Multiplicative Logit Adjustment Approximates Neural-Collapse-Aware Decision Boundary Adjustment

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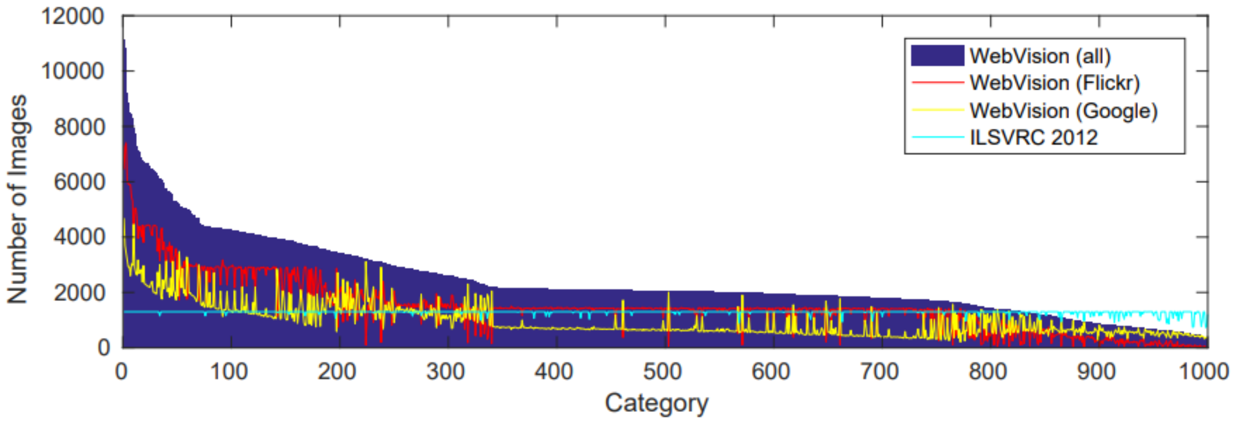
ICLR

Long-Tailed Recognition (LTR)

- Real-world class distributions are often dominated by classes with overwhelmingly small sample sizes (Long-Tailed Data).
  - Zipf's law[Reed, 2001]
- Problems: Naïve training …
  - Biases the classes with large numbers of training samples (Head classes).
  - Degrades accuracy of classes with small number of samples (Tail classes).
  - Degrades overall accuracy.



The number of images per category of the WebVision dataset.
(https://data.vision.ee.ethz.ch/cvl/webvision/dataset2017.html)

Logit Adjustment　(LA)

Post-hoc adjustment methods for LTR that modify the logits.

- Additive LA (ALA) [Menon+, ICLR2020]

$$g_k(\boldsymbol{x}) - \gamma_+ \log n_k$$

  - γ_+: A hyperparameter.
  - Based on a Fisher-consistent loss.
    - Assumes that the sample numbers n_k approach infinity.

- Multiplicative LA (MLA) [Kim+, 2020]

$$n_k^{-\gamma_\times} g_k(\boldsymbol{x})$$

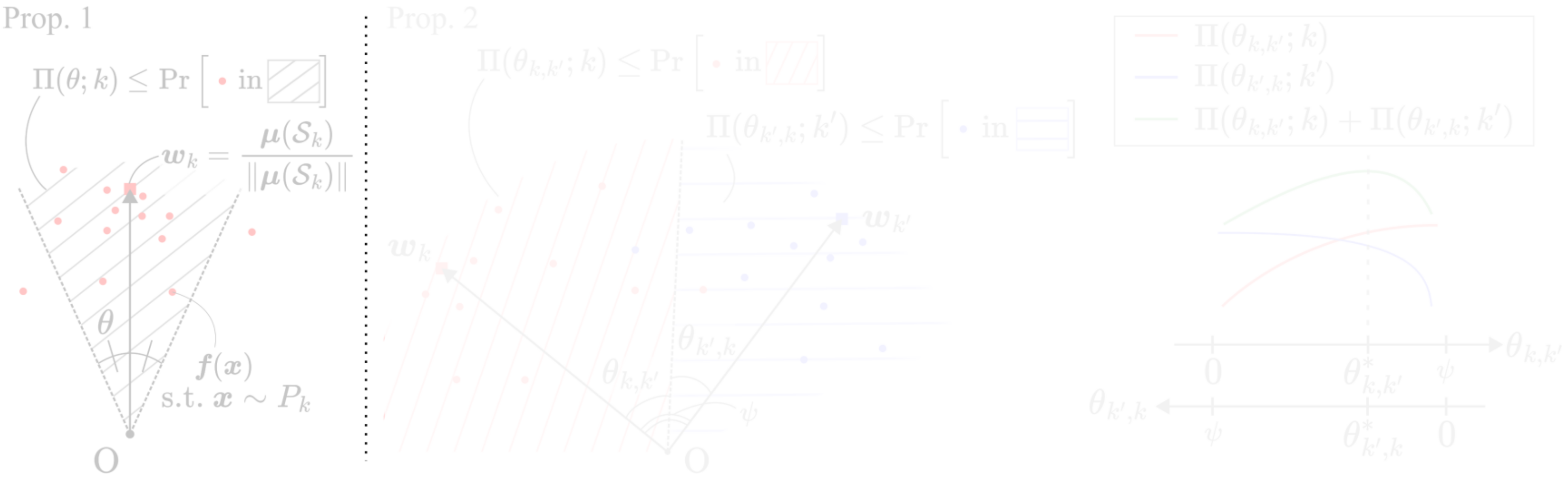
  - $\gamma_\times$: A hyperparameter.
  - Heuristic-based with no theoretical guarantees yet.
  - Outperforms ALA in certain cases [Hasegawa+, ICLR2024].

Our work

- **Research Question**
  - **Is there a theoretical foundation behind the effectiveness of MLA?**
- **Contribution**
  - Provide a near-optimality of MLA with our three-step theory.
  - Validate experimentally that the theory holds under realistic conditions.

Three-Step Theory

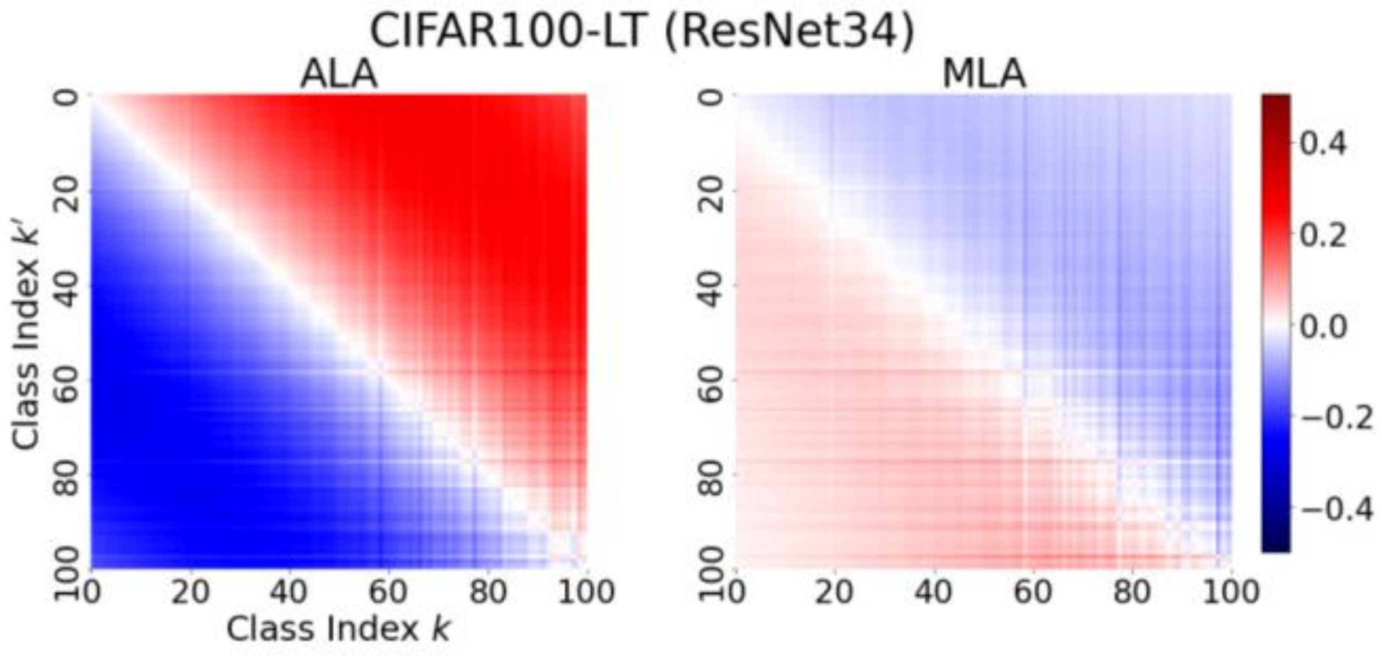
1. Feature spread estimation based on neural collapse [Papyan+, PNAS2020].
2. Decision boundary optimization (1vs1Adjuster).

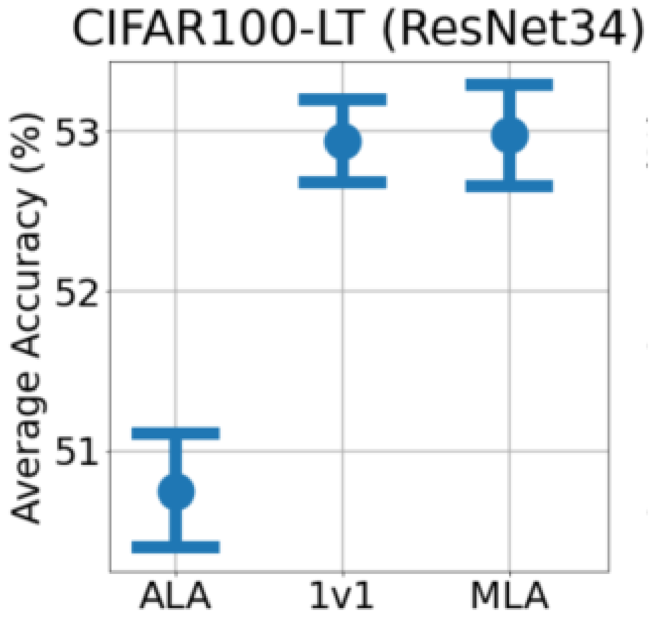


3. Approximation of 1vs1Adjuster to MLA.

Experiment: Validating the Approximation

- Verify the approximation from the following two perspectives.
  1. The angles of decision boundaries.
  2. The test accuracy.





Links

- Our paper
  - https://arxiv.org/abs/2409.17582

- Our github repository
  - https://github.com/HN410/MLA-Approximates-NCDBA