SD-Lora: Scalable Decoupled Low-Rank Adaptation for Class Incremental Learning

Yichen Wu^{1,2}*, Hongming Piao¹*, Long-Kai Huang⁴†, Renzhen Wang³, Wanhua Li², Hanspeter Pfister², Deyu Meng^{3,6}, Kede Ma¹†, Ying Wei^{5†}

¹City University of Hong Kong, ²Harvard University, ³Xi'an Jiaotong University, ⁴Tencent AI Lab, ⁵Zhejiang University, ⁶Pengcheng Laboratory

Presenter: Hongming Piao

Method	Rehearsal-free	Inference Efficiency	End-to-end Optimization
L2P (Wang et al., 2022b)	1	Х	Х
DualPrompt (Wang et al., 2022a)	✓	X	×
CODA-Prompt (Smith et al., 2023)	✓	X	✓
HiDe-Prompt (Wang et al., 2024a)	X	X	✓
InfLoRA (Liang & Li, 2024)	×	✓	✓
SD-LoRA(Ours)	✓	✓	✓

Method	Rehearsal-free	Inference Efficiency	End-to-end Optimization
L2P (Wang et al., 2022b)	1	Х	Х
DualPrompt (Wang et al., 2022a)	✓	X	×
CODA-Prompt (Smith et al., 2023)	✓	X	✓
HiDe-Prompt (Wang et al., 2024a)	X	X	✓
InfLoRA (Liang & Li, 2024)	×	✓	✓
SD-LoRA(Ours)	✓	✓	✓

Sample-dependent inference with foundation models

- Complex designing of the matching mechanism.
- Extra-computation during inference.

Method	Rehearsal-free	Inference Efficiency	End-to-end Optimization
L2P (Wang et al., 2022b)	1	Х	Х
DualPrompt (Wang et al., 2022a)	✓	X	×
CODA-Prompt (Smith et al., 2023)	✓	X	✓
HiDe-Prompt (Wang et al., 2024a)	Х	X	✓
InfLoRA (Liang & Li, 2024)	Х	✓	✓
SD-LoRA(Ours)	✓	✓	✓

Sample-dependent inference with foundation methods

- Require complex designing of the matching mechanism.
- Extra-computation during inference.

Need to store huge learned tasks' features to avoid catastrophic forgetting.

Method	Rehearsal-free	Inference Efficiency	End-to-end Optimization
L2P (Wang et al., 2022b)	1	Х	Х
DualPrompt (Wang et al., 2022a)	✓	X	×
CODA-Prompt (Smith et al., 2023)	✓	X	✓
HiDe-Prompt (Wang et al., 2024a)	Х	X	✓
InfLoRA (Liang & Li, 2024)	X	✓	✓
SD-LoRA(Ours)	✓	✓	✓

Sample-dependent inference with foundation methods

- Require complex designing of the matching mechanism.
- Extra-computation during inference.

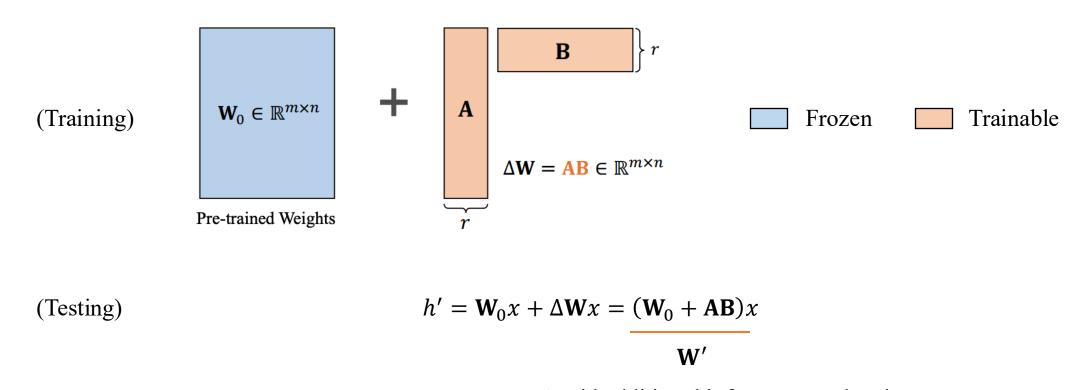
Need to store huge learned tasks' features to avoid catastrophic forgetting.

Motivation

Improve the inference efficiency.

Avoid catastrophic forgetting without the huge storage of the learned tasks' features.

1) Improve the inference efficiency



Avoid additional inference overhead

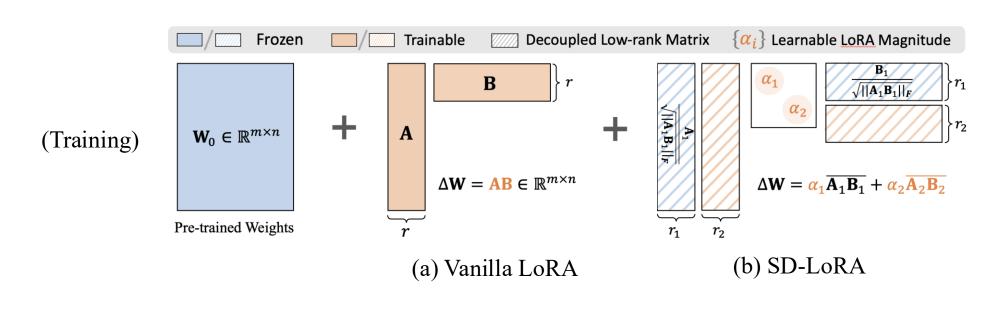
by incorporating the LoRAs into pre-trained weights

2) Avoid catastrophic forgetting without the huge storage of the learned tasks' features

Decouple the magnitude and direction of the learned AB

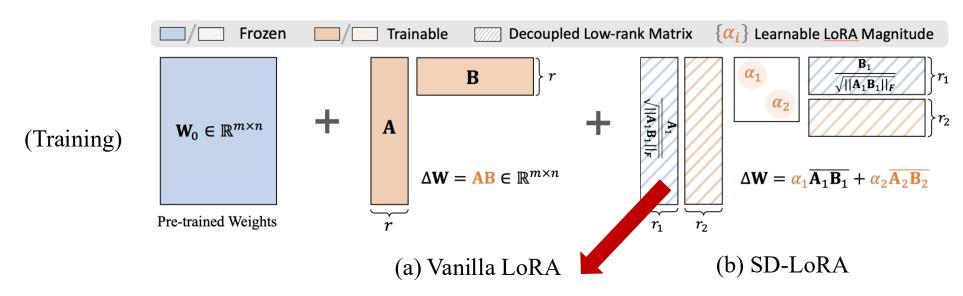
$$\Delta \mathbf{W} = ||\mathbf{A}\mathbf{B}||_F \cdot \overline{\mathbf{A}\mathbf{B}} = ||\mathbf{A}\mathbf{B}||_F \cdot \frac{\mathbf{A}\mathbf{B}}{||\mathbf{A}\mathbf{B}||_F}$$

2) Avoid catastrophic forgetting without the huge storage of the learned tasks' features



(Testing)
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t})x$$
1) Improve the inference efficiency

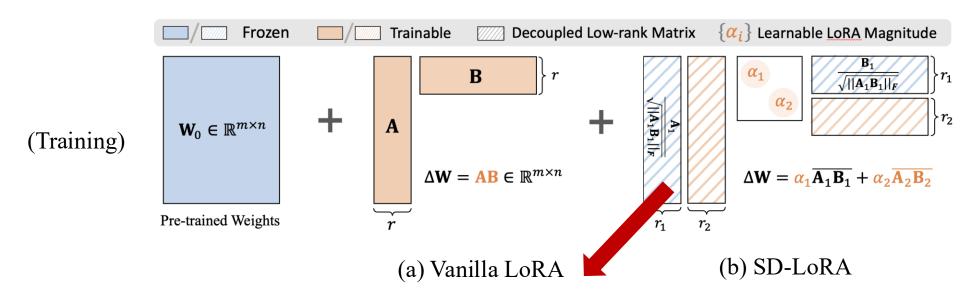
2) Avoid catastrophic forgetting without the huge storage of the learned tasks' features



Task specific knowledge (direction)

(Testing)
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t})x$$
1) Improve the inference efficiency

2) Avoid catastrophic forgetting without the huge storage of the learned tasks' features



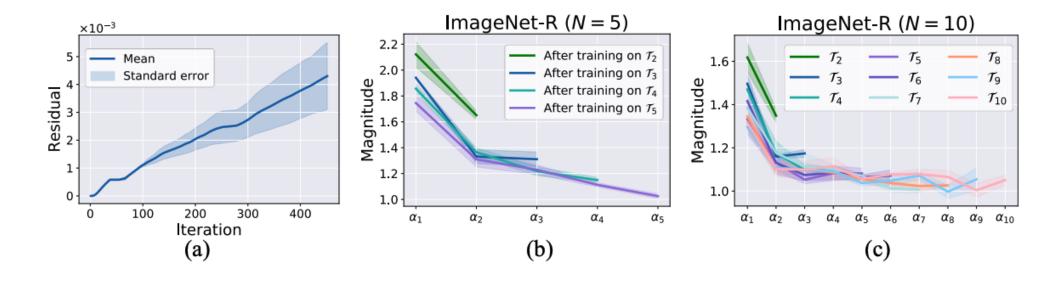
Task specific knowledge (direction)

(Testing)
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t})x$$
1) Improve the inference efficiency

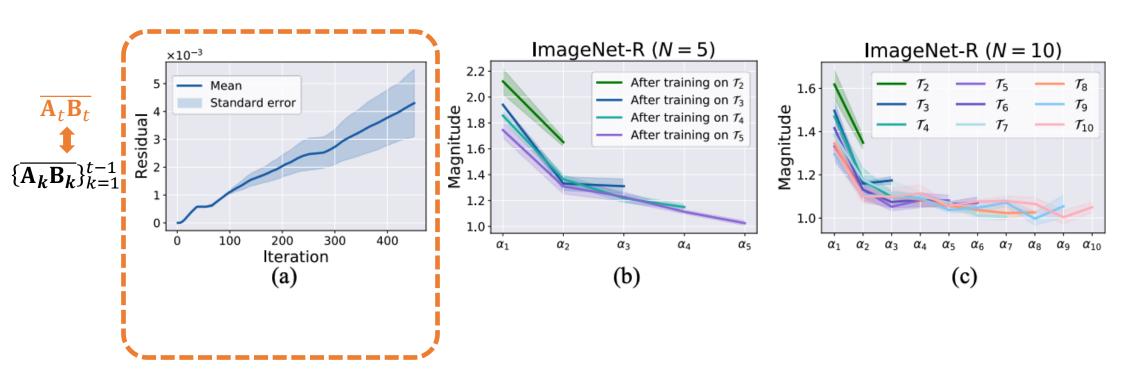


Why SD-LoRA avoid catastrophic forgetting?

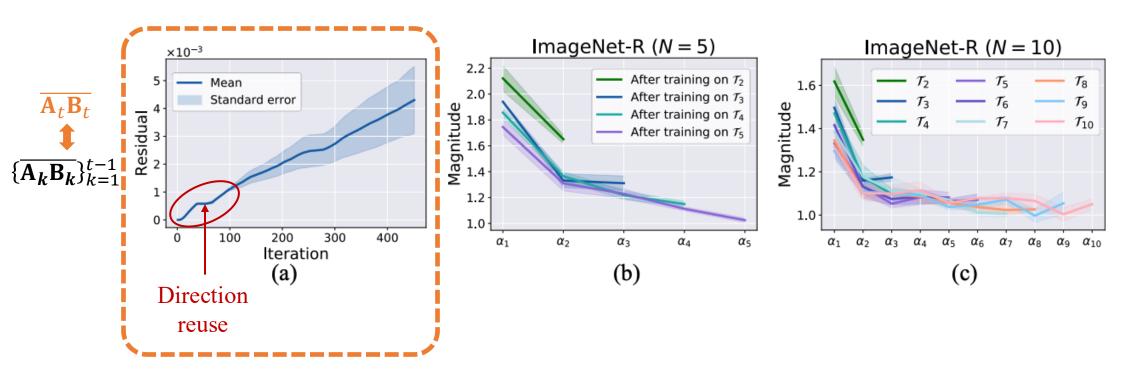
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



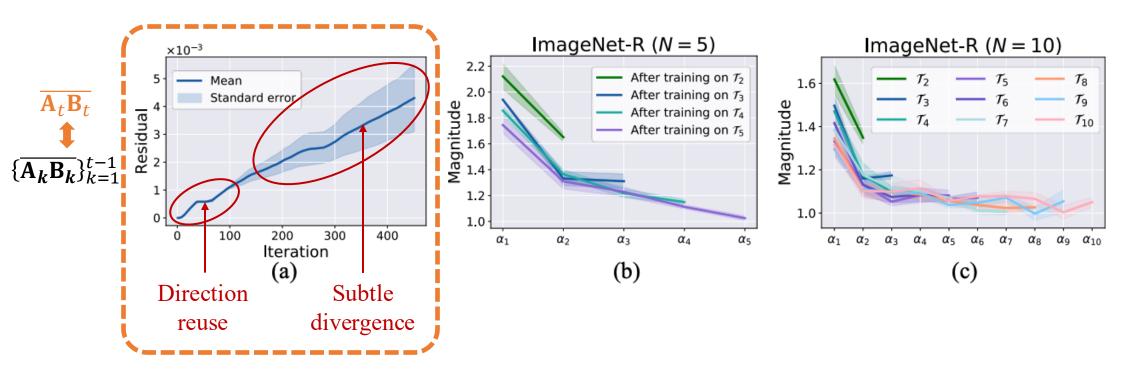
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



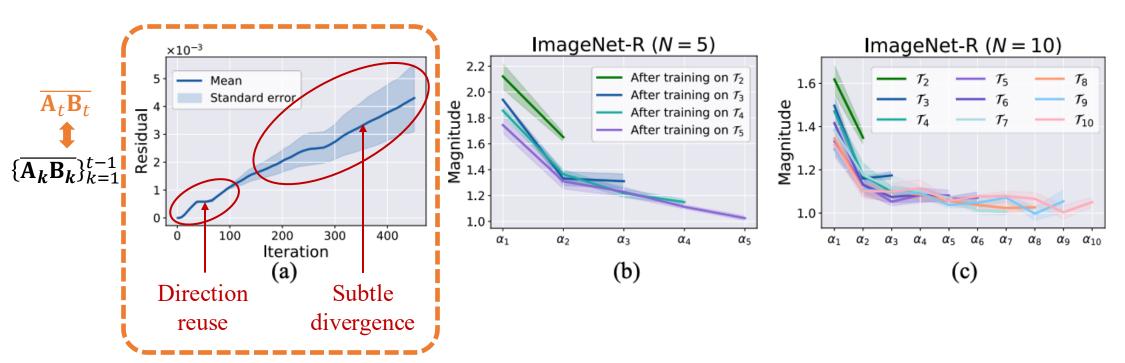
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$

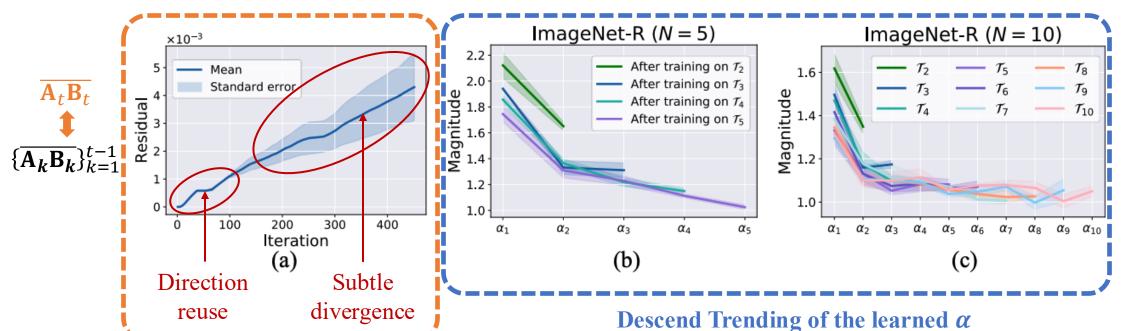


$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



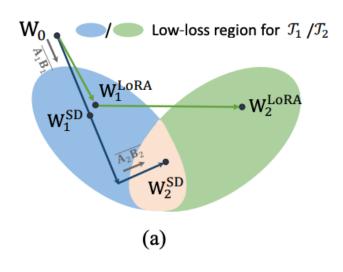
Newly learned direction $\overline{A_tB_t}$ highly related to previously learned ones

$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$

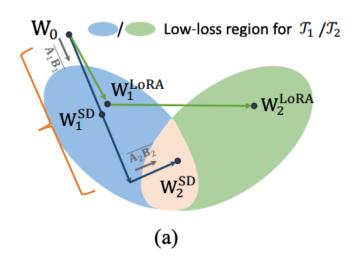


Newly learned direction $\overline{A_tB_t}$ highly related to previously learned ones

$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$

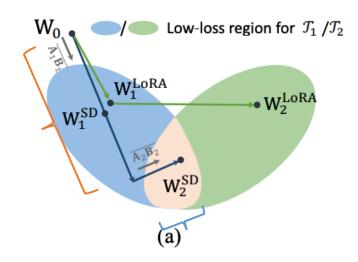


$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



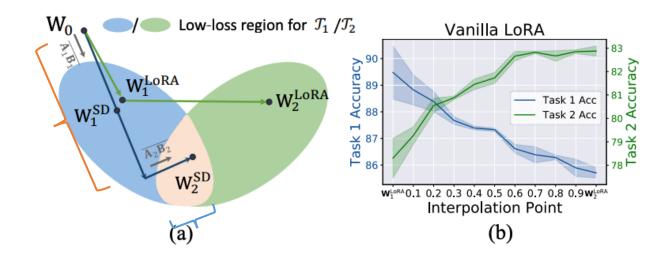
• α encourages updates along the key directions learned from earlier tasks, rapidly approaching the shared low-loss region for multiple tasks.

$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



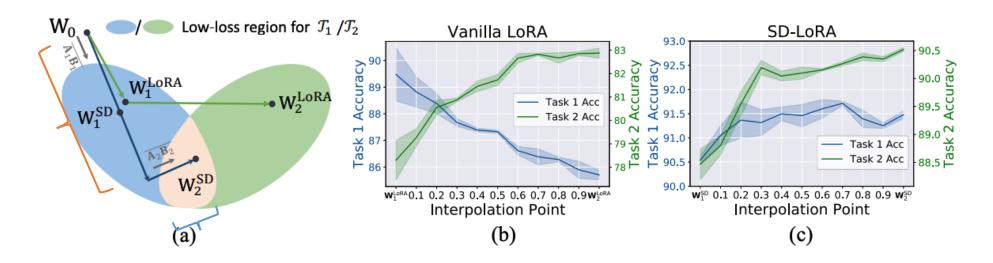
- α encourages updates along the key directions learned from earlier tasks, rapidly approaching the shared low-loss region for multiple tasks.
- By incrementally introducing LoRA, it fine-tunes these directions, allowing the model to accurately converge on the shared low-loss region for different tasks.

$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



- α encourages updates along the key directions learned from earlier tasks, rapidly approaching the shared low-loss region for multiple tasks.
- By incrementally introducing LoRA, it fine-tunes these directions, allowing the model to accurately converge on the shared low-loss region for different tasks.

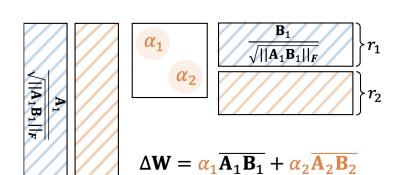
$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$



- α encourages updates along the key directions learned from earlier tasks, rapidly approaching the shared low-loss region for multiple tasks.
- By incrementally introducing LoRA, it fine-tunes these directions, allowing the model to accurately converge on the shared low-loss region for different tasks.



Theoretically explain why the previously learned LoRA directions are so critical.



Theorem 1. Suppose the assumptions stated in Appendix A.1 hold, where ϵ_1 is a small constant. Let $\delta \in (0,1)$ be such that $\delta \leq \min_{k \in \{1,\dots,j\}} \frac{\sigma_k - \sigma_{k+1}}{\sigma_k}$. Fix any tolerance level ϵ_2 satisfying $\epsilon_2 \leq \frac{1}{m+n+r}$. Let η denote the learning rate for updating the matrices \mathbf{A} and \mathbf{B} , and define $\Delta \mathbf{W}^{[:i]}$ as the rank-i approximation of $\Delta \mathbf{W}^*$, obtained by retaining the top-i principal components.

Then, there exist some numerical constants c and c', and a sequence of iteration indices:

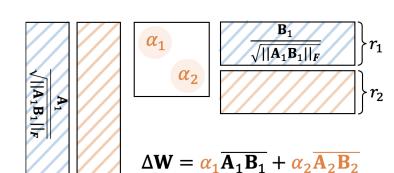
$$i_1 \le i_2 \le \ldots \le i_j \le \frac{c'}{\delta \eta \sigma_j} \log \left(\frac{\kappa_j}{\delta \epsilon_2}\right)$$

such that, with high probability, gradient descent with step size $\eta \leq c \min\{\delta, 1 - \delta\} \frac{\sigma_j^2}{\sigma_1^3}$ and initialization scaling factor $\rho \leq (\frac{c\delta\epsilon_2}{\kappa_j})^{\frac{1}{c\delta}}$ ensures that the approximation error satisfies

$$\left\| \mathbf{A}_{i_k} \mathbf{B}_{i_k} - \Delta \mathbf{W}^{[:k]} \right\|_{\text{op}} \le \epsilon_2 \sigma_1 + \epsilon_1, \quad \forall k = 1, 2, \dots, j.$$



Theoretically explain the previously learned LoRA directions are so critical.



Theorem 1. Suppose the assumptions stated in Appendix A.1 hold, where ϵ_1 is a small constant. Let $\delta \in (0,1)$ be such that $\delta \leq \min_{k \in \{1,\dots,j\}} \frac{\sigma_k - \sigma_{k+1}}{\sigma_k}$. Fix any tolerance level ϵ_2 satisfying $\epsilon_2 \leq \frac{1}{m+n+r}$. Let η denote the learning rate for updating the matrices \mathbf{A} and \mathbf{B} , and define $\Delta \mathbf{W}^{[:i]}$ as the rank-i approximation of $\Delta \mathbf{W}^*$, obtained by retaining the top-i principal components.

Then, there exist some numerical constants c and c', and a sequence of iteration indices:

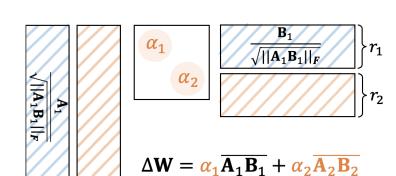
$$i_1 \le i_2 \le \ldots \le i_j \le \frac{c'}{\delta \eta \sigma_j} \log \left(\frac{\kappa_j}{\delta \epsilon_2}\right)$$

such that, with high probability, gradient descent with step size $\eta \leq c \min\{\delta, 1 - \delta\} \frac{\sigma_j^2}{\sigma_1^3}$ and initialization scaling factor $\rho \leq (\frac{c\delta\epsilon_2}{\kappa_j})^{\frac{1}{c\delta}}$ ensures that the approximation error satisfies

$$\left\| \mathbf{A}_{i_k} \mathbf{B}_{i_k} - \Delta \mathbf{W}^{[:k]} \right\|_{\text{op}} \le \epsilon_2 \sigma_1 + \epsilon_1, \quad \forall k = 1, 2, \dots, j.$$



Theoretically explain the previously learned LoRA directions are so critical.



Theorem 1. Suppose the assumptions stated in Appendix A.1 hold, where ϵ_1 is a small constant. Let $\delta \in (0,1)$ be such that $\delta \leq \min_{k \in \{1,\dots,j\}} \frac{\sigma_k - \sigma_{k+1}}{\sigma_k}$. Fix any tolerance level ϵ_2 satisfying $\epsilon_2 \leq \frac{1}{m+n+r}$. Let η denote the learning rate for updating the matrices \mathbf{A} and \mathbf{B} , and define $\Delta \mathbf{W}^{[:i]}$ as the rank-i approximation of $\Delta \mathbf{W}^*$, obtained by retaining the top-i principal components.

Then, there exist some numerical constants c and c', and a sequence of iteration indices:

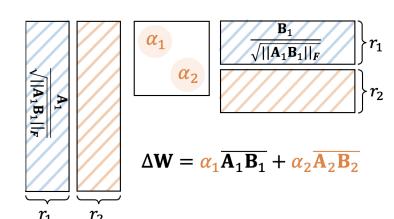
$$i_1 \le i_2 \le \ldots \le i_j \le \frac{c'}{\delta \eta \sigma_j} \log \left(\frac{\kappa_j}{\delta \epsilon_2}\right)$$

such that, with high probability, gradient descent with step size $\eta \leq c \min\{\delta, 1 - \delta\} \frac{\sigma_j^2}{\sigma_1^3}$ and initialization scaling factor $\rho \leq (\frac{c\delta\epsilon_2}{\kappa_j})^{\frac{1}{c\delta}}$ ensures that the approximation error satisfies

$$\left\|\mathbf{A}_{i_k}\mathbf{B}_{i_k} - \Delta\mathbf{W}^{[:k]}\right\|_{\mathrm{op}} \leq \epsilon_2\sigma_1 + \epsilon_1, \quad \forall k = 1, 2, \dots, j.$$



Theoretically explain the previously learned LoRA directions are so critical.



Theorem 1. Suppose the assumptions stated in Appendix A.1 hold, where ϵ_1 is a small constant. Let $\delta \in (0,1)$ be such that $\delta \leq \min_{k \in \{1,\dots,j\}} \frac{\sigma_k - \sigma_{k+1}}{\sigma_k}$. Fix any tolerance level ϵ_2 satisfying $\epsilon_2 \leq \frac{1}{m+n+r}$. Let η denote the learning rate for updating the matrices \mathbf{A} and \mathbf{B} , and define $\Delta \mathbf{W}^{[:i]}$ as the rank-i approximation of $\Delta \mathbf{W}^*$, obtained by retaining the top-i principal components.

Then, there exist some numerical constants c and c', and a sequence of iteration indices:

$$i_1 \le i_2 \le \ldots \le i_j \le \frac{c'}{\delta \eta \sigma_j} \log \left(\frac{\kappa_j}{\delta \epsilon_2}\right)$$

such that, with high probability, gradient descent with step size $\eta \leq c \min\{\delta, 1 - \delta\} \frac{\sigma_j^2}{\sigma_1^3}$ and initialization scaling factor $\rho \leq (\frac{c\delta\epsilon_2}{\kappa_j})^{\frac{1}{c\delta}}$ ensures that the approximation error satisfies

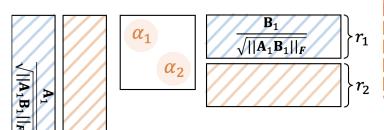
$$\left\|\mathbf{A}_{i_k}\mathbf{B}_{i_k} - \Delta\mathbf{W}^{[:k]}\right\|_{\mathrm{op}} \leq \epsilon_2\sigma_1 + \epsilon_1, \quad \forall k = 1, 2, \dots, j.$$

As the continual training progress, the learned matrix AB gradually approximate the principal components of ΔW^*

Efficient Variants of SD-LoRA

$$h' = (\mathbf{W}_0 + \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2} + \dots + \alpha_t \overline{\mathbf{A}_t \mathbf{B}_t}) x$$

Reduce the rank of the newly introduced LoRA



$$\Delta \mathbf{W} = \alpha_1 \overline{\mathbf{A}_1 \mathbf{B}_1} + \alpha_2 \overline{\mathbf{A}_2 \mathbf{B}_2}$$

SD-LoRA-RR

$$r_1 = r_2 = \ldots > r_{\mu} = r_{\mu+1} = \ldots > r_{\nu} = r_{\nu+1} = \ldots = r_N$$

Don't need to introduce the extra LoRA part

SD-LoRA-KD (Knowledge Distillation)

$$\{\Delta lpha_k\}_{k=1}^{t-1} = rg\min_{\{lpha_k'\}_{k=1}^{t-1}} \left\| \overline{\mathbf{A}_t \mathbf{B}_t} - \sum_{k=1}^{t-1} lpha_k' \overline{\mathbf{A}_k \mathbf{B}_k}
ight\|_F^2$$

$$\boldsymbol{h}' = \left(\mathbf{W}_0 + (\alpha_1 + \Delta \alpha_1)\overline{\mathbf{A}_1\mathbf{B}_1} + (\alpha_2 + \Delta \alpha_2)\overline{\mathbf{A}_2\mathbf{B}_2} + \ldots + (\alpha_{t-1} + \Delta \alpha_{t-1})\overline{\mathbf{A}_{t-1}\mathbf{B}_{t-1}}\right)\boldsymbol{x}$$

Represent new directions by the subspace spanned by previously learned directions

Experimental Results

The performance on different task lengths.

Method	ImageNet-R ($N=5$)		ImageNet-R ($N = 10$)		ImageNet-R ($N=20$)	
	Acc ↑	$AAA\uparrow$	Acc ↑	AAA ↑	Acc ↑	AAA ↑
Full Fine-Tuning	$64.92_{(0.87)}$	$75.57_{(0.50)}$	$60.57_{(1.06)}$	$72.31_{(1.09)}$	$49.95_{(1.31)}$	$65.32_{(0.84)}$
L2P	$73.04_{(0.71)}$	$76.94_{(0.41)}$	$71.26_{(0.44)}$	$76.13_{(0.46)}$	$68.97_{(0.51)}$	$74.16_{(0.32)}$
DualPrompt	$69.99_{(0.57)}$	$72.24_{(0.41)}$	$68.22_{(0.20)}$	$73.81_{(0.39)}$	$65.23_{(0.45)}$	$71.30_{(0.16)}$
CODA-Prompt	$76.63_{(0.27)}$	$80.30_{(0.28)}$	$74.05_{(0.41)}$	$78.14_{(0.39)}$	$69.38_{(0.33)}$	$73.95_{(0.63)}$
HiDe-Prompt	$74.77_{(0.25)}$	$78.15_{(0.24)}$	$74.65_{(0.14)}$	$78.46_{(0.18)}$	$73.59_{(0.19)}$	$77.93_{(0.19)}$
InfLoRA	$76.95_{(0.23)}$	$81.81_{(0.14)}$	$74.75_{(0.64)}$	$80.67_{(0.55)}$	$69.89_{(0.56)}$	$76.68_{(0.57)}$
SD-LoRA	79.15 _(0.20)	83.01 _(0.42)	77.34 _(0.35)	82.04 _(0.24)	75.26 _(0.37)	80.22 _(0.72)
SD-LoRA-RR	$79.01_{(0.26)}$	$82.50_{(0.38)}$	$77.18_{(0.39)}$	$81.74_{(0.24)}$	$74.05_{(0.51)}$	80.65 (0.35)
SD-LoRA-KD	$78.85_{(0.29)}$	$82.47_{(0.58)}$	$77.03_{(0.67)}$	$81.52_{(0.26)}$	$74.12_{(0.66)}$	$80.11_{(0.75)}$

The performance on different continual learning benchmarks.

Method	ImageNet-	A (N = 10)	DomainNet $(N = 5)$		
	Acc ↑	AAA ↑	Acc ↑	AAA ↑	
Full Fine-Tuning	$16.31_{(7.89)}$	$30.04_{(13.18)}$	$51.46_{(0.47)}$	$67.08_{(1.13)}$	
L2P (Wang et al., 2022b)	$42.94_{(1.27)}$	$51.40_{(1.95)}$	$70.26_{(0.25)}$	$75.83_{(0.98)}$	
DualPrompt (Wang et al., 2022a)	$45.49_{(0.96)}$	$54.68_{(1.24)}$	$68.26_{(0.90)}$	$73.84_{(0.45)}$	
CODA-Prompt (Smith et al., 2023)	$45.36_{(0.78)}$	$57.03_{(0.94)}$	$70.58_{(0.53)}$	$76.68_{(0.44)}$	
HiDe-Prompt (Wang et al., 2024a)	$42.70_{(0.60)}$	$56.32_{(0.40)}$	$72.20_{(0.08)}$	$77.01_{(0.04)}$	
InfLoRA (Liang & Li, 2024)	$49.20_{(1.12)}$	$60.92_{(0.61)}$	$71.59_{(0.23)}$	$78.29_{(0.50)}$	
SDLoRA	55.96 _(0.73)	64.95 _(1.63)	72.82 _(0.37)	78.89 _(0.50)	
SD-LoRA-RR	$55.59_{(1.08)}$	$64.59_{(1.91)}$	$72.58_{(0.40)}$	$78.79_{(0.78)}$	
SD-LoRA-KD	$54.24_{(1.12)}$	$63.89_{(0.58)}$	$72.15_{(0.50)}$	$78.44_{(0.66)}$	

The detailed performance on the streaming tasks and the results on different backbones

