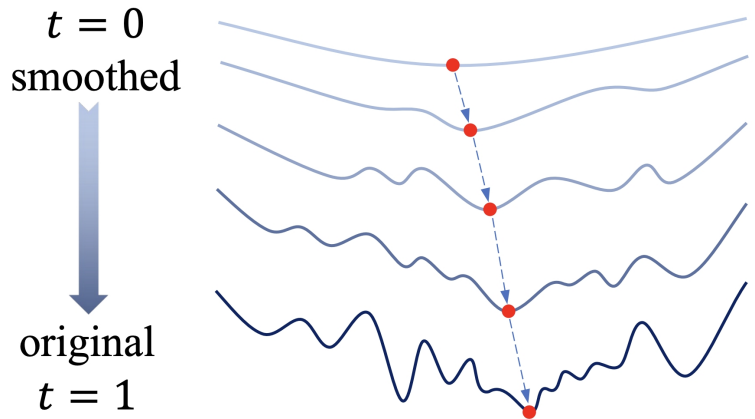
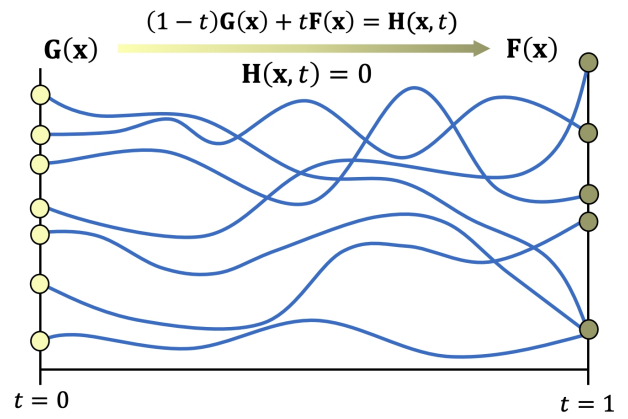


Neural Predictor-Corrector: Solving Homotopy Problems with Reinforcement Learning

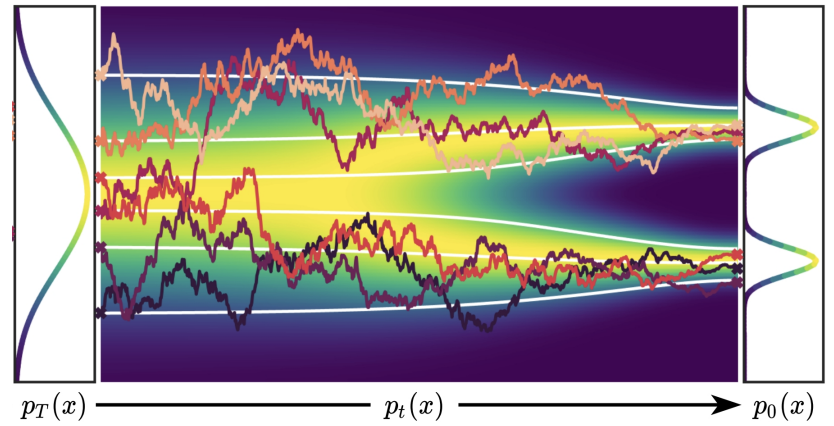
Jiayao Mai^{1*}, Bangyan Liao^{2*}, Zhenjun Zhao³, Yingping Zeng¹,
Haoang Li⁴, Javier Civera³, Tailin Wu², Yi Zhou^{1†}, Peidong Liu^{2†}



Optimization^[1]



Polynomial Root Finding^[2]



Sampling^[3]

A general principle for solving challenging problems

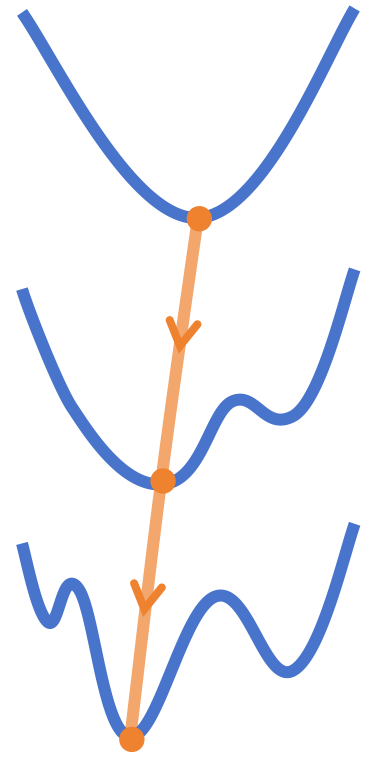
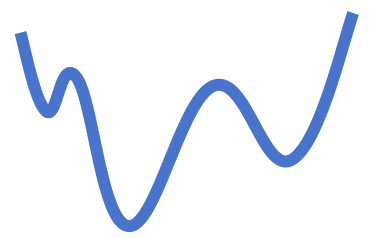
[1] Lin X, Yang Z, Zhang X, et al. Continuation path learning for homotopy optimization[C]//International Conference on Machine Learning. PMLR, 2023: 21288-21311.
[2] Zhang, Xinyue, et al. "Simulator HC: Regression-based Online Simulation of Starting Problem-Solution Pairs for Homotopy Continuation in Geometric Vision." 2025 IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR). IEEE, 2025.
[3] Song Y, Sohl-Dickstein J, Kingma D P, et al. Score-based generative modeling through stochastic differential equations[J]. arXiv preprint arXiv:2011.13456, 2020.



 **Easy Problem
Known Solutions**



 **Hard Problem
Unknown Solutions**



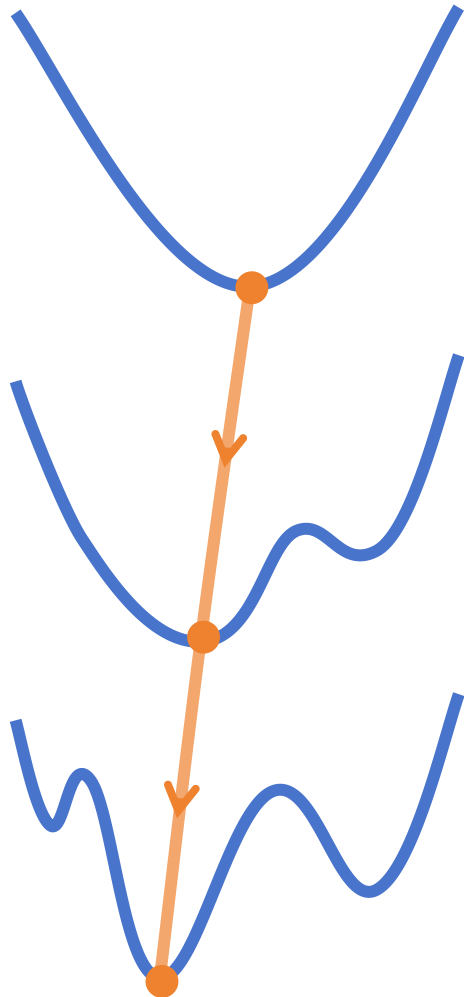
**Explicit Homotopy
Interpolation Construction**



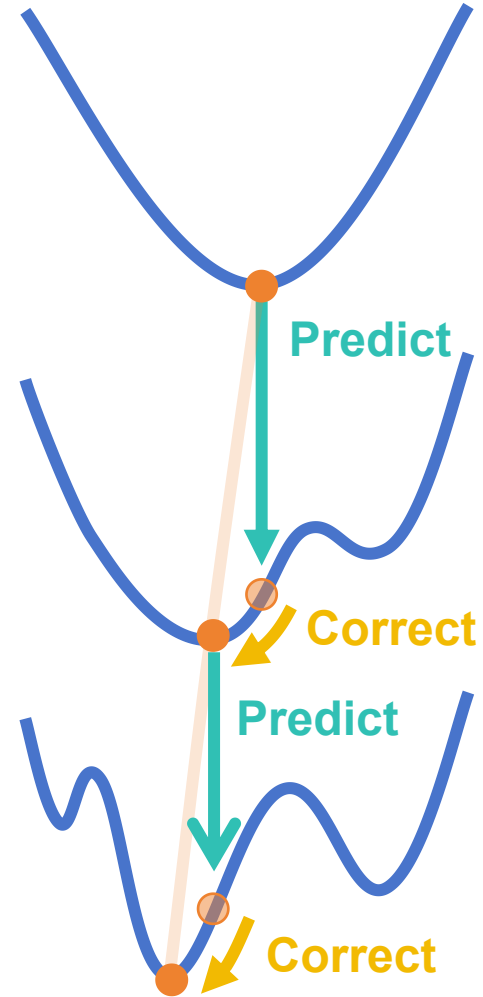
**Implicit Solution
Trajectory Tracing**

Our Scope

Implicit Solution Trajectory



Classical Predictor-Corrector



Algorithm

while $t_n < T$ **do**

Predictor: Predict next level t_n

Predictor: Predict \mathbf{x}_{t_n}

while $H(\mathbf{x}_{t_n}, t_n) \leq \epsilon$ **do**

Corrector: Correct \mathbf{x}_{t_n}

end

end

Classical PC relies on hand-crafted heuristics

Classical Predictor-Corrector

$\Delta t, \epsilon$

Algorithm 1

```
while  $t_n < T$  do
  Predictor: Predict next level  $t_n = t_{n-1} + \Delta t$ 
  Predictor: Predict  $\mathbf{x}_{t_n}$ 
  while  $H(\mathbf{x}_{t_n}, t_n) \leq \epsilon$  do
    Corrector: Correct  $\mathbf{x}_{t_n}$ 
  end
end
end
```

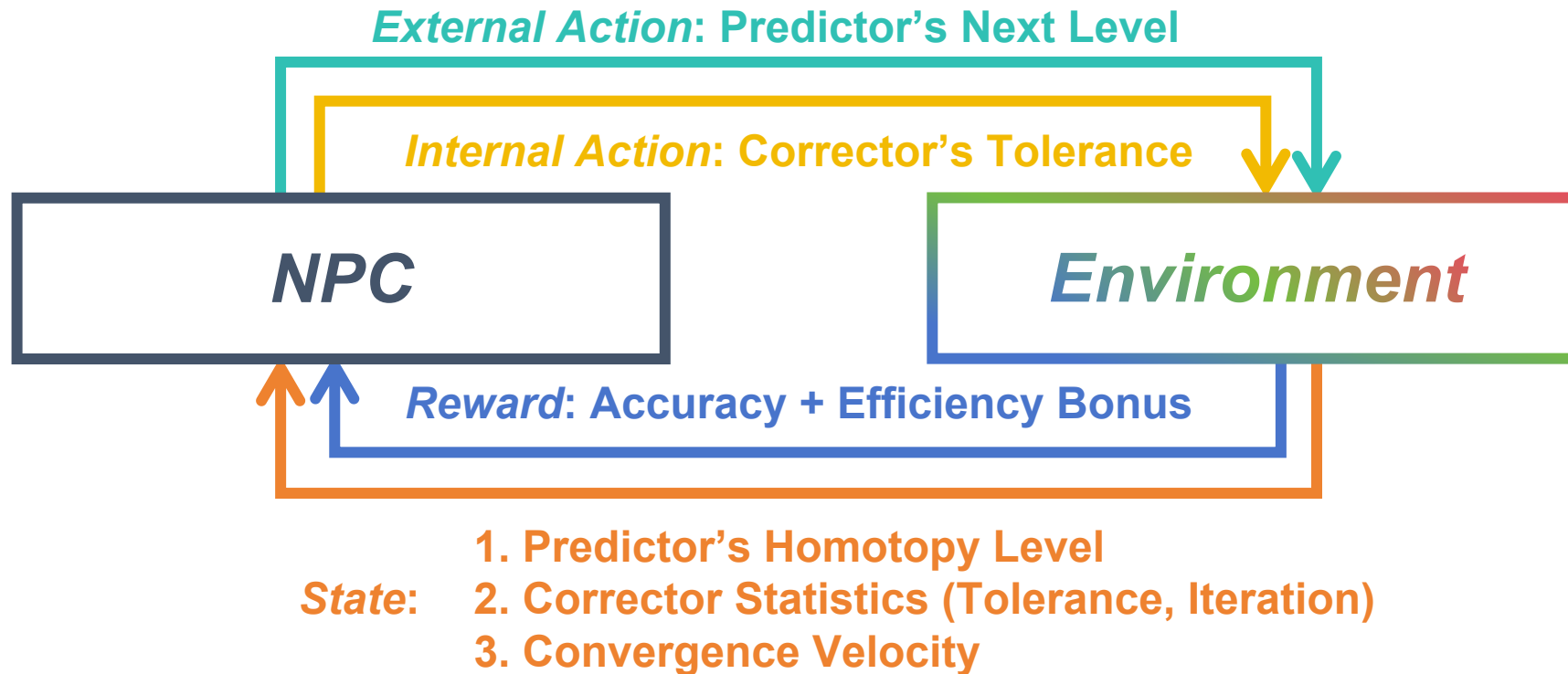
Neural Predictor-Corrector

$\Delta t_n, \epsilon_n$

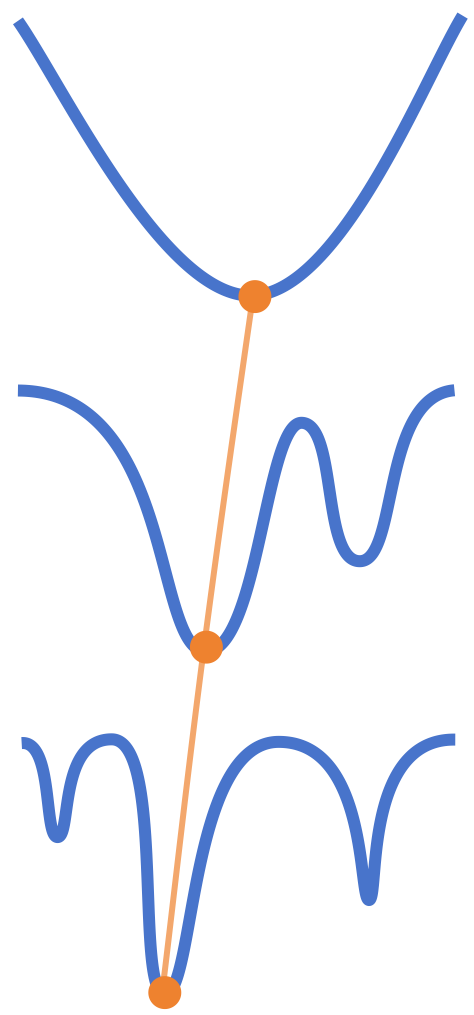
Algorithm 2

```
while  $t_n < T$  do
  NPC:  $\{\Delta t_n, \epsilon_n\} = \mathbf{NN}(t_{n-1}, \epsilon_{n-1}, \tau_{n-1})$ 
  Predictor: Predict next level  $t_n = t_{n-1} + \Delta t_n$ 
  Predictor: Predict  $\mathbf{x}_{t_n}$ 
  while  $H(\mathbf{x}_{t_n}, t_n) \leq \epsilon_n$  do
    Corrector: Correct  $\mathbf{x}_{t_n}$ 
  end
  Collect corrector statistics  $\epsilon_n$ 
  Collect convergence velocity  $\tau_n$ 
end
```

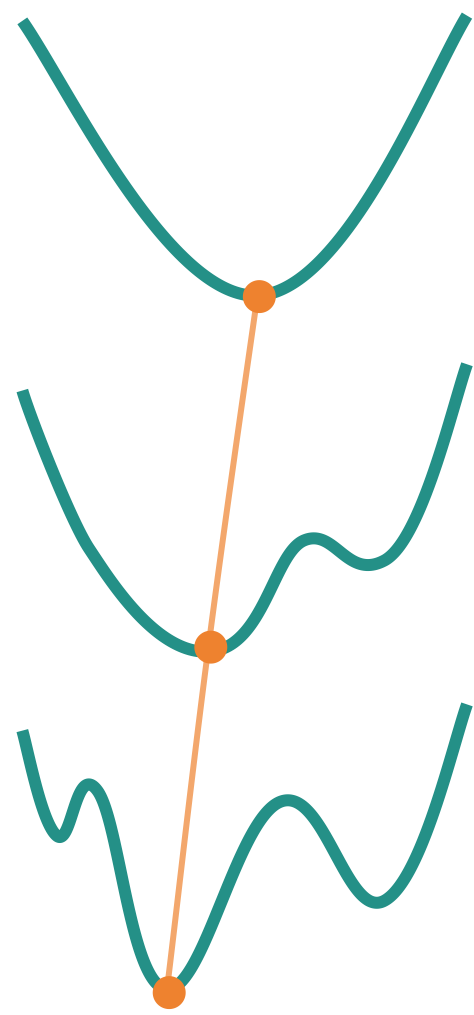
NPC replaces manual heuristics with a neural network



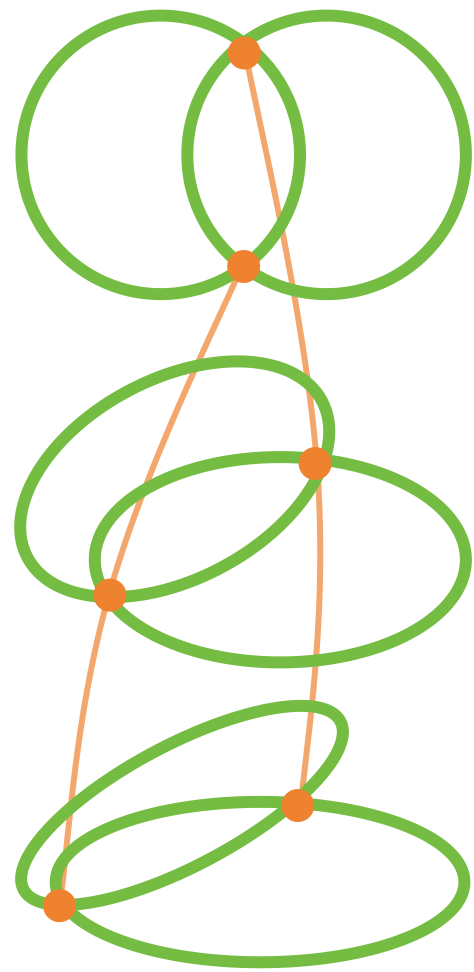
Training NPC via RL to discover optimal policies



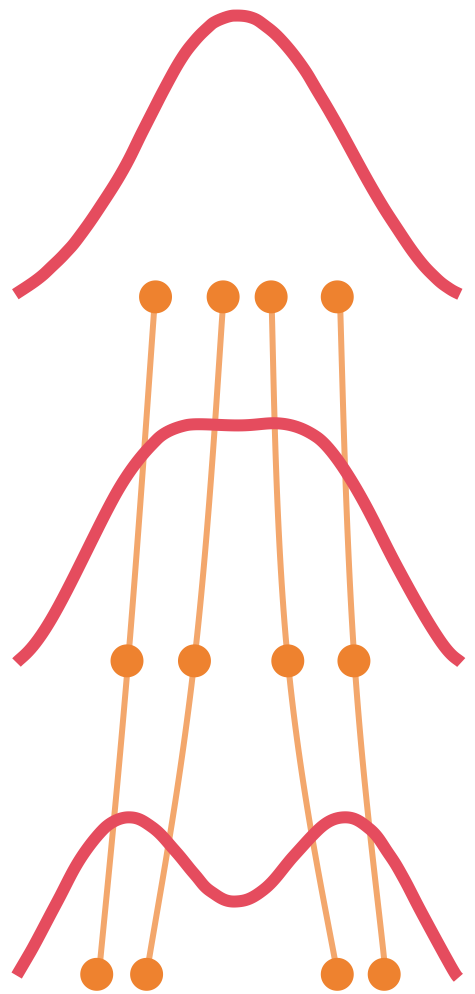
Robust Optimization



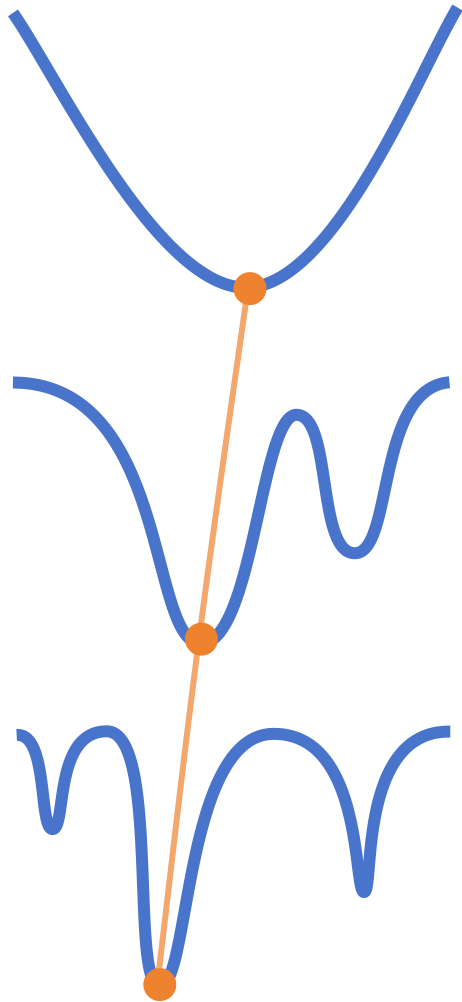
Global Optimization



Polynomial Root Finding



Probabilistic Sampling

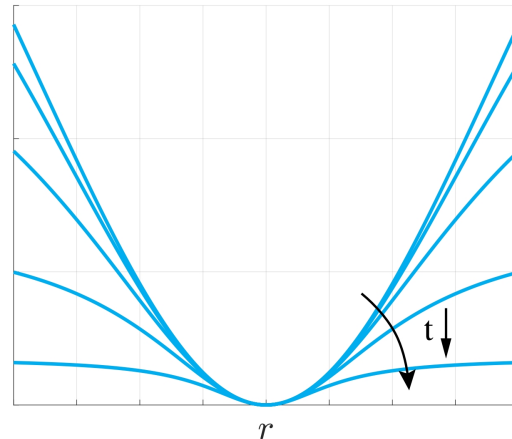


**Robust
Optimization**

Graduated Non-Convexity (GNC)^[1]

Outer Interpolation (Geman-McClure)

$$H(\mathbf{x}, t) = \sum_i \frac{\bar{c}^2 r(\mathbf{x}, y_i)^2}{\bar{c}^2 + tr(\mathbf{x}, y_i)^2}$$

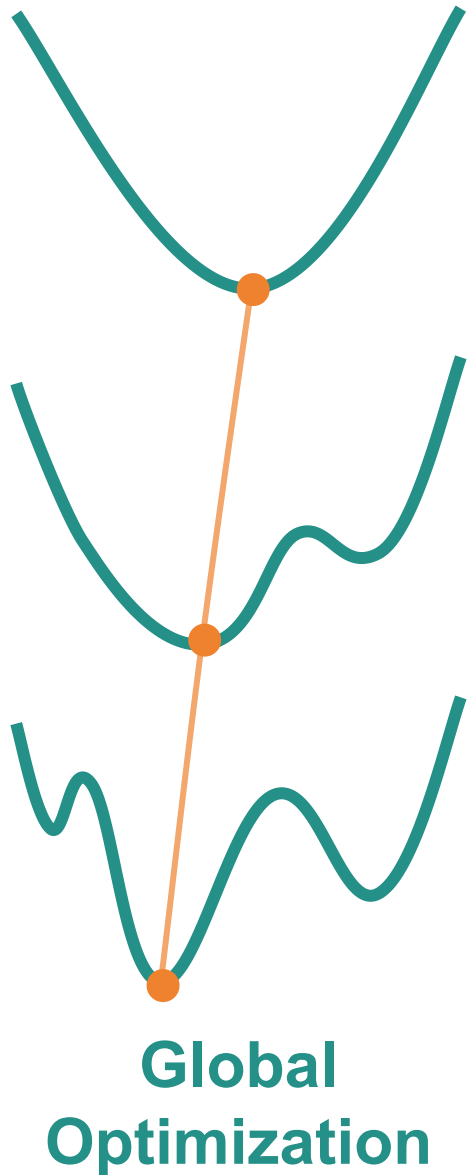


Predictor

$$t_{i+1} = t_i + \Delta t$$
$$\mathbf{x}_{t_{i+1}} = \mathbf{x}_{t_i}^*$$

Corrector (*Gauss-Newton*)

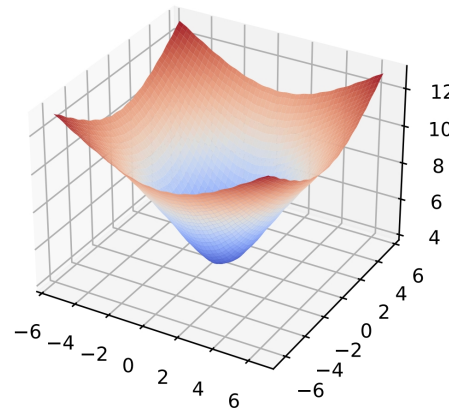
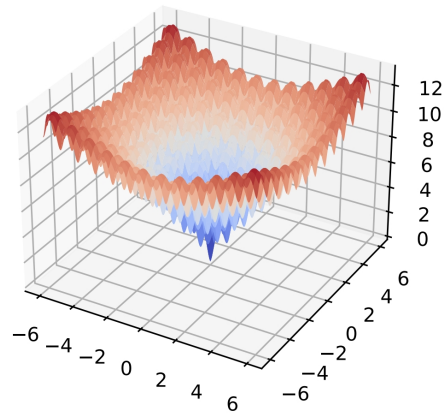
$$\mathbf{x}_t^* = \arg \min_{\mathbf{x}} H(\mathbf{x}, t)$$



Gaussian Homotopy (GH)^[1]

Outer Interpolation

$$H(\mathbf{x}, t) = g(\mathbf{x}) * \mathcal{N}(0, t\sigma^2)$$



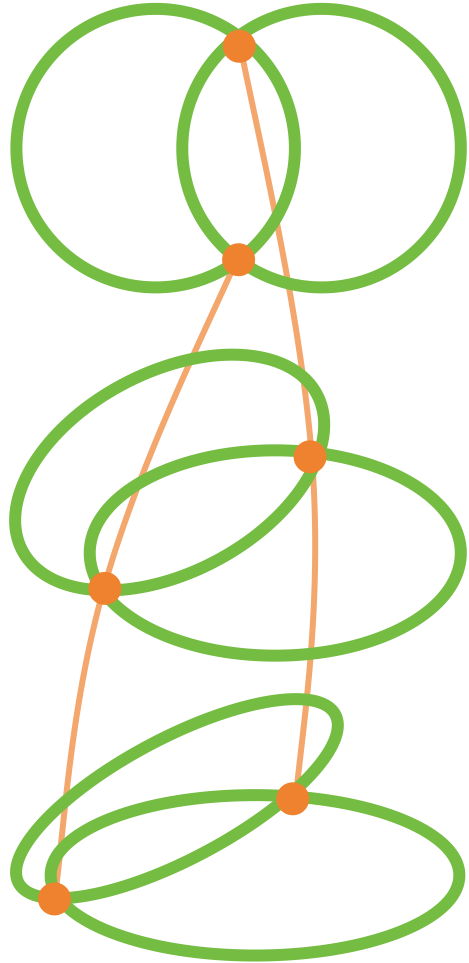
Predictor

$$t_{i+1} = t_i + \Delta t$$

$$\mathbf{x}_{t_{i+1}}^* = \mathbf{x}_{t_i}^*$$

Corrector

$$\mathbf{x}_t^* = \arg \min_{\mathbf{x}} H(\mathbf{x}, t)$$

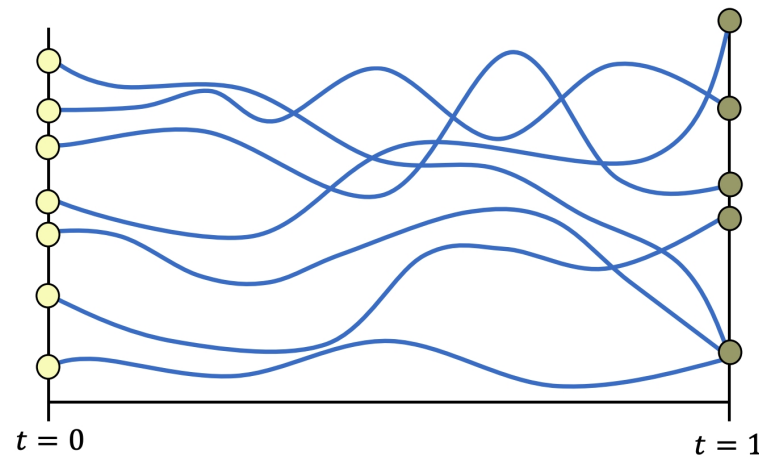


Polynomial Root Finding

Homotopy Continuation (HC)^[1]

Outer Interpolation

$$H(\mathbf{x}, t) = g(\mathbf{x})(1 - t) + f(\mathbf{x})t$$

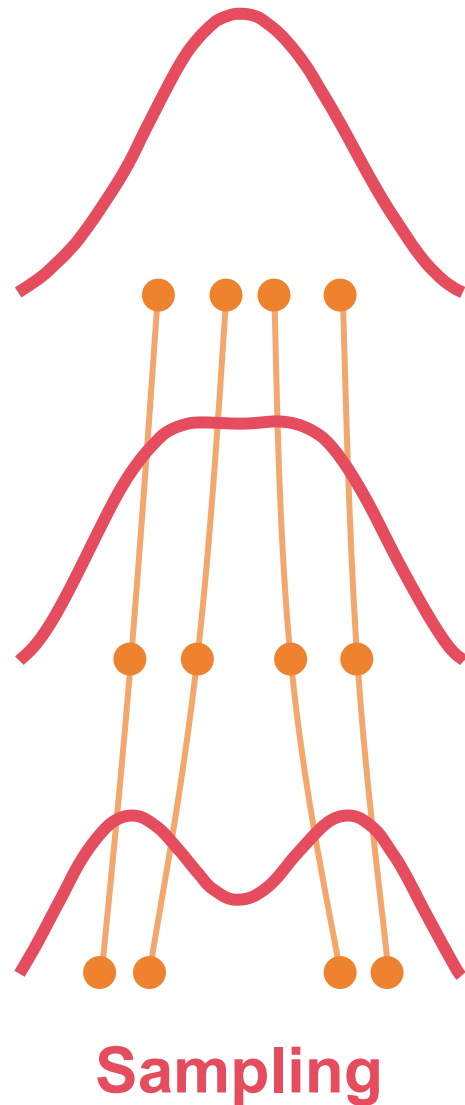


Predictor

$$t_{i+1} = t_i + \Delta t$$
$$\{\mathbf{x}_{t_{i+1}}\} = \{\mathbf{x}_{t_i}^* + \partial_t H(\mathbf{x}_{t_i}^*, t_i) \Delta t\}$$

Corrector (*Gauss-Newton*)

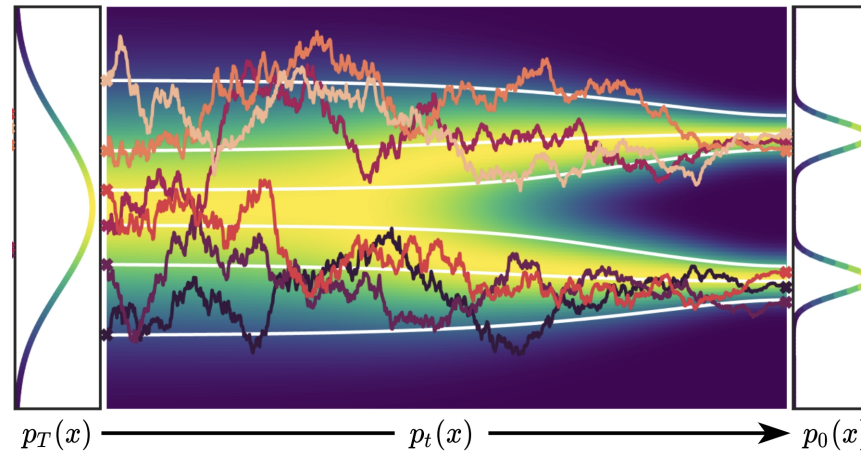
$$\{\mathbf{x}_t^*\} = \{H(\mathbf{x}, t) = 0\}$$



Annealed Langevin Dynamics (ALD)^[1]

Outer Interpolation

$$H(\mathbf{x}, t) \propto \exp(-g(\mathbf{x})(1-t) - f(\mathbf{x})t)$$



Predictor

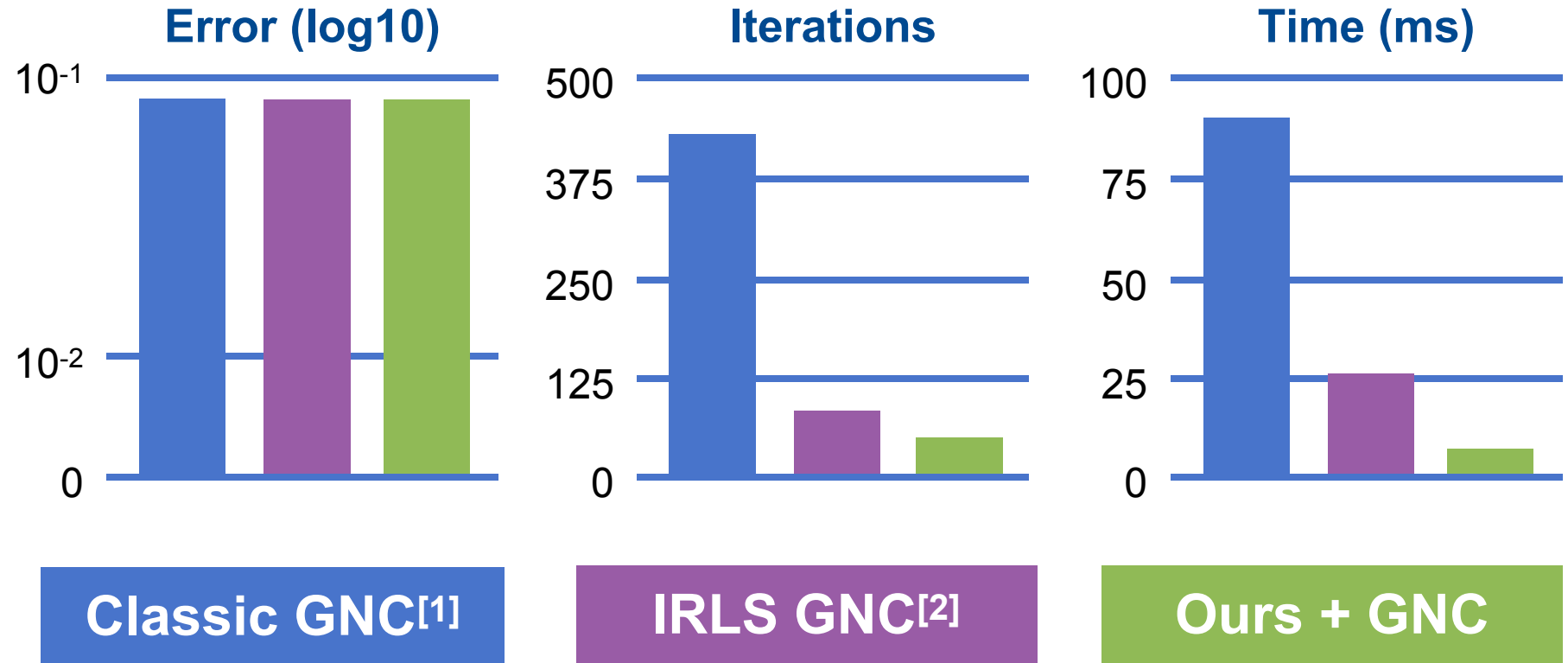
$$t_{i+1} = t_i + \Delta t$$

$$\{X_{t_{i+1}}^0\} = \{X_{t_i}^*\}$$

Corrector (LD)

$$dX_t^S = \nabla \log H(\mathbf{x}, t) ds + dW_s$$

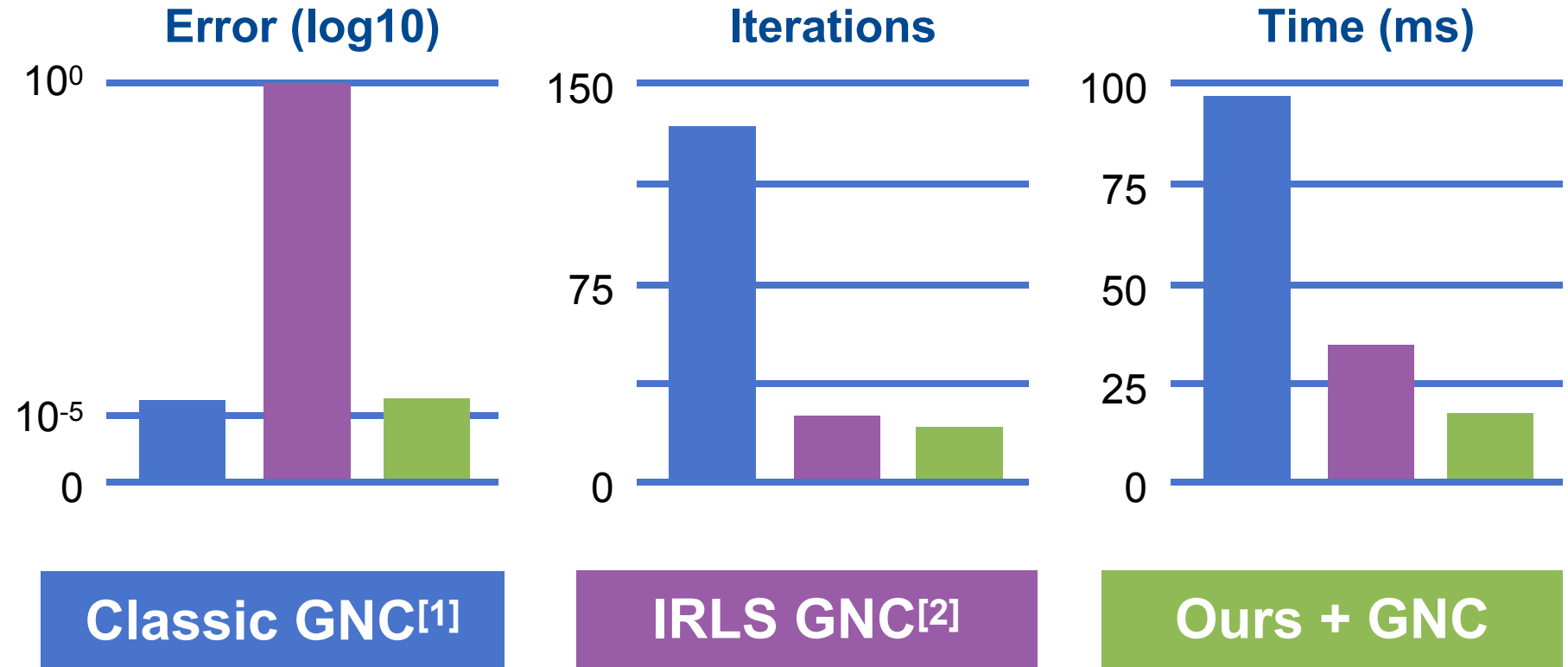
Point Cloud Registration (Seq cube)



[1] Yang H, Antonante P, Tzoumas V, et al. Graduated non-convexity for robust spatial perception: From non-minimal solvers to global outlier rejection[J]. IEEE Robotics and Automation Letters, 2020, 5(2): 1127-1134.

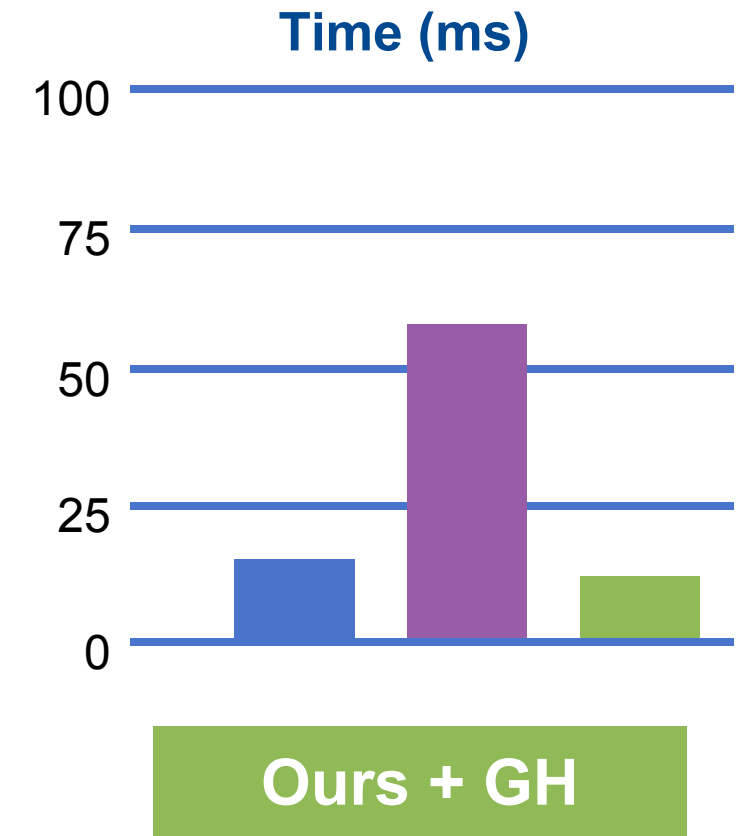
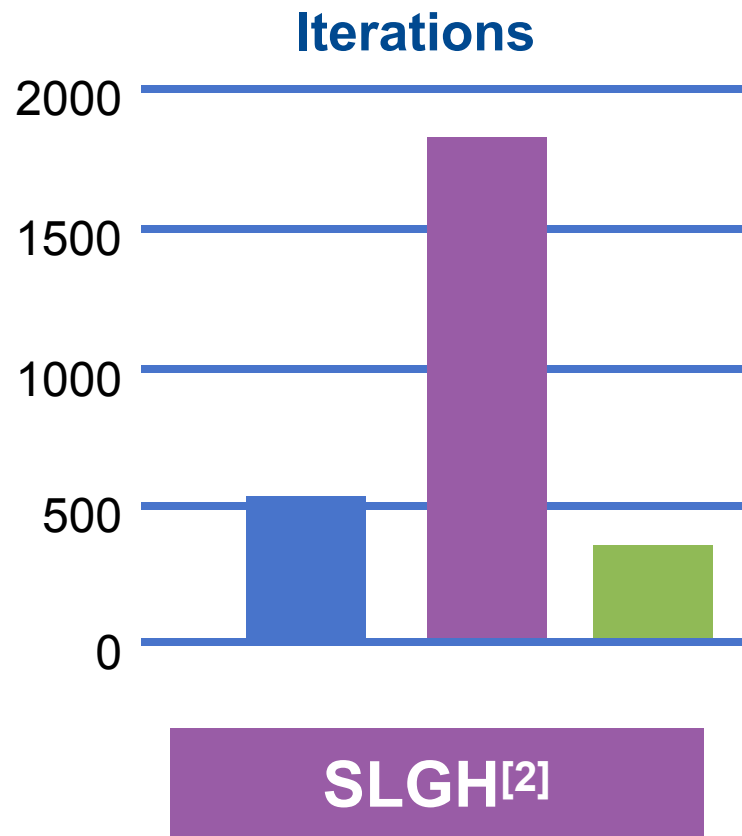
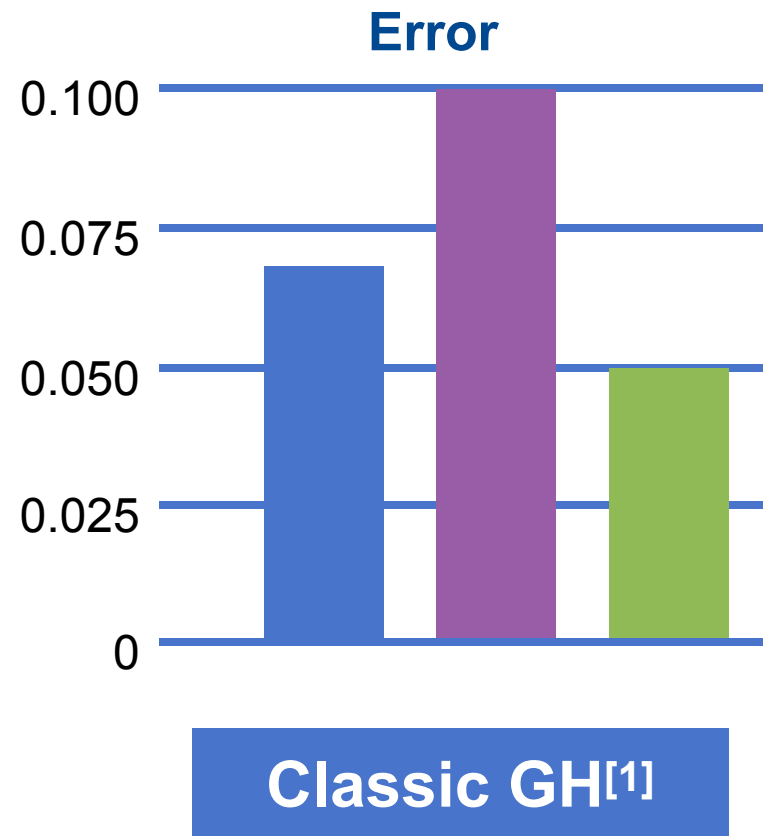
[2] Peng L, Kümmerle C, Vidal R. On the convergence of IRLS and its variants in outlier-robust estimation[C]//Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. 2023: 17808-17818.

Multi-view Triangulation (Seq st_pt_sq)



[1] Yang H, Antonante P, Tzoumas V, et al. Graduated non-convexity for robust spatial perception: From non-minimal solvers to global outlier rejection[J]. IEEE Robotics and Automation Letters, 2020, 5(2): 1127-1134.

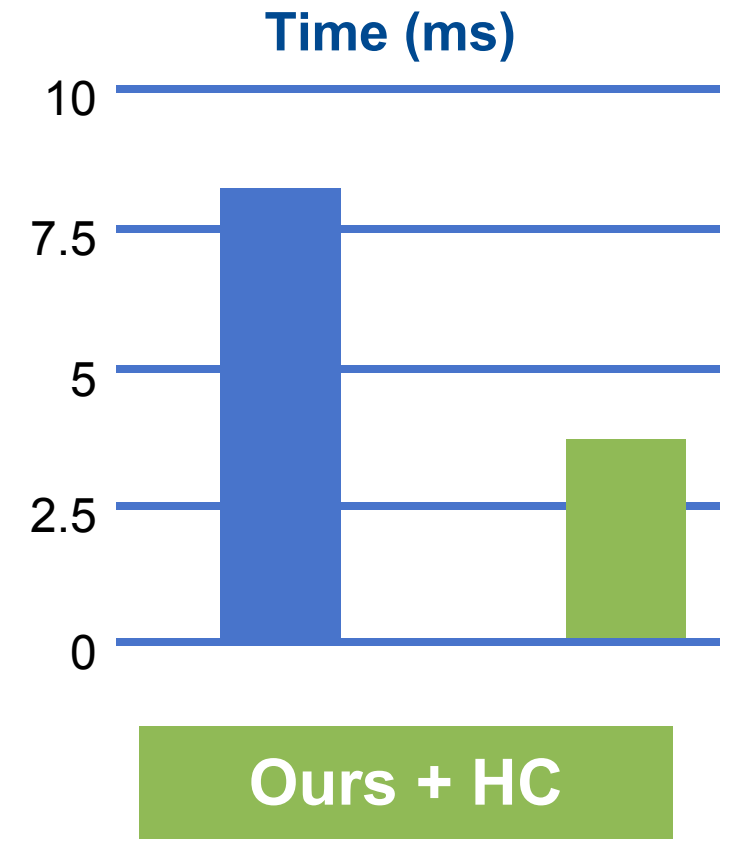
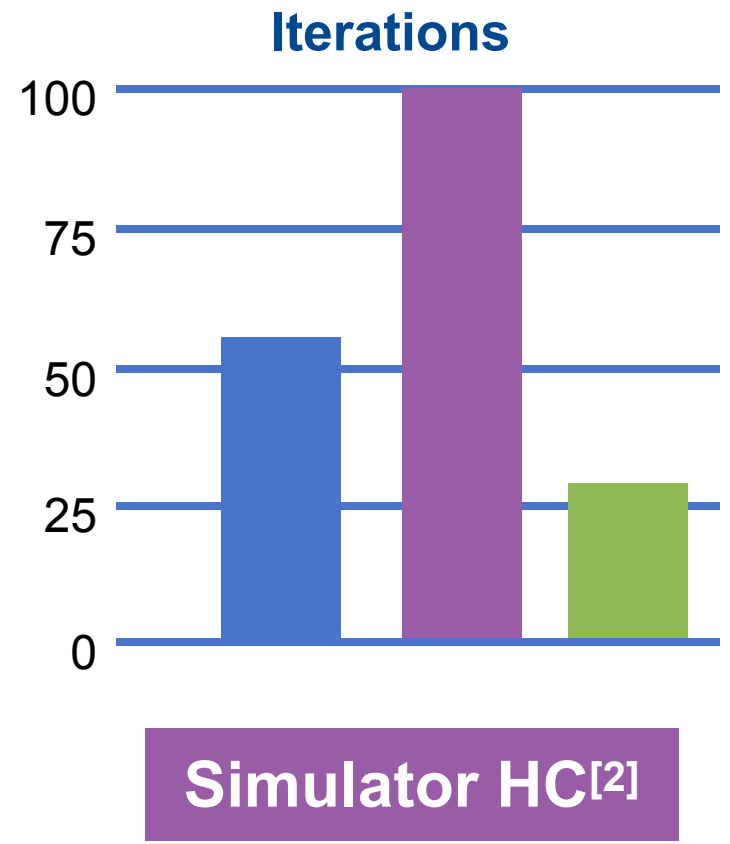
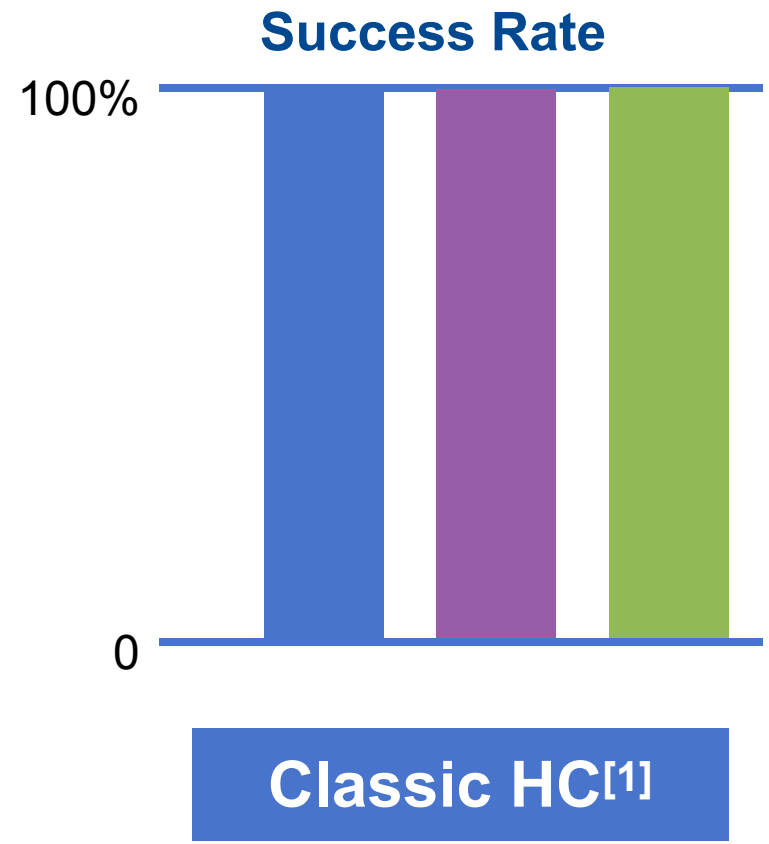
[2] Peng L, Kümmerle C, Vidal R. On the convergence of IRLS and its variants in outlier-robust estimation[C]//Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. 2023: 17808-17818.



(Seq 2d_Ackley)

[1] Mobahi H, Fisher III J W. On the link between gaussian homotopy continuation and convex envelopes[C]//International Workshop on Energy Minimization Methods in Computer Vision and Pattern Recognition. Cham: Springer International Publishing, 2015: 43-56.

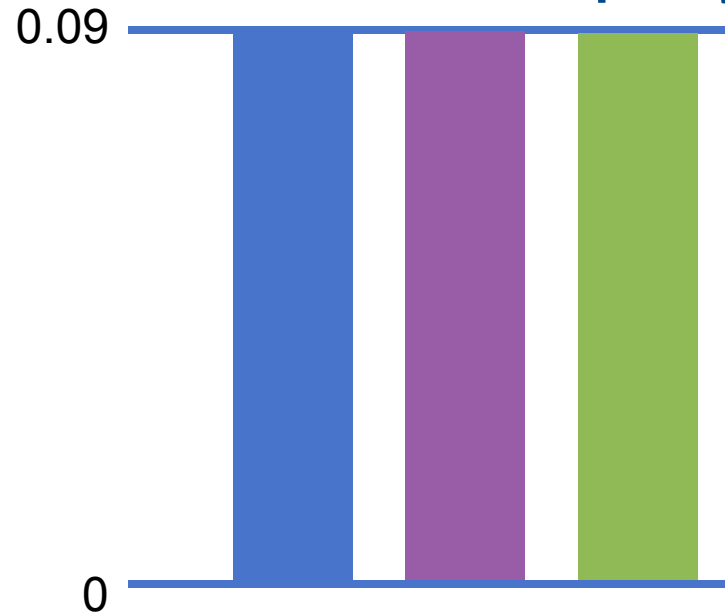
[2] Iwakiri H, Wang Y, Ito S, et al. Single loop gaussian homotopy method for non-convex optimization[J]. Advances in Neural Information Processing Systems, 2022, 35: 7065-7076.



(Seq UPnP)

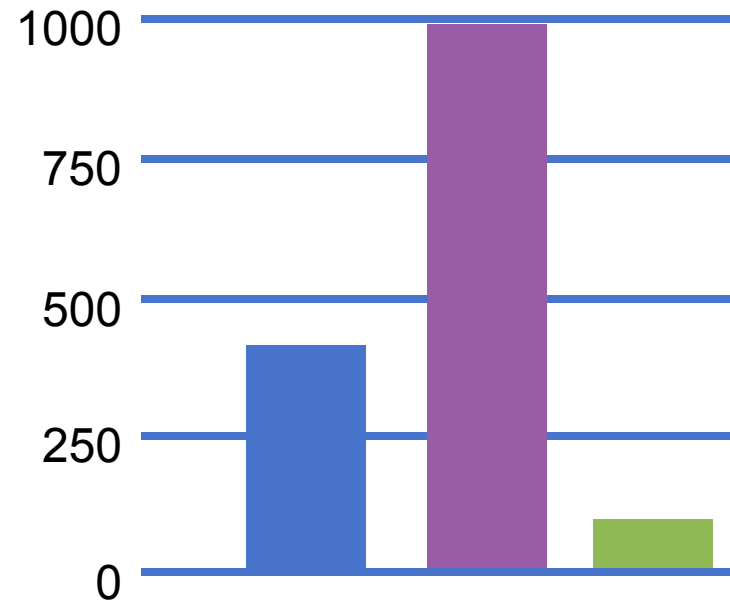
[1] Bates D J, Sommese A J, Hauenstein J D, et al. Numerically solving polynomial systems with Bertini[M]. Society for Industrial and Applied Mathematics, 2013.
[2] Zhang X, Dai Z, Xu W, et al. Simulator hc: Regression-based online simulation of starting problem-solution pairs for homotopy continuation in geometric vision[C]//Proceedings of the Computer Vision and Pattern Recognition Conference. 2025: 27103-27112.

Kernel Stein Discrepancy



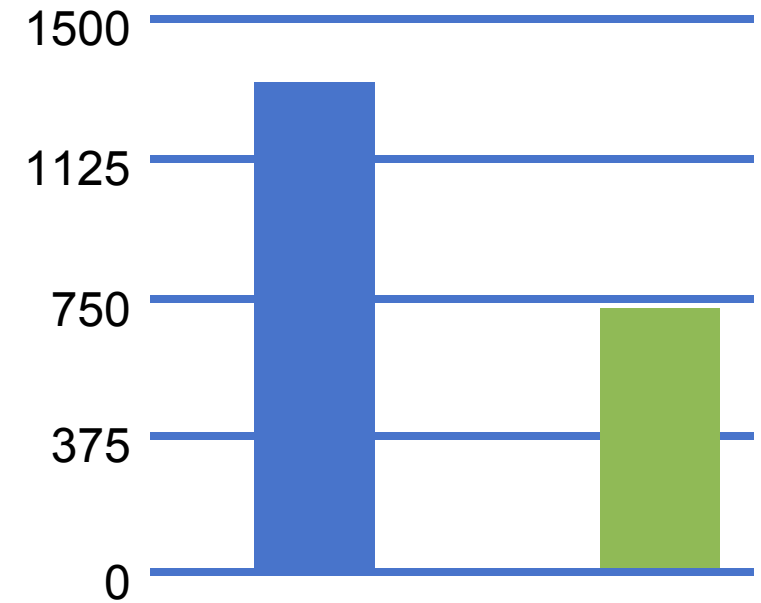
Classic ALD^[1]

Iterations



IDEM^[2]

Time (ms)



Ours + ALD

(Seq DW-4)

[1] Song Y, Sohl-Dickstein J, Kingma D P, et al. Score-based generative modeling through stochastic differential equations[J]. arXiv preprint arXiv:2011.13456, 2020.

[2] Akhound-Sadegh T, Rector-Brooks J, Bose A J, et al. Iterated denoising energy matching for sampling from boltzmann densities[J]. arXiv preprint arXiv:2402.06121, 2024.

Neural Predictor-Corrector: Solving Homotopy Problems with Reinforcement Learning



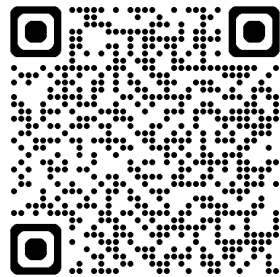
***Comparable
Performance***



***Less
Iterations***

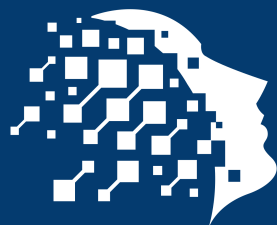


***Unified
Framework***



 **GitHub**

<https://github.com/maijiayao1/NPC>



Thanks for watching