HyperPLR: Hypergraph Generation through Projection, Learning, and Reconstruction

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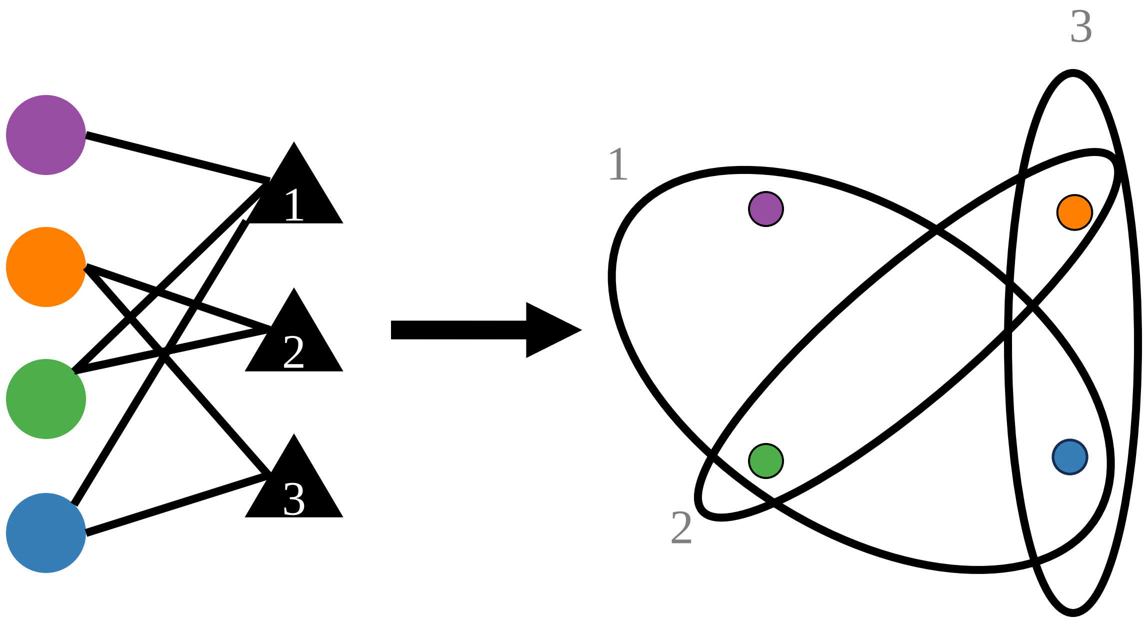
ICLR

Motivation

- Hypergraphs naturally capture **higher-order interactions**, which are crucial in domains like collaboration networks, biological systems, and drug modeling.
- Existing generative models focus on simple graphs or assume specific hypergraph structures, **limiting expressiveness and generalizability**.
- **Goal**: Build a **general, scalable, and learnable** framework to generate realistic hypergraphs.



(Authors) (Publications) **Hypergraph**

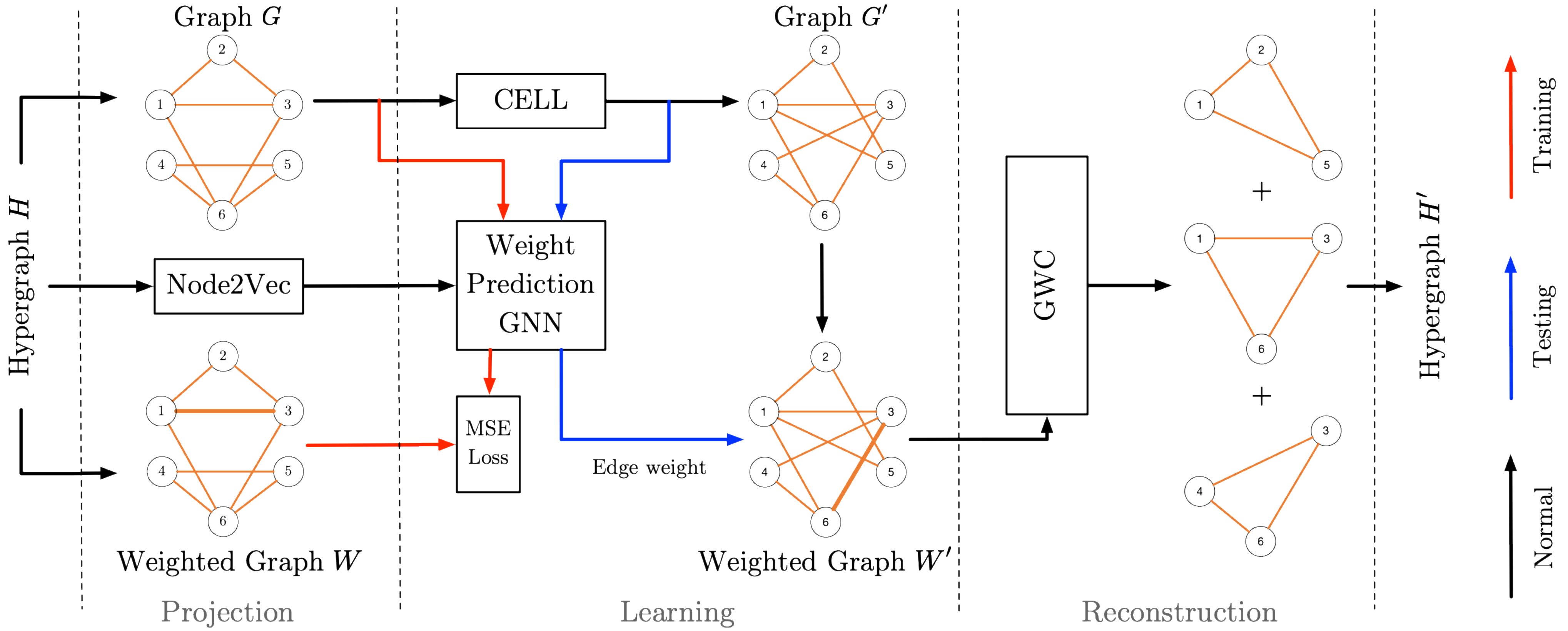
Our Contributions

In this paper, we propose **HyperPLR**, a novel framework for hypergraph generation, with the following key contributions:

1. **Formulation of a new problem**: We introduce the Weighted Clique Edge Cover (WCEC) problem for reconstructing hypergraphs from weighted projections, and propose a fast heuristic solution.
2. **Unified three-stage framework**: HyperPLR integrates Projection, Learning, and Reconstruction phases, enabling end-to-end generation of hypergraphs that preserve high-order structures.
3. **Superior performance**: Extensive experiments on five real-world datasets demonstrate that HyperPLR outperforms state-of-the-art methods across multiple hypergraph-level and projection-level metrics.

HyperPRL Framework

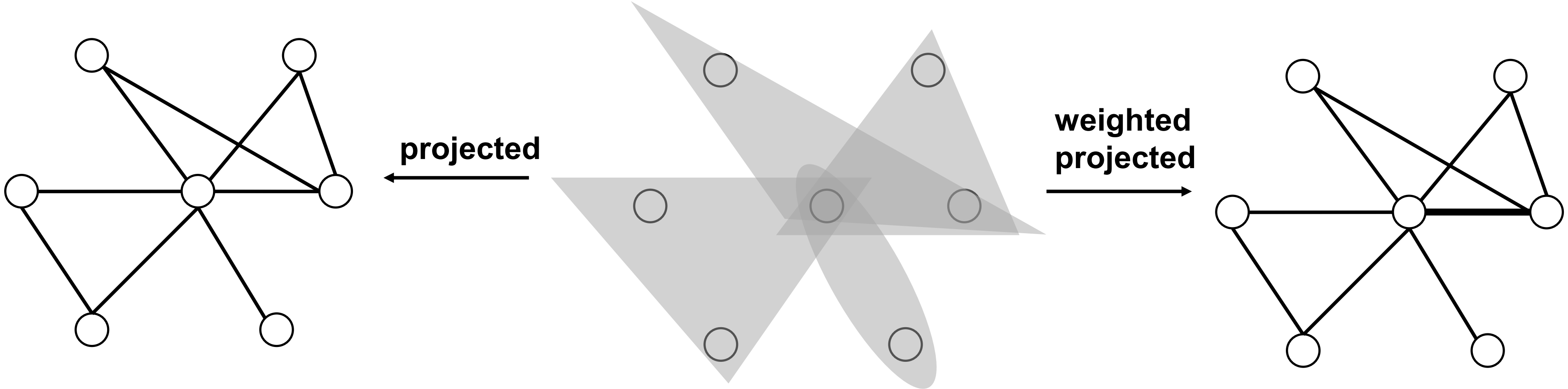
As shown, a high-level overview of the HyperPLR framework consists of three core blocks: **Projection**, **Learning**, and **Reconstruction**, acting in the presented order.



Hypergraph Projection

Graph Projection. A graph projection, or clique expansion of a hypergraph is a transformation process that converts a hypergraph into a traditional graph. Formally, if $\mathcal{H} = (V, \mathcal{E})$ is a hypergraph, the projection graph $G = \text{Proj}(\mathcal{H}) = (V, \mathcal{E}')$, where $\mathcal{E}'$ consists all pairs $(v_i, v_j) \in e_i$ for hyperedge $e_i \in \mathcal{E}$.

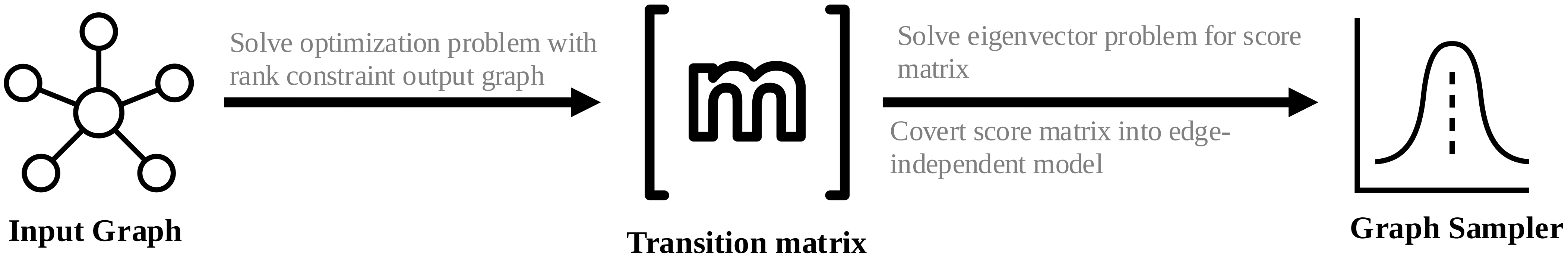
Weighted Graph Projection. The weighted graph projection preserve how frequently vertices co-occur within hyperedges. Formally, given a hypergraph $\mathcal{H} = (V, \mathcal{E})$, the weighted graph projection is to project $\mathcal{H}$ into a weighted graph $G_w = \text{Proj}_\text{w}(\mathcal{H}) = (V, \mathcal{E}, w)$.The w is a weight function that assigns a weight to each edge $e' \in \mathcal{E}'$.



(a) Graph projection (b) Hypergraph (c) Weighted graph projection

Hypergraph Learning

Graph Structure Learning. HyperPLR employ CELL[1] to learn the pattern of the original graph projection and generate new projection. CELL is a graph generative model working in the spectral space by removing redundant computation from NetGAN[2].
Graph Weight Learning. HyperPLR extend CELL by providing predicted weights on the sampled edges, where the edge weight predictor is a GCN. During the training phase, this predictor takes the vertex embeddings $X^{(0)}$ and the adjacency matrix A as input, and outputs a set of new embedding $X^{(l)}$: $X^{(l)} = \mathrm{GCN}(X^{(0)}, A)$, where l is the number of basic GCN layers. The loss is the MSE between the predicted weights $X^{(l)} \cdot X^{(l)}$.



Hypergraph Reconstruction

Reconstructing hypergraphs from their graph projections is a challenging task[3]. HyperPLR raises a novel **Weighted Clique Edge Cover (WCEC)** problem and an associated efficient algorithm Greedy Weighted Cover (GWC) in a hill-climbing manner.

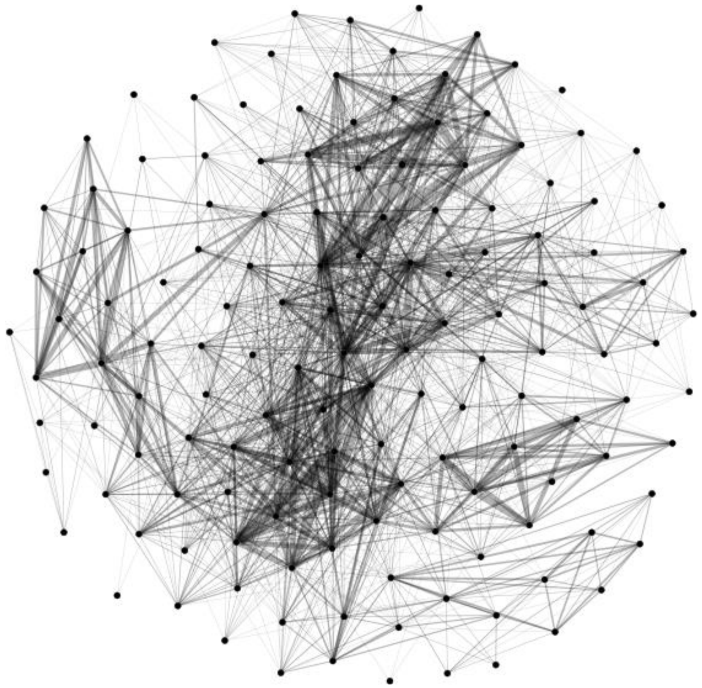
WCEC Problem: Given a weighted graph $\mathcal{G}$ with weight matrix W, it asks to select k cliques from $\mathcal{G}$, such that the distance $\mathrm{d}(W, W') \geq 0$ between W and W' is minimized, where weight matrix W' is derived from a new weighted graph $\mathcal{G}'$ by stacking k selected cliques.

In our setting, we let $\mathrm{d}(W, W')$ be the L1-distance:

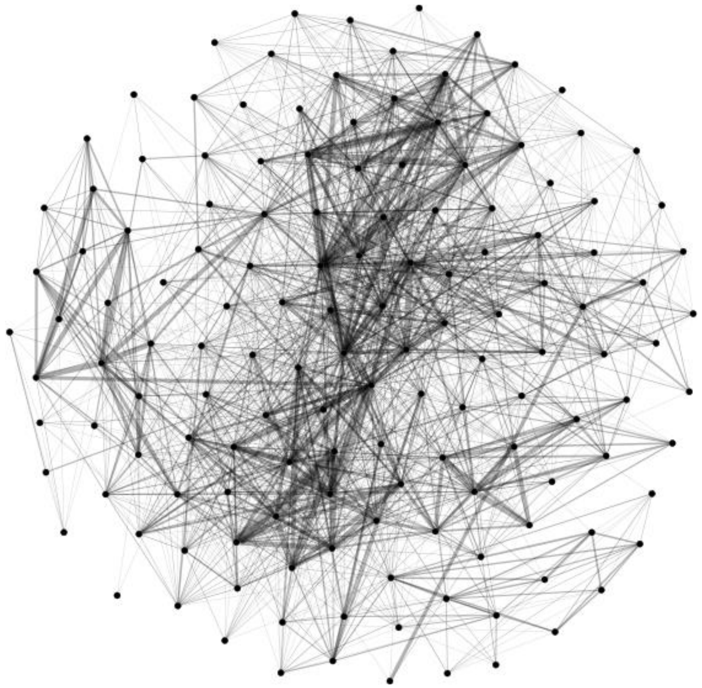
$$\mathrm{d}(W, W') = \sum_{i=1}^{n} \sum_{j=1}^{n} \left| W_{ij} - W'_{ij} \right| .$$

where W_{ij} represents the entry of W at i-th row and j-th column.

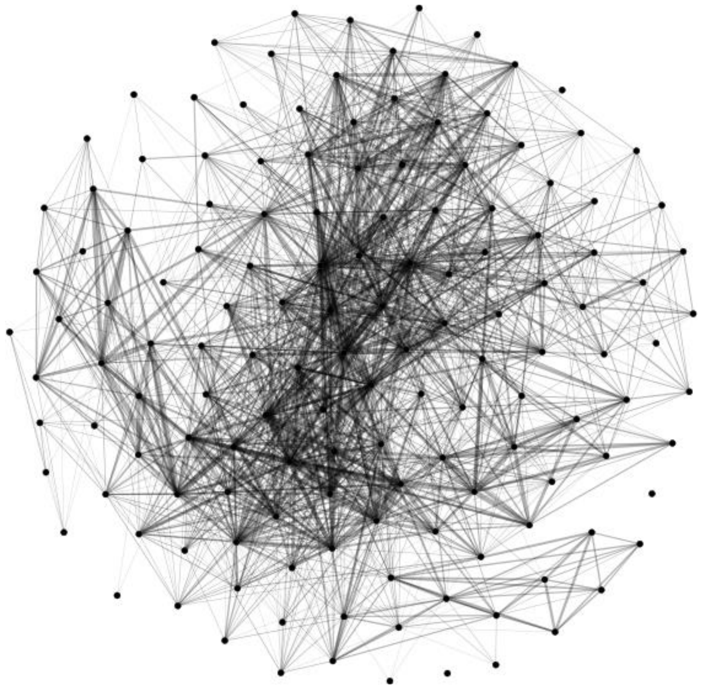
Experiment Result in visualization



Original Graph Weighted Projection



Generating Graph Weighted Projection



Reconstructing Graph Weighted Projection

Experiment Result in graph metrics

	contact-high-school	contact-primary-school	email-Enron	email-Eu	NDC-classes
Diameter					
HyperDK00	0.500	0.333	0.500	0.667	0.778
HyperDK11	0.250	0.333	0.250	0.333	0.533
Hyperlap	**0.200**	0.067	0.250	0.333	0.422
Hyperlap+	0.250	0.333	0.050	0.133	0.289
TheRA	0.250	**0.000**	0.250	0.667	0.222
HyperPLR	0.250	**0.000**	**0.000**	**0.033**	**0.000**
Triangles number					
HyperDK00	167.117	21.456	41.427	156.738	184.785
HyperDK11	84.440	15.988	10.867	41.227	79.077
Hyperlap	0.599	0.409	3.315	8.998	3.536
Hyperlap+	0.942	0.284	1.517	2.981	2.124
TheRA	0.468	**0.198**	2.451	3.725	1.638
HyperPLR	**0.316**	0.354	**0.055**	**0.742**	**0.281**
degree distribution					
HyperDK00	23.026	23.026	23.026	23.026	23.026
HyperDK11	21.002	22.635	12.406	8.490	**3.960**
Hyperlap	5.958	8.072	9.294	6.946	4.415
Hyperlap+	5.336	8.812	9.023	**6.448**	4.474
TheRA	6.634	10.530	7.986	15.315	14.325
HyperPLR	**5.065**	**7.657**	**4.639**	7.424	4.020
Singular value distribution					
HyperDK00	6.212	5.730	4.653	6.422	6.750
HyperDK11	5.462	5.147	4.513	6.345	7.008
Hyperlap	5.369	5.043	4.470	6.293	6.589
Hyperlap+	5.372	5.056	4.520	6.319	6.560
TheRA	5.377	5.054	4.465	6.339	6.620
HyperPLR	**5.347**	**5.032**	**4.451**	**6.278**	**6.432**

References

[1] NetGAN without GAN: From Random Walks to Low-Rank Approximations. ICML 2020.

[2] NetGAN: Generating Graphs via Random Walks. ICML 2018.

[3] From Graphs to Hypergraphs: Hypergraph Projection and its Remediation. ICLR 2024.

[4] Simplicial closure and higher-order link prediction. PNAS 2018.

Thanks for your attention :)

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