

Unifying Renormalization with Markov Categories

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Idea

Renormalization

Central concept of modern physics, it is a mathematical technique of “coarse-graining” over certain degrees of freedoms.

Markov kernels

A mathematical structure modeling notions of “averaging”, including probabilistic, and can be used as renormalization steps. Mathematically, a kernel $(X, \mathcal{A}) \rightarrow (Y, \mathcal{B})$ is a map $X \times \mathcal{B} \rightarrow [0, 1]$ which is integrable in X , and a probability measure in \mathcal{B} .

Markov categories

Markov kernels, with their composition, form a category, which can be used to model subsequent steps of coarse-graining.

Markov categories are an abstraction of that, and allow rigorous calculations by manipulating diagrams (cf. Feynman diagrams).

Example: Ising spin chain

Setting: Ising chain of spins $\{+1, -1\}$ with periodic boundary

$$H(\sigma) = -J \sum_{i=1}^N \sigma_i \sigma_{i+1},$$

Construct a kernel e.g. by majority rule

$$P(\tilde{\sigma}_k = +1 \mid \sigma_{(k-1)b+1}, \dots, \sigma_{kb}) = \frac{e^{-\beta \Delta E_+}}{e^{-\beta \Delta E_+} + e^{-\beta \Delta E_-}} \quad \text{etc.}$$

Compose renormalization steps (in diagrams)

$$X - \boxed{\varphi_1} - X^{(1)} - \boxed{\varphi_2} - X^{(2)} - \dots$$

References



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