Certified Robustness Under Bounded Levenshtein Distance

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Adversarial Attacks in Sentence Classification

Elections in Spain: Spanish citizens choose their next president this sunday.

Jordan scored a tripledouble in today's match against Celtics.

Tesla stock went up 20% after Elon Musk's last tweet.

PyTorch 3.0 just released, is there any future for TensorFlow?



Sports	Business	Sci/Tech		
0	0	0		
1	0	0		
0	0.966	0.026		
0	0.001	0.999		

textattack/bert-base-uncased-ag-news

World

0.008



Adversarial Attacks in Sentence Classification

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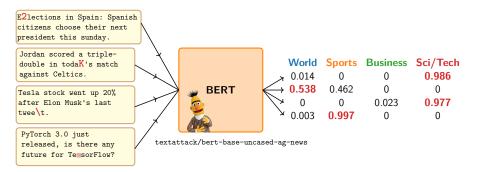


	World	Sports	Business	Sci/Tech
Х	0.014	0	0	0.986
\rightarrow	0.538	0.462	0	0
×	0	0	0.023	0.977
Ā	0.003	0.997	0	0

textattack/bert-base-uncased-ag-news



Adversarial Attacks in Sentence Classification



How can we obtain certifiably robust models? 🤔





Robustness Verification

• The worst case margin needs to be positive:

$$g_{y,\hat{y}}(\boldsymbol{S'}) = f_{\boldsymbol{\theta}}(\boldsymbol{S'})_y - \max_{\hat{y} \neq y} f_{\boldsymbol{\theta}}(\boldsymbol{S'})_{\hat{y}} > 0, \ \forall \boldsymbol{S'} : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S'}) \leq k \,,$$

where f_{θ} is our classifier, (S, y) the original sentence and its label, d_{Lev} the Levenshtein distance and k our perturbation budget.



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O How can we check this condition?



Brute Force: An illustrative example

 $\emph{\textbf{S}}=$ "Elections in Spain: Spanish citizens choose their next president this Sunday."

$$y = 1$$
 ("World")

o Can we just try all of the sentences?

$$\arg\max_{j} f(\boldsymbol{S}')_{j} = 1 \qquad \forall \boldsymbol{S}' \in \left\{ \begin{array}{l} \text{"Elec!tions in SpaiT: Spanish} \cdots \text{"} \\ \text{"3lections in Spain:4Spanish} \cdots \text{"} \\ \vdots \\ \text{"Elections.in Sain: Spanish} \cdots \text{"} \end{array} \right\}$$



Existing approaches

• The number of sentences to test grows exponentially with k



$$\left| \left\{ \begin{array}{l} \text{"Elec!tions in SpaiT: Spanish} \cdots \text{"} \\ \text{"3lections in Spain: 4Spanish} \cdots \text{"} \\ \vdots \\ \text{"Elections.in Sain: Spanish} \cdots \text{"} \end{array} \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \left\{ \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right\} \right| = \left| \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \leq k \right| = \left| \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \right| = \left|$$

Existing approaches also require a large number of forward passes:



Huang et al., Achieving verified robustness to symbol substitutions via interval bound propagation. EMNLP 2019.



Huang et al., Edit distance robustness certificates for sequence classifiers via randomized deletion. NeurIPS 2023.



Our idea

Lipschitzness + large enough margin ⇒ Robustness

$$|g_{y,\hat{y}}(\boldsymbol{S}) - g_{y,\hat{y}}(\boldsymbol{S}')| \le G \cdot d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \\ + \\ g_{y,\hat{y}}(\boldsymbol{S}) > k \cdot G \end{cases} \Rightarrow g_{y,\hat{y}}(\boldsymbol{S}') > 0 \quad \forall \boldsymbol{S}' : d_{\mathsf{Lev}}(\boldsymbol{S}, \boldsymbol{S}') \le k$$

where
$$g_{y,\hat{y}}(\boldsymbol{S}) = f(\boldsymbol{S})_y - f(\boldsymbol{S})_{\hat{y}}$$
.

This paper: We are able to compute G for convolutional classifiers.



Some results

Table: Verified accuracy in AG-News: IBP, proposed by (Huang et al., 2019)

р	1.	k Acc.(%)	Charmer		BruteF		IBP		LipsLev	
	K		Adv. Acc.(%)	Time(s)	Ver.(%)	Time(s)	Ver.(%)	Time(s)	Ver.(%)	Time(s)
∞	$\frac{1}{2}$	65.23	47.90 32.97	5.70 5.70	47.87	16.15)T	27.77	16.76	32.33 11.60	$0.0015 \\ 0.0015$
1	$\frac{1}{2}$	69.63	54.47 37.77	5.43 5.43	54.43 O (15.33 T	18.93	17.56	$34.50 \\ 12.53$	$0.00140 \\ 0.00140$
2	$\begin{array}{ c c } 1 \\ 2 \end{array}$	74.80	62.20 46.47	$7.32 \\ 7.32$	62.07 O (29.12)T	29.10	31.54	$38.80 \\ 13.93$	$\begin{array}{c} 0.00970 \\ 0.00970 \end{array}$

- \circ Lipslev is the only method able to verify for k > 1.
- o Lipslev is 4 orders of magnitude faster than other methods.



Final remarks

- Conclusion
 - ▷ **LipsLev:** We introduce Lipschitz certification in NLP.
 - > First to verify Levenshtein constraints in a single forward pass!
- Some interesting challenges 🤔
 - ▶ Tokenizers: Can we compute the Lips. constant of tokenizers?
 - > Transformers: Can we compute the Lips. constant of Self-Attention?
 - > **Scalability:** Can we scale the approach to larger models?



Thank you!

• Check our paper: More details, datasets and ablations!

Thanks for your attention!

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