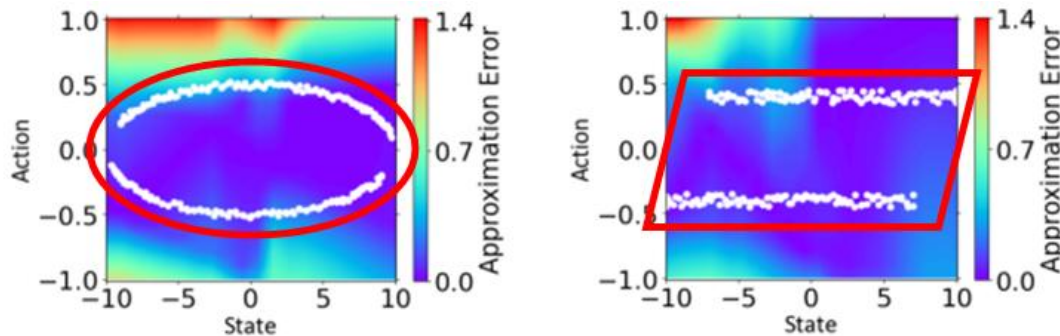
## Contributions

- ✓ Under the safety guarantees of the Convex Hull and its Neighborhood (CHN), we propose the **Smooth Bellman Operator (SBO)**, which enhances Q-function generalization in OOD regions and approximates the true Q-values.
- ✓ Building on SBO, we design an effective algorithm, **SQOG**, which **alleviates the over-constraint issue** and **obtains SOTA results** on D4RL benchmarks.

# Methods

## Safety guarantees of the generalization within CHN

- Safety guarantee 1: Q-value difference is controlled within CHN
  - Previous work (DOGE, 2023) demonstrated that Q-value difference is controlled within the convex hull, we extend this result to the CHN.



- Safety guarantee 2: Q-function is uniformly continuous within CHN

These two guarantees ensure safer and more reliable Q-function generalization in **OOD regions within CHN!**

# Methods

## Smooth Bellman Operator

### Definition of SBO

$$\tilde{\mathcal{B}}^\pi Q(s, a) = (\mathcal{G}_1 \hat{\mathcal{B}}_2^\pi) Q(s, a)$$

### Base Bellman operator

$$\hat{\mathcal{B}}_2^\pi Q(s, a) = \begin{cases} \hat{\mathcal{B}}^\pi Q(s, a), & \hat{\mu}(a|s) > 0 \\ Q(s, a), & \hat{\mu}(a|s) = 0 \text{ and } (s, a) \in CHN \end{cases}$$

empirical Bellman operator (CQL, 2020)

### Smooth generalization operator

$$\mathcal{G}_1 Q(s, a) = \begin{cases} Q(s, a), & \hat{\mu}(a|s) > 0 \\ Q(s, a_{neighbor}^{in}), & \hat{\mu}(a|s) = 0 \text{ and } (s, a) \in CHN \end{cases}$$

in-sample neighbor action of the OOD action  $a$

OOD action within CHN

# Methods

## Theoretical justification for SBO

- Why  $\hat{Q}_{\theta}^{\pi}(s, a_{neighbor}^{in})$  is an appropriate OOD target ?
- Goal: let  $\hat{Q}_{\theta}^{\pi}(s, a^{ood})$  (the output of value network) **approximate the true OOD Q-value**  $Q^{\pi}(s, a^{ood})$

- Proposition 3: if  $\hat{Q}_{\theta}^{\pi}(s, a^{in}) \approx Q^{\pi}(s, a^{in})$  , then

$$\|Q^{\pi}(s, a^{ood}) - \hat{Q}_{\theta}^{\pi}(s, a_{neighbor}^{in})\| < \varepsilon$$

- Theorem 1 shows  $\hat{Q}_{\theta}^{\pi}(s, a^{in}) \approx Q^{\pi}(s, a^{in})$  , when the KL-divergence of learned policy and behavior policy is bound
- $\|\hat{\mathcal{B}}^{\pi} Q_{\theta} - \mathcal{B}^{\pi} Q_{\theta}\|$  is bound, for all  $(s, a) \in \mathcal{D}$
- $\hat{Q}_{\theta}^{\pi}(s, a^{in})$  closely approximates  $Q_{\theta}^{\pi}(s, a^{in})$
- $Q_{\theta}^{\pi}(s, a^{in})$  is close to true in-sample Q-value  $Q^{\pi}(s, a^{in})$  (PRDC, 2023)
- $\hat{Q}_{\theta}^{\pi}(s, a^{in}) \approx Q^{\pi}(s, a^{in})$



# Methods

## Effects of the SBO

- ▣ SBO achieves better Q-value estimation
  - For in-sample evaluation, SBO introduces negligible changes to the empirical Bellman operator. (Theorem 2)
  - For OOD evaluation, SBO helps mitigate underestimation and overestimation. (Theorem 3)
- ▣ Based on the SBO, we develop the algorithm SQOG.

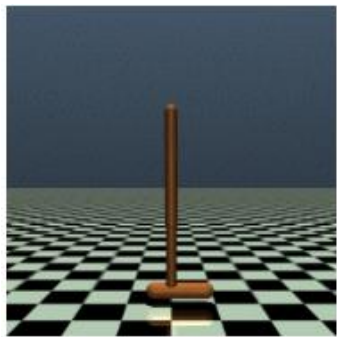
# Experiments

## Results on D4RL Benchmarks

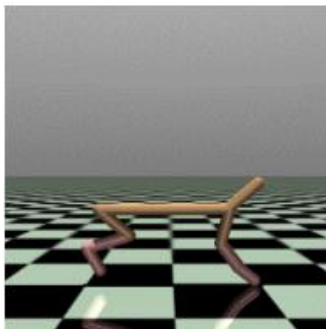
### ▣ D4RL benchmarks

- D4RL is the **most widely** used benchmarks in offline RL.
- **Gym-Mujoco** are the **locomotion** tasks (Hopper, Halfcheetah, Walker2d)
- **Maze2D** is a **navigation** task requiring a 2D agent to reach a fixed goal location.
- **Adroit** involves **controlling Hand robot** tasked with hammering a nail, opening a door, twirling a pen, or picking up and moving a ball.
- Performance Evaluation: **Normalized score** (100 → expert, 0 → random).

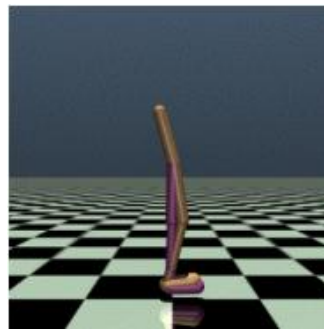
$$\text{normalized score} = 100 * \frac{\text{score} - \text{random score}}{\text{expert score} - \text{random score}}$$



Hopper



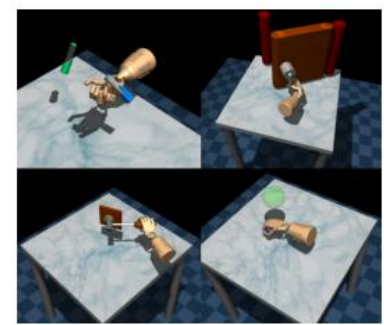
Halfcheetach



Walker2d



Maze2d



Adroit

# Experiments

## Results on D4RL Benchmarks

### □ SQOG obtains SOTA results on benchmark datasets

Dataset	BC	TD3+BC	CQL	IQL	DOGE	MCQ	SQOG
halfcheetah-r	2.2±0.0	11.0±1.1	17.5±1.5	13.1±1.3	17.8±1.2	23.6±0.8	<b>25.6±0.4</b>
hopper-r	3.7±0.6	8.5±0.6	7.9±0.4	7.9±0.2	21.1±12.6	<b>31.0±1.7</b>	15.6±3.3
walker2d-r	1.3±0.1	1.6±1.7	5.1±1.3	5.4±1.2	0.9±2.4	10.3±6.8	<b>17.7±3.5</b>
halfcheetah-m	43.2±0.6	48.3±0.3	47.0±0.5	47.4±0.2	45.3±0.6	58.3±1.3	<b>59.2±2.4</b>
hopper-m	54.1±3.8	59.3±4.2	53.0±28.5	66.2±5.7	98.6±2.1	73.6±10.3	<b>100.6±0.7</b>
walker2d-m	70.9±11.0	83.7±2.1	73.3±17.7	78.3±8.7	86.8±0.8	<b>88.4±1.3</b>	82.9±0.8
halfcheetah-m-r	37.6±2.1	44.6±0.5	45.5±0.7	44.2±1.2	42.8±0.6	<b>51.5±0.2</b>	46.4±1.2
hopper-m-r	16.6±4.8	60.9±18.8	88.7±12.9	94.7±8.6	76.2±17.7	99.5±1.7	<b>100.9±5.1</b>
walker2d-m-r	20.3±9.8	81.8±5.5	81.8±2.7	73.8±7.1	87.3±2.3	83.3±1.9	<b>88.3±3.5</b>
halfcheetah-m-e	44.0±1.6	90.7±4.3	75.6±25.7	86.7±5.3	78.7±8.4	85.4±3.4	<b>92.6±0.4</b>
hopper-m-e	53.9±4.7	98.0±9.4	105.6±12.9	91.5±14.3	102.7±5.2	106.1±2.3	<b>109.2±2.8</b>
walker2d-m-e	90.1±13.2	110.1±0.5	107.9±1.6	109.6±1.0	<b>110.4±1.5</b>	110.3±0.1	109.0±0.3
Mujoco Average	36.5	58.2	61.8	59.9	64.1	68.4	<b>70.7</b>
Maze2d Average	-2.0	35.0	19.6	37.2	-	102.2	<b>124.7</b>
Adroit Total	93.9	0.0	93.6	110.7	-	123.3	<b>149.6</b>
Runtime (h)	0.3	0.4	10.8	0.4	0.9	8.0	0.4

- SQOG **consistently attains the highest scores** on most datasets (8/12) and achieves the **highest average scores (bold)** across the Mujoco, Maze2d, and Adroit tasks, with low computational cost.

# Experiments

## Sanity Check: alleviation of the over-constraint issue

### □ SQOG alleviates the over-constraint issue

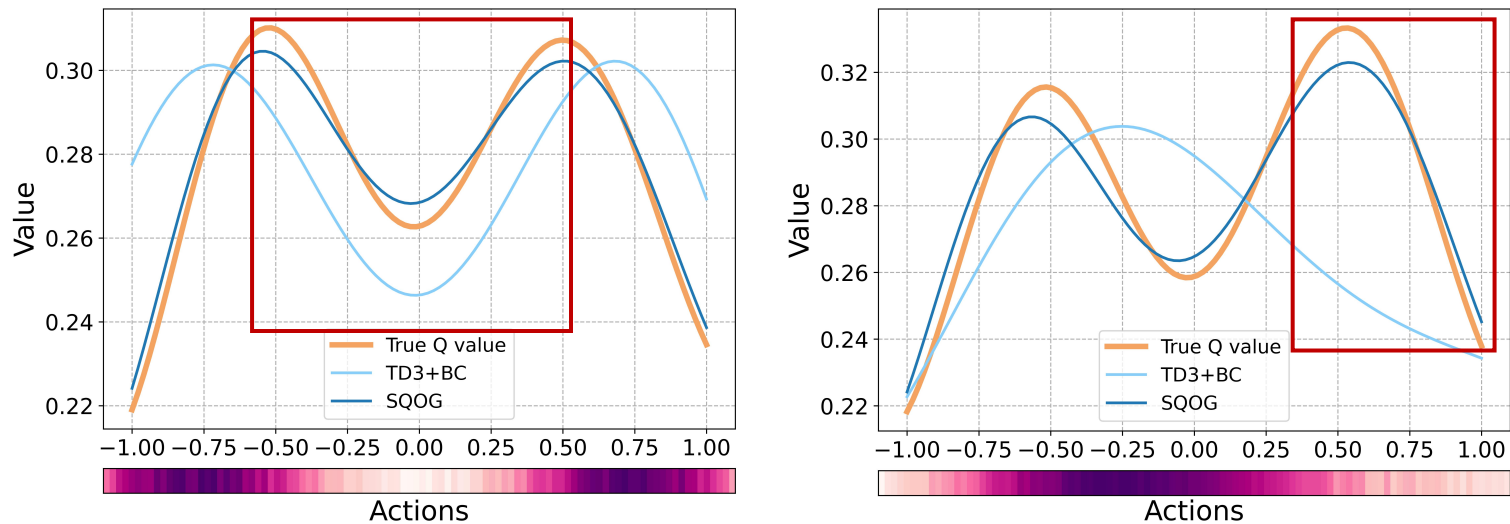


Fig. Q values estimation of actions in two key states.

- The highest true value exists in  $[-0.50, 0.50]$  (left), which corresponds to OOD regions within the **convex hull**.
- The highest true value exists in  $[0.30, 1.00]$  (right), corresponding to OOD regions in **the neighborhood** of the convex hull.

# Experiments

## Sanity Check: alleviation of the over-constraint issue

### □ SQOG alleviates the over-constraint issue

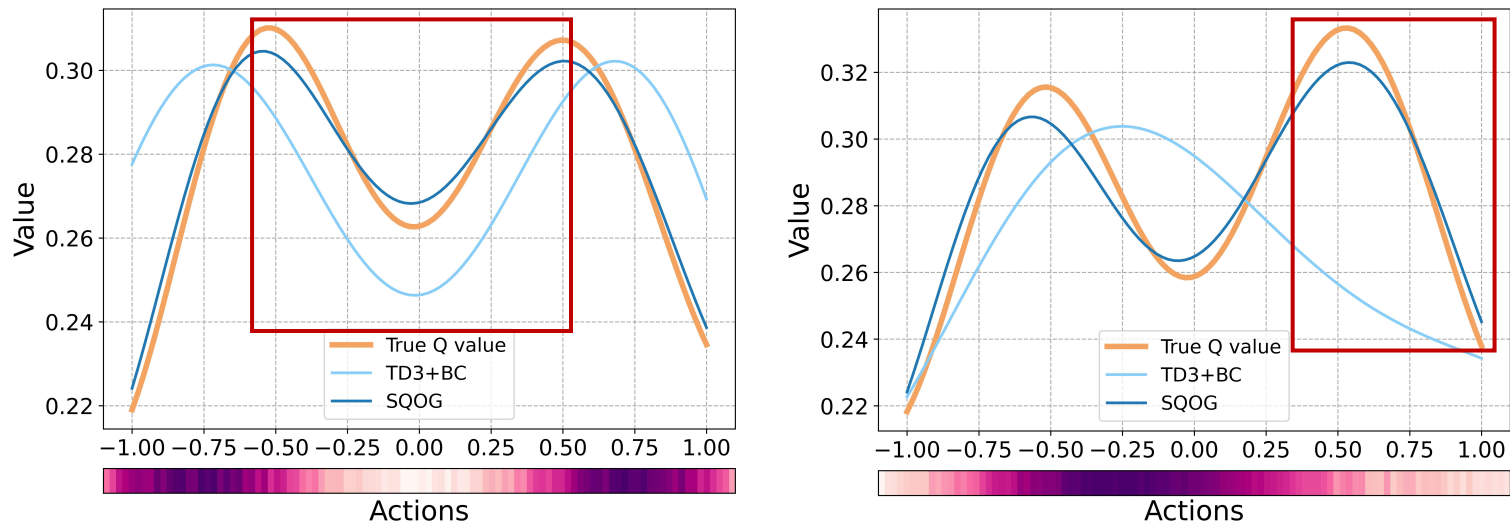


Fig. Q values estimation of actions in two key states.

- TD3+BC encounters the **over-constraint issue** in these OOD regions, failing to leverage **implicit OOD information** within CHN.
- SQOG **accurately** estimates Q-values through **smooth OOD generalization** within the CHN (convex hull and its neighborhood).

# Experiments

## Generalizability of SBO

- SBO is a versatile plug-in for policy constraint methods.

Dataset	BRAC	BRAC+SBO
halfcheetah-medium	49.8±1.2	<b>54.3±1.2</b>
hopper-medium	3.6±3.1	<b>90.9±2.9</b>
walker2d-medium	7.8±8.1	<b>85.6±4.3</b>
halfcheetah-medium-replay	41.8±6.2	<b>47.8±2.0</b>
hopper-medium-replay	28.8±20.3	<b>61.1±11.9</b>
walker2d-medium-replay	8.5±3.0	<b>67.6±11.0</b>
Mujoco Average	23.4	<b>67.9</b>
Improvement	-	<b>190.2%</b>
pen-human	19.2±16.3	<b>69.7±8.7</b>
pen-cloned	28.4±23.4	<b>69.0±14.8</b>
Adroit Average	23.8	<b>69.4</b>
Improvement	-	<b>191.6%</b>

- A **significant** performance improvement when SBO is added to BRAC.
- SBO serves as a valuable **complement** to policy constraint methods.



# Summary and Takeaways

- ▣ We present a method that broadly alleviates the **over-constraint** issue in policy constraint methods, achieving SOTA performance with **low computational cost**.
- Better Q-value **estimation** leads to better policy **performance**.
- **Neighboring in-sample** Q-values serve as appropriate targets for over-constrained OOD Q-values.

Thank you!