Deep Random Features for Interpolation of Spatiotemporal Data

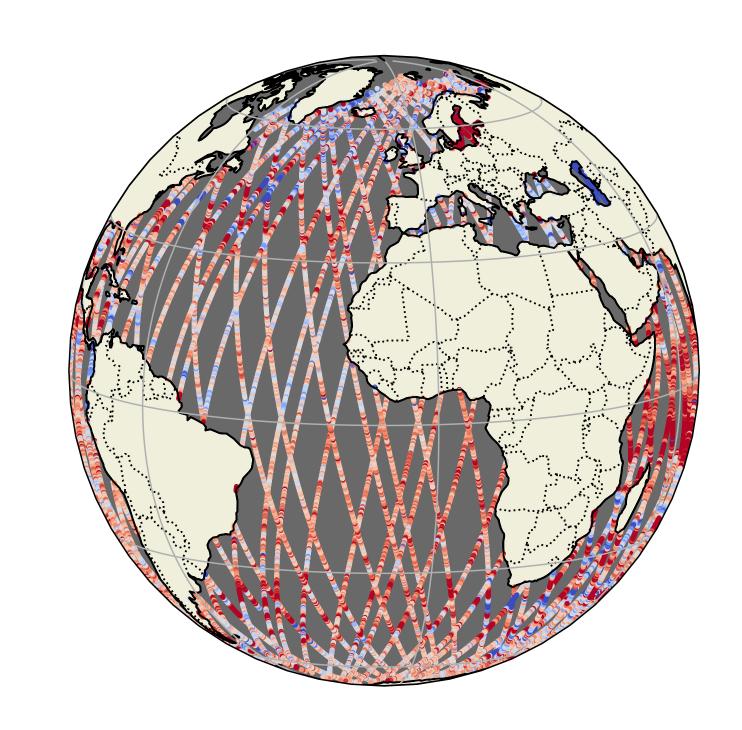




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Motivation

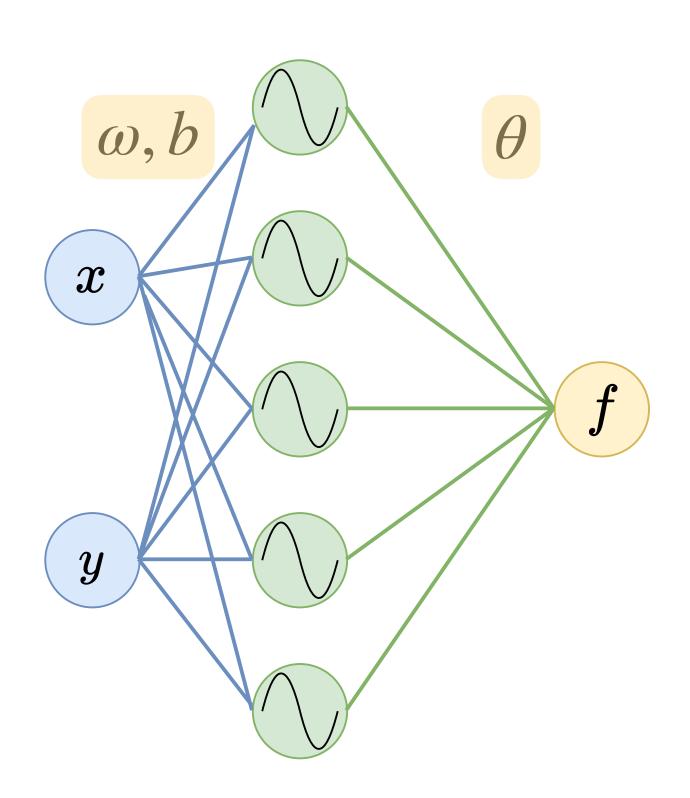
- Remote sensing observations are inherently sparse
- Exact Gaussian process has poor scalability
- Typical kernels are too simple
 - Struggles to learn from non-stationary fields



Random Fourier Features (RFFs)

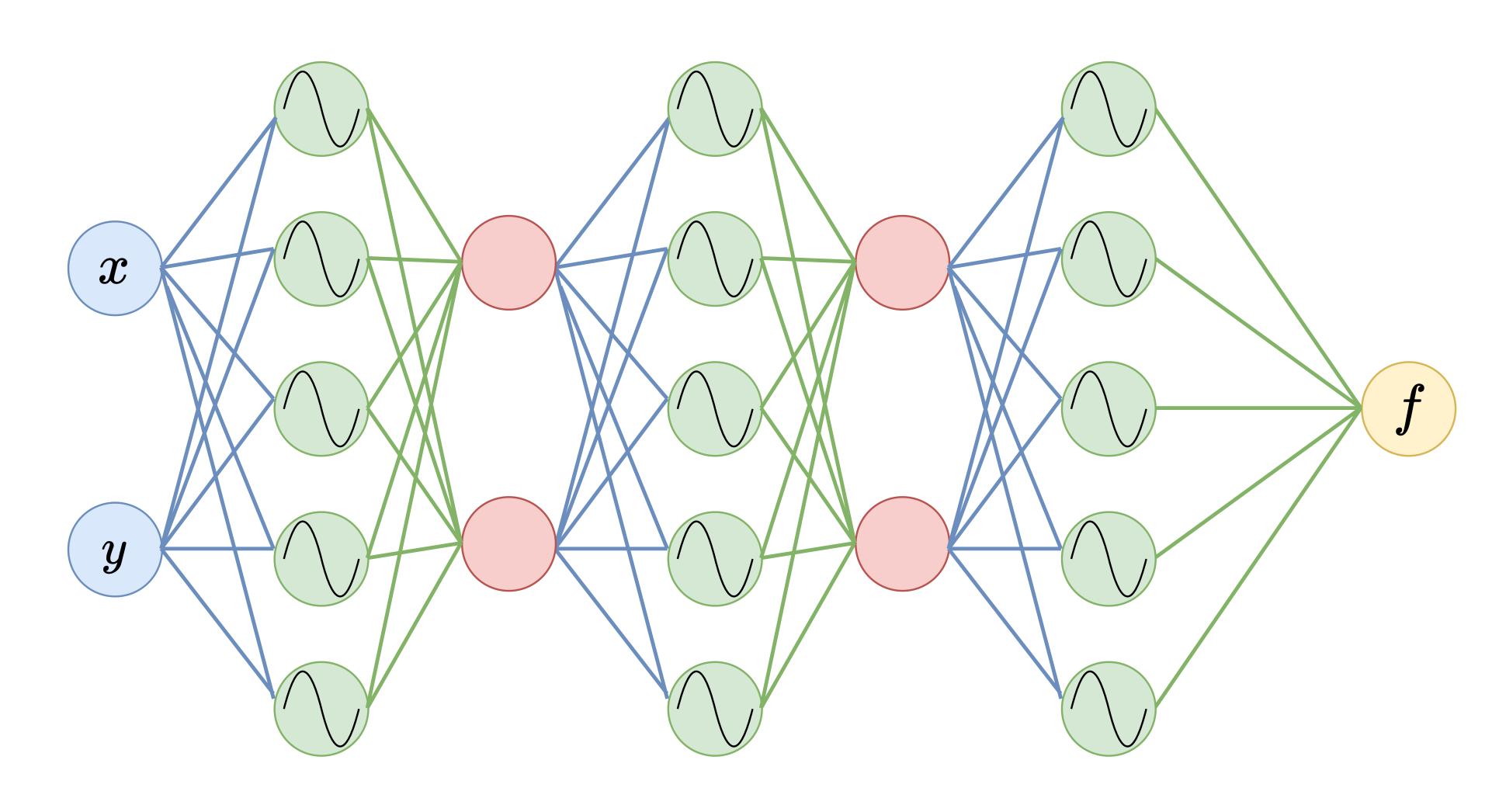
 Any stationary kernel can be approximated using Random Fourier Features (RFFs)*

$$egin{aligned} f(x) &pprox \sum_{h=1}^H heta_h \phi_h(x), \quad heta_h \sim \mathcal{N}(0,1), \ \end{aligned} \ ext{where} \quad \phi_h(x) = \sqrt{2\sigma^2/H}\cosig(\omega_h^ op x + b_hig), \quad h = 1,\dots, H, \ \end{aligned} \ ext{with} \ \omega_h \sim p(\omega) ext{ and } b_h \sim U([0,2\pi])$$



^{*}Rahimi et al. "Random features for large-scale kernel machines." NeurIPS 2007

Deep Random Features (DRF)



Spherical Random Features

$$f(z)pprox \sum_{i=1}^{M} heta_i\,\phi_i(z), \quad heta_i\sim \mathcal{N}(0,1)$$

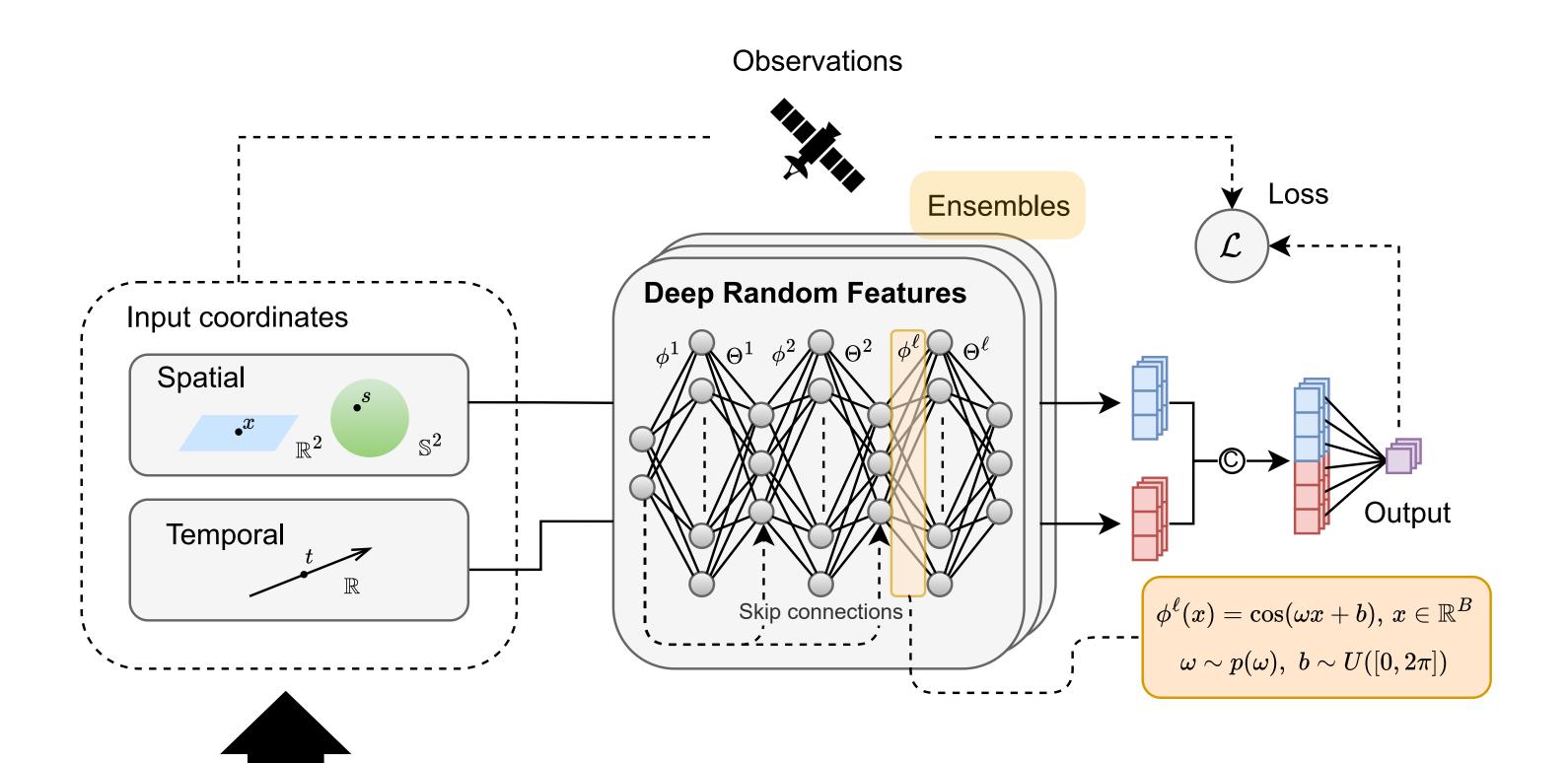
Euclidean case ($z=x\in\mathbb{R}^d$)

$$\phi_i(x) = \sqrt{rac{2\sigma^2}{M}}\cos\left(\omega_i^ op x + b_i
ight) \ \omega_i \sim p(\omega), \quad b_i \sim U([0,2\pi])$$

 $\textbf{Spherical case } (z=s\in\mathbb{S}^2)$

$$egin{aligned} \phi_i(s) &= \sqrt{rac{1}{M}} c_{\omega_i} \mathcal{G}_{\omega_i}^{1/2}(d_{\mathbb{S}^2}(s,b_i)) \ \omega_i &\sim \operatorname{Multinomial}(C_{\Phi}^{-1}\Phi(\lambda_1),\ldots,C_{\Phi}^{-1}\Phi(\lambda_J)) \ b_i &\sim U(\mathbb{S}^2) \end{aligned}$$

General training settings

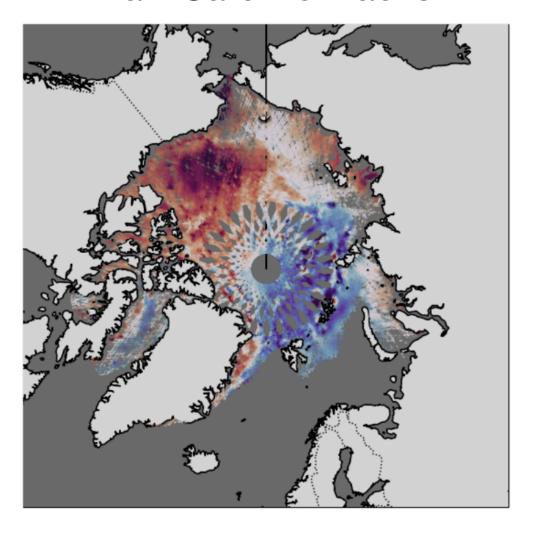


- Deep ensembles for uncertainty quantification
- Inner loop: Train ensemble of models
- Outer loop: Tune hyperparameters via Bayesian optimisation

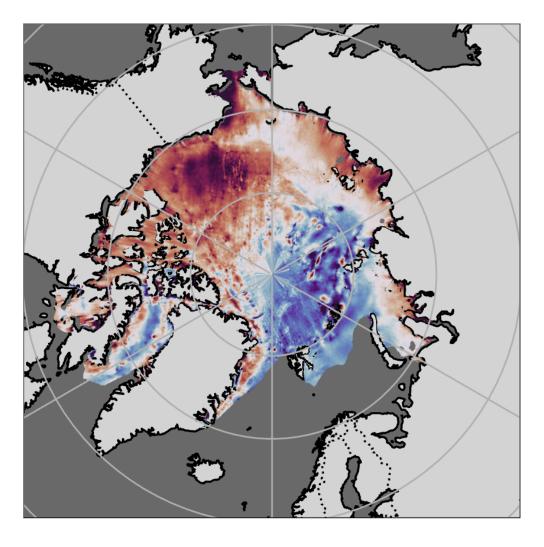
Results

Local scale interpolation of mean sea surface

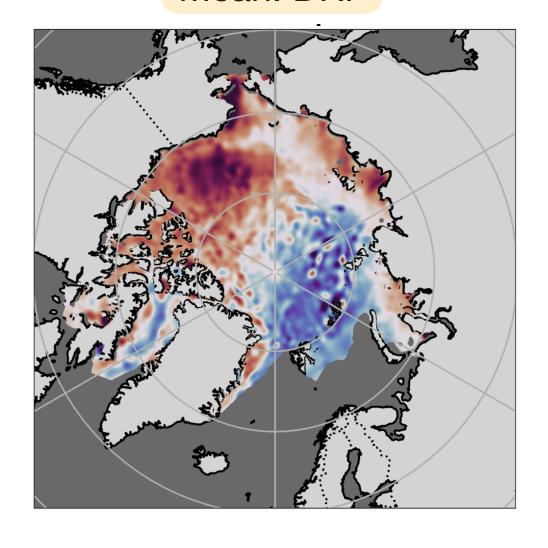
Raw Satellite Tracks



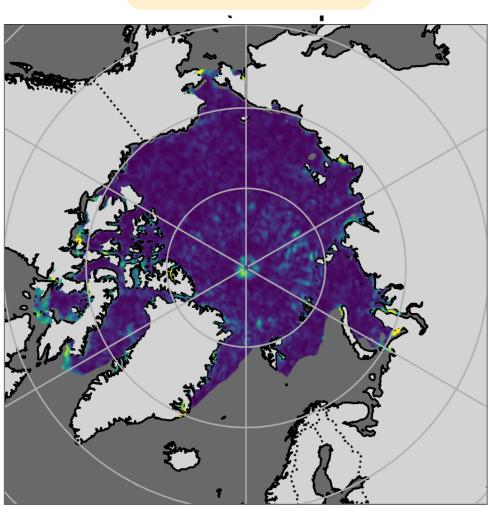
Ground Truth



Mean: DRF

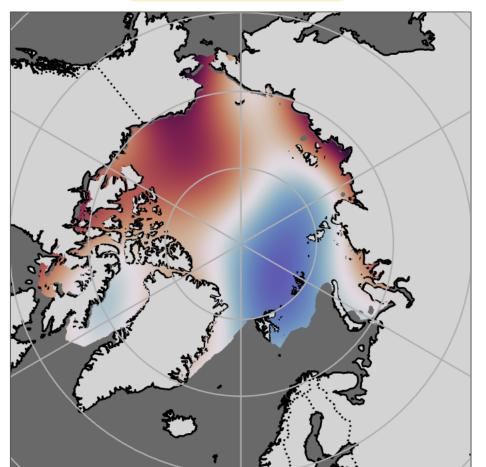


Variance: DRF

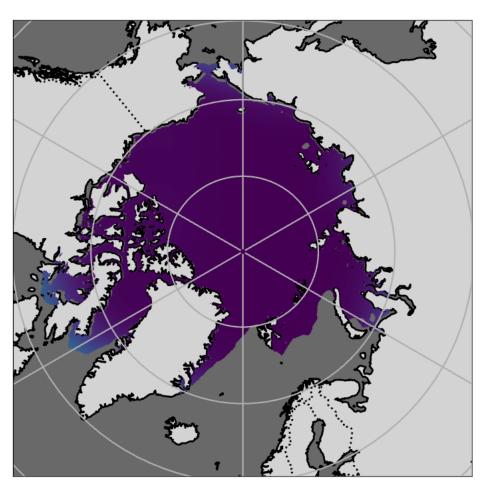


Comparisons with baselines

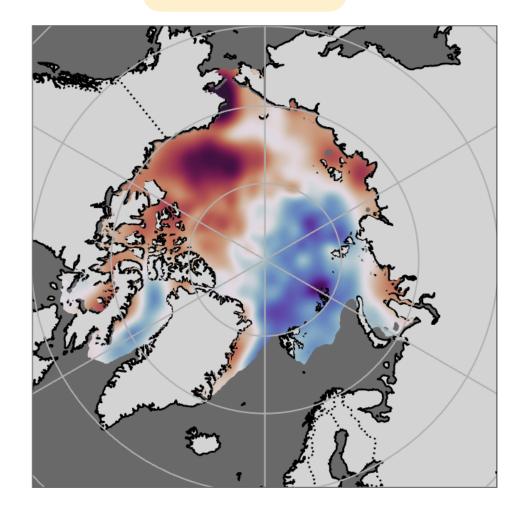
Mean: SVGP



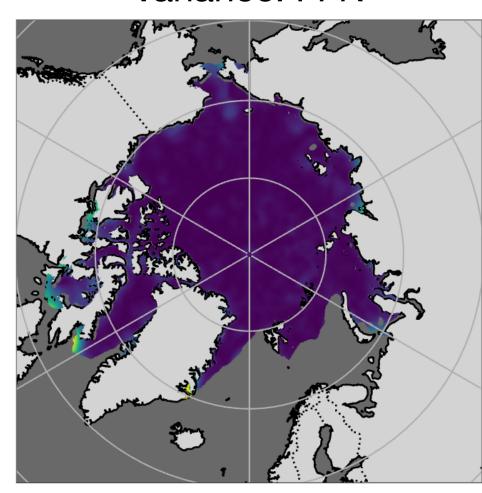
Variance: SVGP



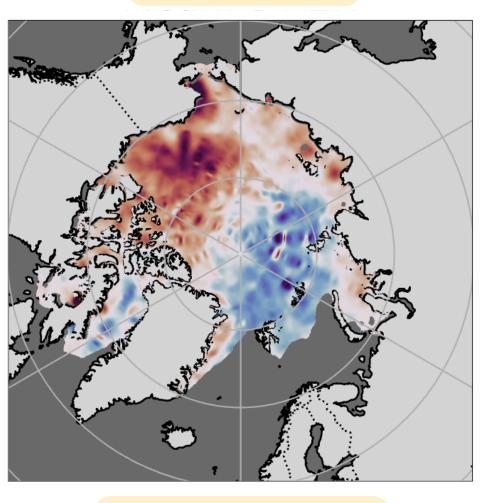
Mean: FFN



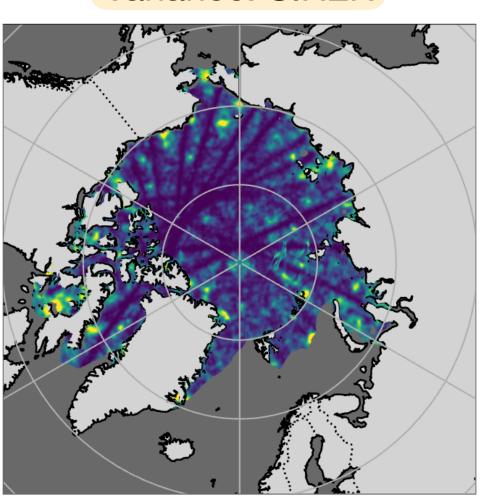
Variance: FFN



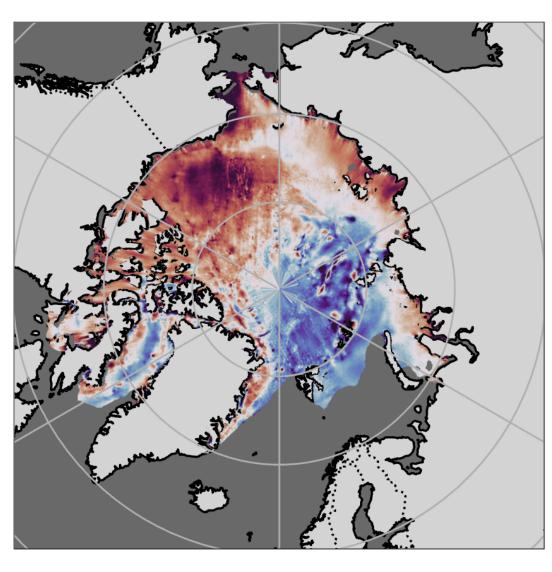
Mean: SIREN



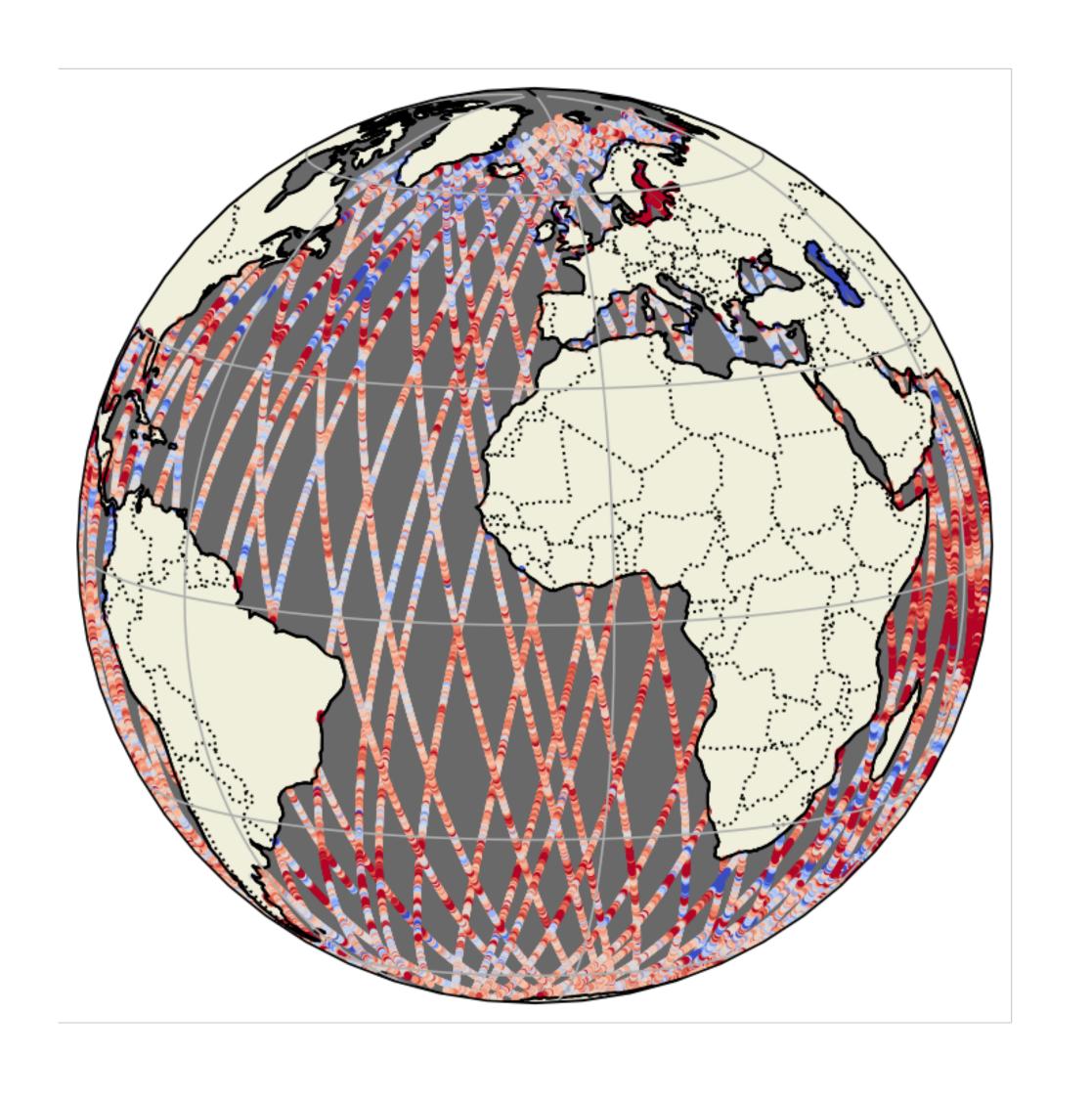
Variance: SIREN

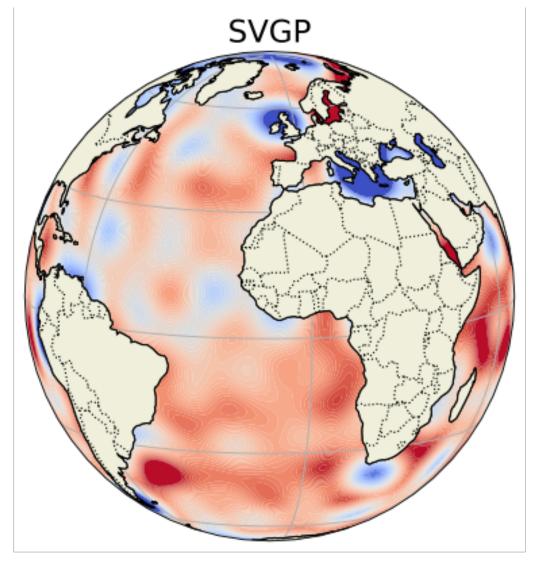


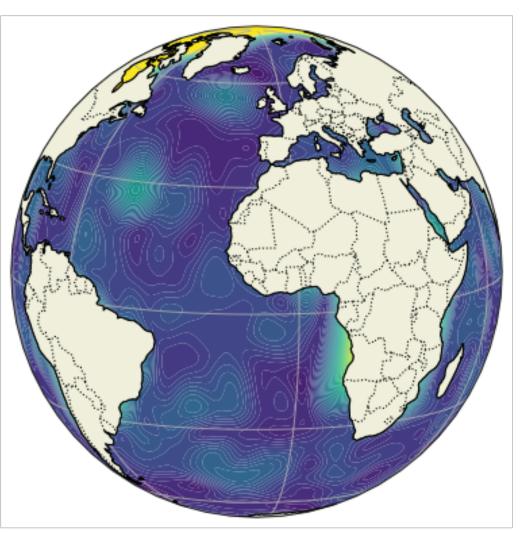
Ground Truth

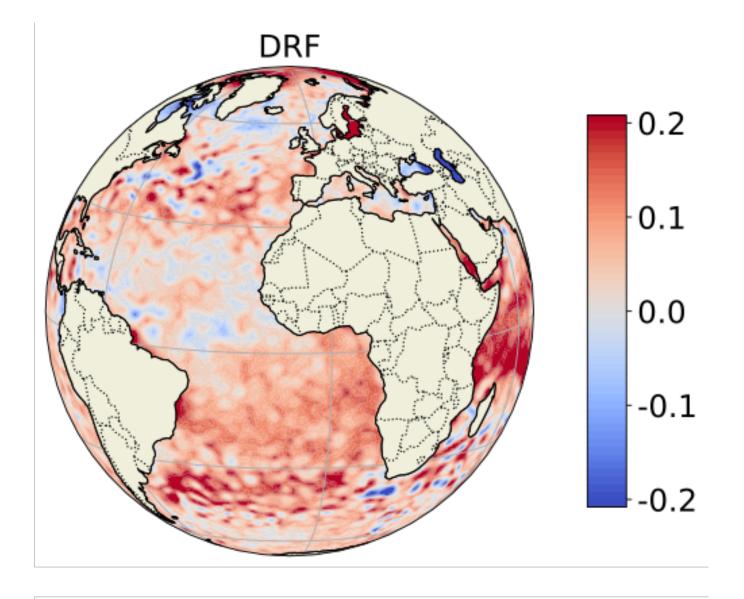


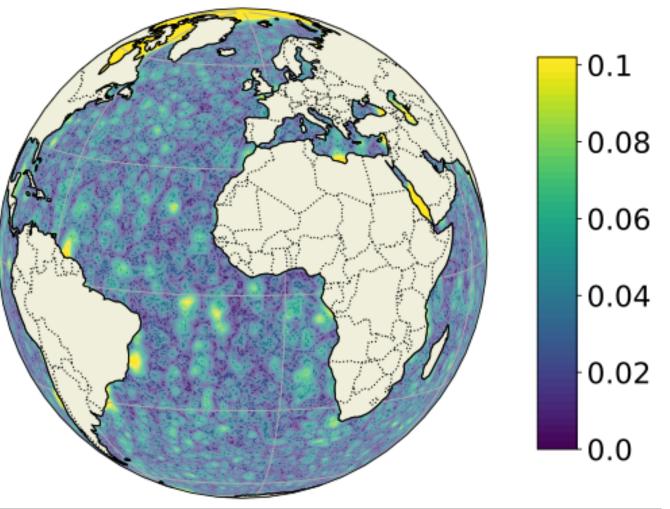
Results: Global scale











Conclusion

+ Scalability

+ Learn non-stationary fields

- Hyperparameter tuning is challenging

