On Fourier Analysis in SO(3) Space: The EquiLoPO Network

Dmitrii Zhemchuzhnikov[1,2], **Sergei Grudinin**[1]

[1]Univ. Grenoble Alpes, CNRS, Grenoble INP, LJK

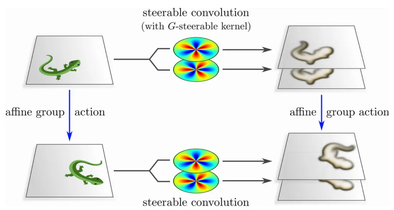
[2]AIRI

Summary

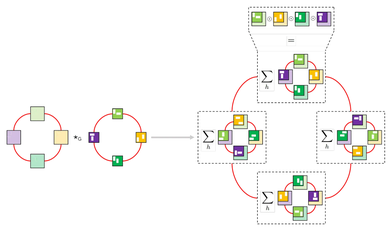
- **Challenge:** Recognition of arbitrary 3D patterns regardless of their orientations
- **Solution:** Group convolution in the continuous rotation space using Wigner decomposition
- **Contributions:**
  - **Convolution operation** providing analytical equivariance w.r.t. continuous SO(3)
  - **Novel local activation in Fourier** that has point-wise effect in real space

Background

- **Data augmentation** is often **infeasible in 3D** due to **3 angles of rotation**.
- **Equivariant methods**:
  - **Steerable networks**, which may fail to recognize **arbitrarily shaped patterns**.
  - **Group convolution networks**, which lack equivariance to **continuous SO(3)**



Steerable networks

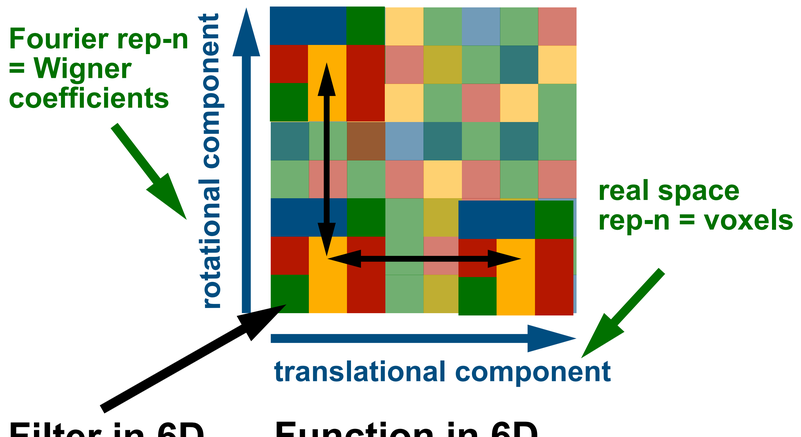


Group convolution networks

Convolution

Translational component: real space rep-n

Rotational component: Fourier rep-n



Formula:

$$h(\vec{\Delta}, \mathcal{R}) = \int\int d\vec{r}\; d\mathcal{R}_0\; f(\vec{\Delta} + \vec{r}, \mathcal{R}_0) g(\mathcal{R}^{-1}\vec{r}, \mathcal{R}^{-1}\mathcal{R}_0)$$

Formula in Fourier space:

$$h^{l_1}_{m_1 m_2}(\vec{\Delta}) = \int d\vec{r} \sum_{l_2, m_3, m_4} f^{l_2}_{m_3 m_4}(\vec{\Delta} + \vec{r}) S^{l_1 l_2}_{m_1 m_2 m_3 m_4}(\vec{r})$$

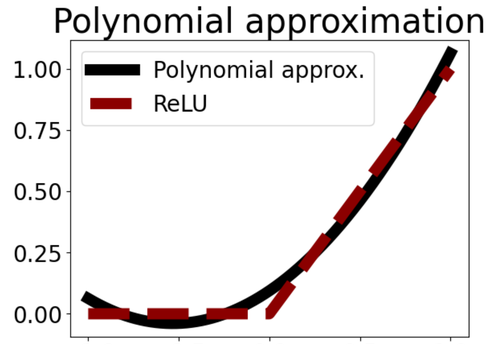
Results

MedMNIST (6 subsets, 28^3 voxels)

Method	# of params	Mean AUC	Mean ACC
ResNet-18 +3D	15M	0.86	0.84
ILPONet	38 k	0.89	0.81
EquiLoPONet	418 k	**0.94**	**0.94**

Activation

- **Challenge:** Propose a Fourier activation that has a local effect in real space
- **Solution:** polynomial approximation of **ReLU**
- **Advantages:**
  - **Close form expression** for Fourier space
  - **Doesn't** lead to infinite **resolution** in Fourier space



Ablation study

Method	# of params	Mean AUC	Mean ACC
Local activation	418 k	**0.94**	**0.94**
Global (gated) activation	113k	0.68	0.63
Local activation, low resolution	548k	0.69	0.63

Links