PRIVATE MECHANISM DESIGN VIA QUANTILE ESTIMATION
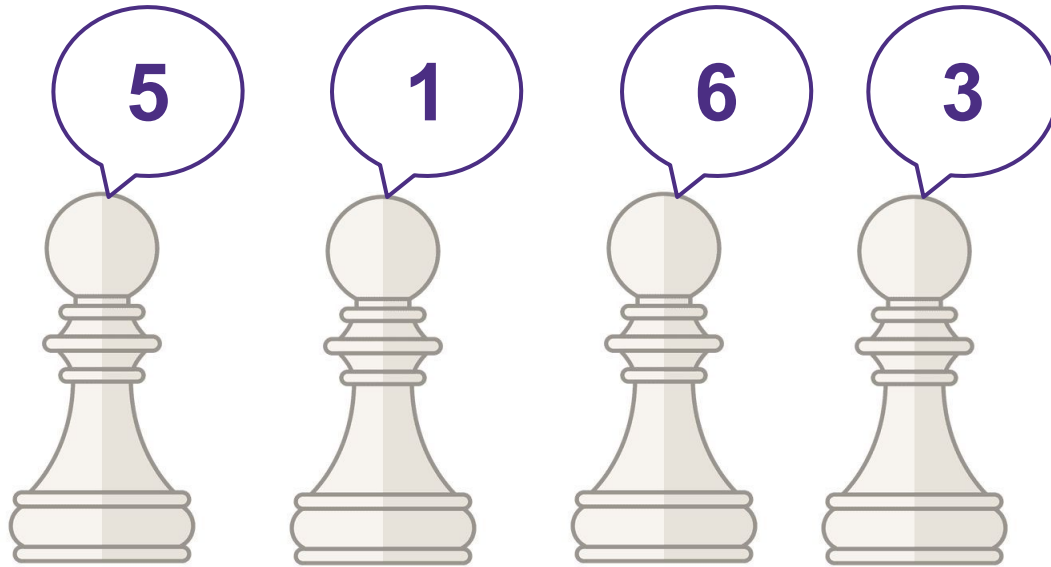
Yuanyuan (Chloe) Yang (Looking for Full-Time Jobs)
w./ Tao Xiao, Bhuvesh Kumar, & Jamie Morgenstern

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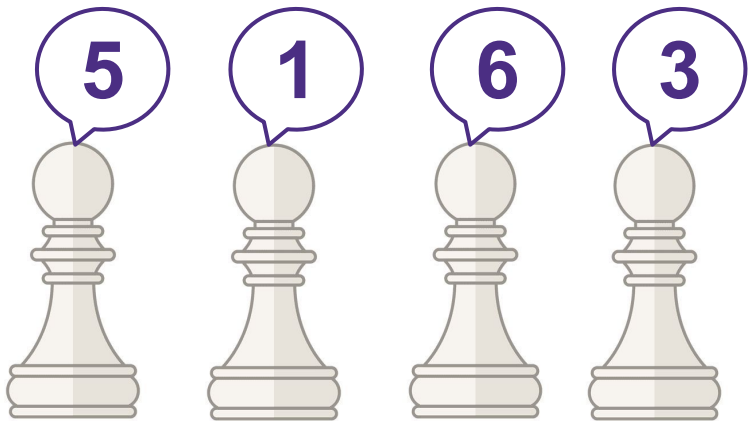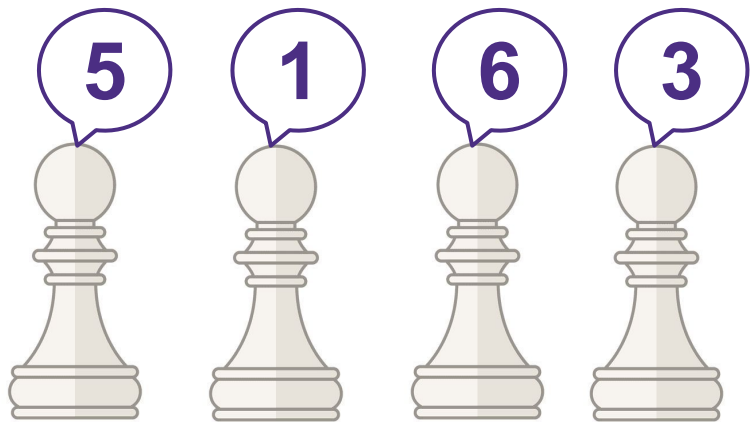
Auction Design

Bid: 5 1 6 3

Auction Design

- **Allocation:** $(x_1(\mathbf{b}), \ldots, x_n(\mathbf{b}))$
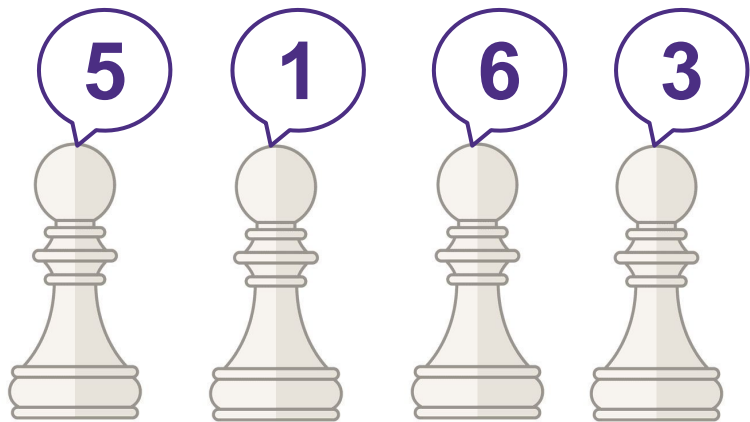- **Payment:** $(p_1(\mathbf{b}), \ldots, p_n(\mathbf{b}))$

Auction Design

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Both Depends on **Bid**, not Values

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Auction Design

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5 1 6 3

Revenue optimality requires knowledge of **bidder value distributions.**

Differentially Private(DP) Mechanism Design

- [F McSherry, K Talwar, 07] Pure DP, Exponential Mechanisms
- [Z Huang, J Liu, X Wang, 18] Approx. DP, Online Against Optimal Reserve
- [JD Abernethy et al, 19] Approx. DP, Online Against Myerson

Ours: A near optimal single-item auction with **pure** DP, Offline Against Optimal Myerson, Online Against Optimal Myerson.

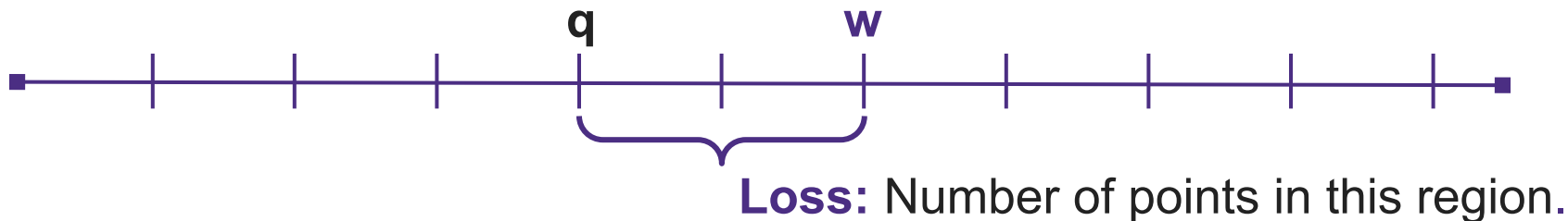# Our Focus: Single Item Auction with *Pure* Privacy

(Offline) Model:
- Samples from value distributions: $\mathbf{v}_1, \ldots, \mathbf{v}_n$
- Learner publish functions: $\mathbf{x} : \mathbb{R}_+^k \to [0, 1]^k, \mathbf{p} : \mathbb{R}_+^k \to \mathbb{R}_+^k$

(Online) Model: For t = 1,... T:
- Bidders **bid** as a function of their value: $\mathbf{b}_t = (b_1, \ldots, b_k)$
- Learner publish payments and allocations: $\mathbf{x}_t(\mathbf{b}_t), \mathbf{p}_t(\mathbf{b}_t)$
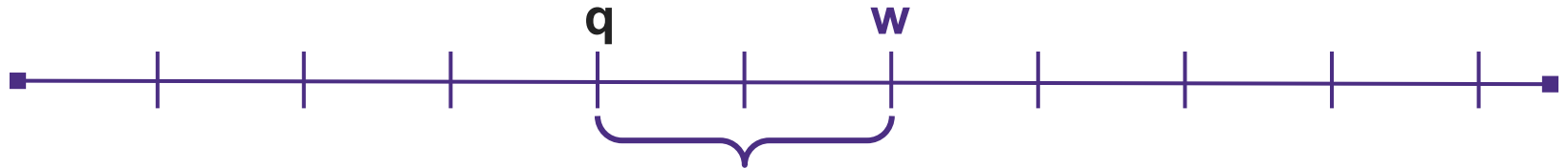
Technical Overview

Private Quantile Estimations, when applied to samples following a distribution, does NOT shift the distribution by much.

Loss: Number of points in this region.

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1. Create ϵ-net for values.
2. Private estimating quantiles on the samples
3. Myerson's auction is insensitive to small distribution shift.

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Conclusions

Quantile Estimation with pure privacy works for designing revenue optimal single item auction.

Future Directions:

1) Extension to more general mechanism design problems
2) Extension to correlated value distributions
3) Privacy acts as a regularizer, preventing overfitting to sample-specific revenue peaks.