Provable Accuracy Bounds for Hybrid Dynamical Optimization and Sampling

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Post-Moore's Law

Motivation: Slowing Digital Scaling→Novel Computing Substrates

Analog "Dynamical Accelerators" (DXs)

Examples

- Oscillators [15, 18]
- Resistively-Coupled Capacitors [1, 19]
- Memristors [9, 4]

Applications

- Combinatorial Optimization [8]
- Graph Neural Network Inference [12, 17]
- Energy Based Model Training/Inference [14]
- Linear Algebra [7]

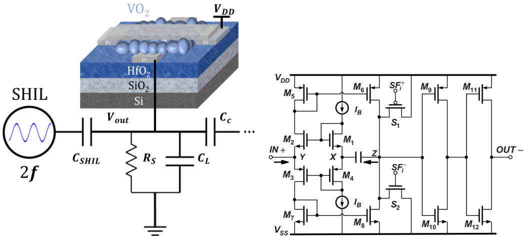


Figure: Left: From [6]. Right: From [19]

Capacity Limitations

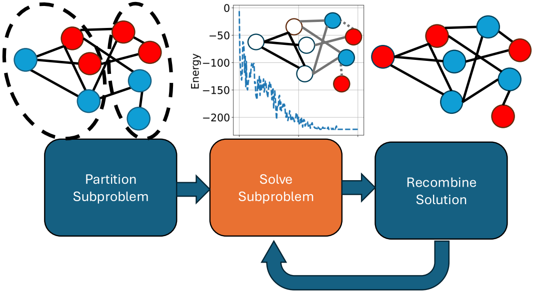
DX designed with fixed capacity limits
Common solution: *Large-Neighborhood Local Seach* (LNLS)†

(Static) LNLS

1. Decompose problem into *b* disjoint "blocks"
2. Sample/optimize over blocks
3. Repeat until convergence/limit

Gaps

- Dependence on device parameters?
- Convergence rate?
- Impact of analog non-idealities?



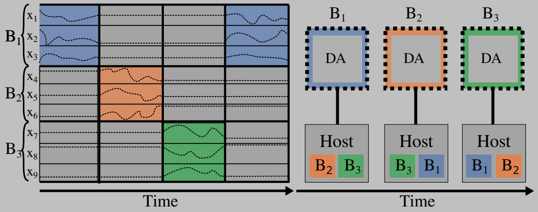
† AKA alternating minimization

Our Approach

Analyze LNLS as a sampling algorithm

Device Model

- Overdamped Langevin dynamics [15, 16, 11]
- LNLS = *Block coordinate sampling* with Langevin processes
  "Block Langevin Dynamics" (BLD)



BLD model allows us to draw from a rich sampling analysis toolbox [13, 2, 3, 10, 5]

Results (Ideal Devices)

Assuming a γ-LSI target $\pi(x) \propto e^{-\beta f(x)}$

Randomized Block Selection (Theorem 1 Informally)

Randomized BLD (RBLD) with block distribution ϕ and an *ideal* DX achieves

$$\mathrm{D}_{\mathrm{KL}}(\mu_{t_k} \| \pi) \leq e^{-2\gamma\beta^{-1}\phi_{\min}\lambda_{\min}k}\, \mathrm{D}_{\mathrm{KL}}(\mu_0 \| \pi). \tag{1}$$

after each iteration

Cyclic Block Selection (Theorem 2 Informally)

Cyclic BLD (CBLD) with an *ideal* DX achieves

$$\mathrm{D}_{\mathrm{KL}}(\mu_{t_{kb}} \| \pi) \leq e^{-2\gamma\beta^{-1}\lambda_{\min}k}\, \mathrm{D}_{\mathrm{KL}}(\mu_0 \| \pi). \tag{2}$$

after each complete cycle of b iterations

Results (Non-Ideal Devices)

Sampling with a "perturbed" oracle g_δ. Assuming

- γ-LSI target $\pi(x) \propto e^{-\beta f(x)}$
- f and g_δ are smooth and dissipative

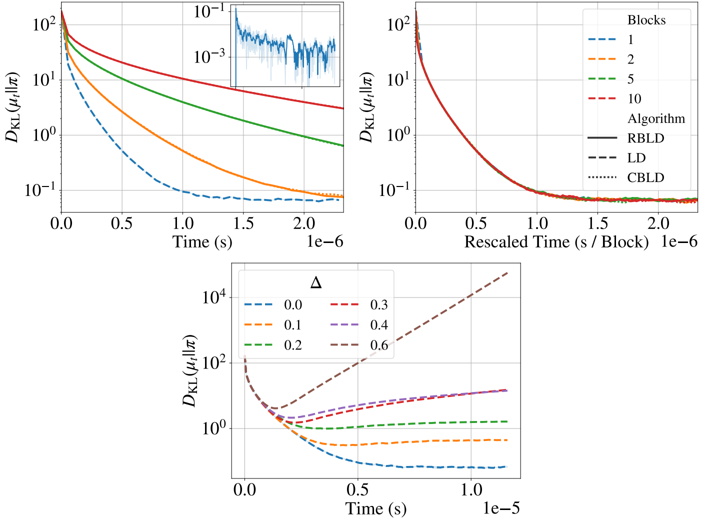
Non-Ideal Device (Theorem 3 Informally)

BLD (either cyclic or uniform random) with a Gaussian-perturbed DX achieves

$$W_2(\mu_{t_{bk}}, \pi_\beta) \leq \left(\frac{2}{\gamma}\right)^{1/2} e^{-\gamma\beta^{-1}\lambda k} \sqrt{\mathrm{D_{KL}}(\mu_0 \| \pi)} + C_{\mathrm{DX},\lambda} k\lambda. \tag{3}$$

- ***Biased algorithm***
- $C_{\mathrm{DX},\lambda} k\lambda$ determined by problem and device constants

Illustration ($d = 50$ Gaussian)



Conclusion and Future Work

Contributions

1. Analytically tractable model for LNLS
2. Exponential convergence for cyclic and randomized orderings under LSI
3. Characterization of bias for non-ideal devices

Future work

1. Compact Spaces, more general distributions
2. Alternate non-ideality sources

Thank You!

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