Continuous Multinomial Logistic Regression for neural decoding

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Background: Multinomial Logistic Regression (MLR) for neural decoding

neural population activity

Neural decoding →

sensory/behavioral variables

continuous

↑ **CMLR**

discrete

Multinomial Logistic Regression (MLR)

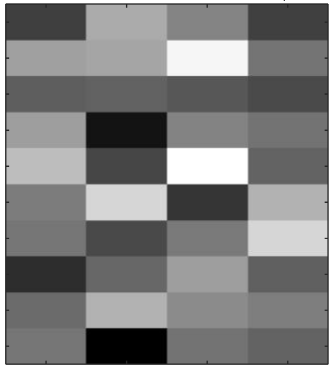
input feature vector

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix}$$

×

decoding weight matrix

class 1 … class K

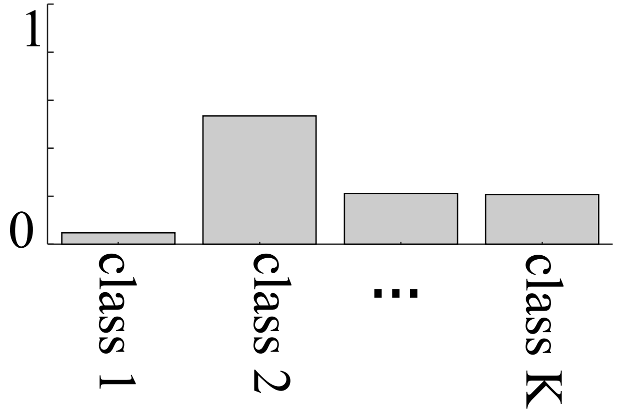


$\boldsymbol{w}_1$ … $\boldsymbol{w}_K$

exp ⇒

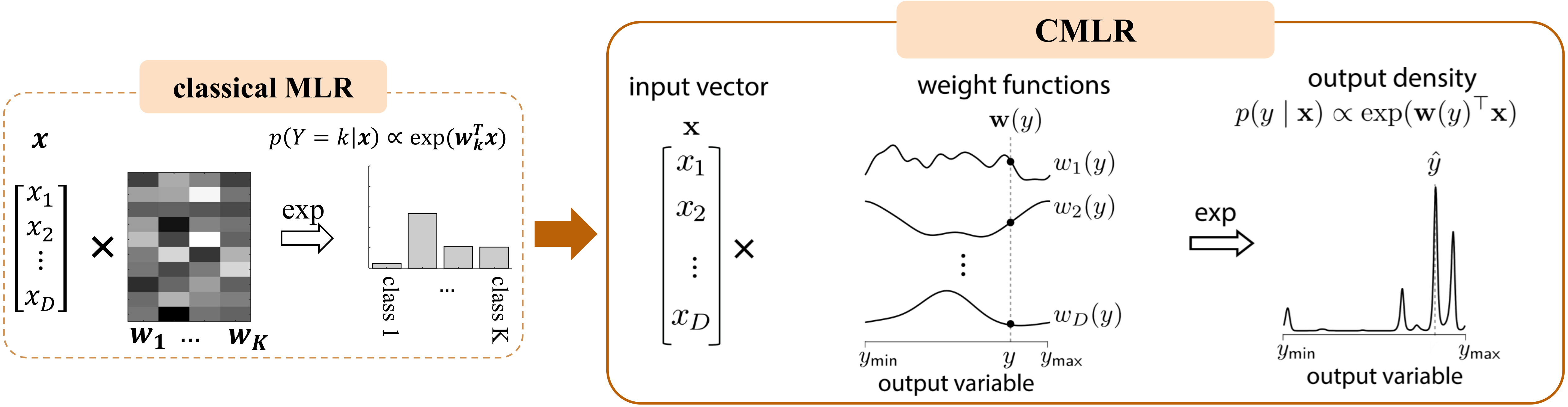
output distribution

$$p(Y = k|\boldsymbol{x}) \propto \exp(\boldsymbol{w}_k^T \boldsymbol{x})$$



Limitation: MLR requires **output discretization**, reducing accuracy

Continuous Multinomial Logistic Regression (CMLR) model



- **Gaussian Process (GP) priors:** induce smoothness in the decoding weight functions

$$w_d(y) \sim \mathcal{GP}\left(\mathbf{0}, K_d\right) \qquad K_d\left(y', y''\right) = \rho_d \exp\left(-\left(y' - y''\right)^2 / 2\ell_d^2\right)$$

- **Conditional Density Estimation (CDE):** CMLR defines an output conditional density

$$p(Y = y \mid \mathbf{x}) = \frac{\exp\left(\mathbf{w}\left(y\right)^\top \mathbf{x}\right)}{\int_\Omega \exp\left(\mathbf{w}(y')^\top \mathbf{x}\right) dy'}$$

CMLR model inference and output prediction

- **Stochastic Variational Inference (SVI)**: scalable frequency domain implementation
  - observations $\{\boldsymbol{x}_n, y_n\}_{n=1:N} \rightarrow$ infer weight functions $\boldsymbol{w}(y) = \{w_d(y)\}_{d=1:D}$
- **Output prediction**: decode outputs at any desired resolution
  - test sample $\boldsymbol{x}_n \rightarrow$ compute the posterior over an output grid $\{\tilde{y}_j\}_{j=1:J}$

$$p\left(y_n = \tilde{y}_j \mid \mathbf{x}_n, \mathbf{w}(y)\right) = \frac{\exp\left(\mathbf{w}(\tilde{y}_j)^\top \mathbf{x}_n\right)}{\sum_{j'=1}^{J} \exp\left(\mathbf{w}(\tilde{y}_{j'})^\top \mathbf{x}_n\right)}$$

posterior mean

$$\widehat{y}_{\text{mean}} = \sum_{j=1}^{J} \tilde{y}_j \cdot p\left(y_n = \tilde{y}_j \mid \mathbf{x}_n, \mathbf{w}(y)\right)$$

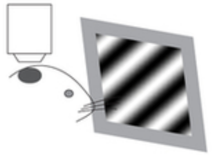
regression-style tasks

posterior mode

$$\widehat{y}_{\text{mode}} = \arg \max_{j \in \{1:J\}} p\left(y_n = \tilde{y}_j \mid \mathbf{x}_n, \mathbf{w}(y)\right)$$

classification-style tasks

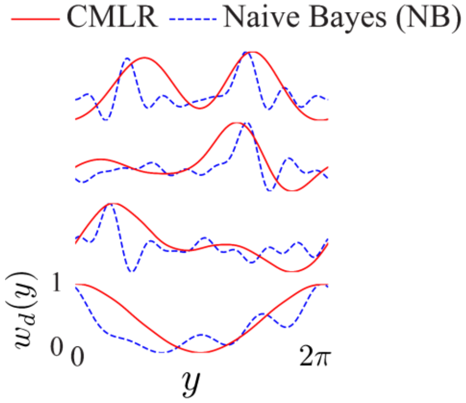
Mouse primary visual cortex (V1): decoding drifting grating orientation



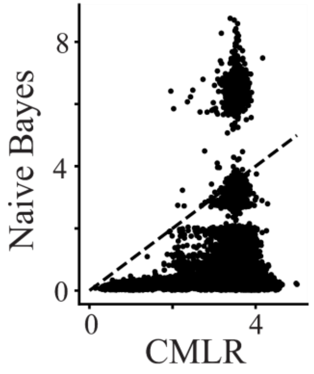
- **Input**: 3 calcium imaging datasets of mouse V1 [Stringer et al., 2021]
- **Output**: orientations $y \in [0, 2\pi)$, $D \sim 10^4$ neurons, $N \sim 4000$ samples

Comparing with Naïve Bayes

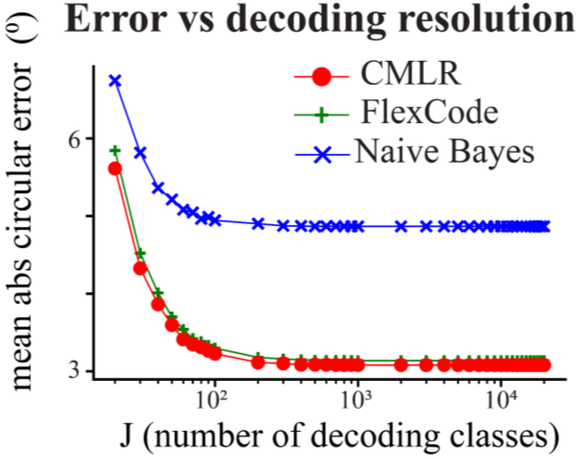
Inferred weights



Length-scale comparisons



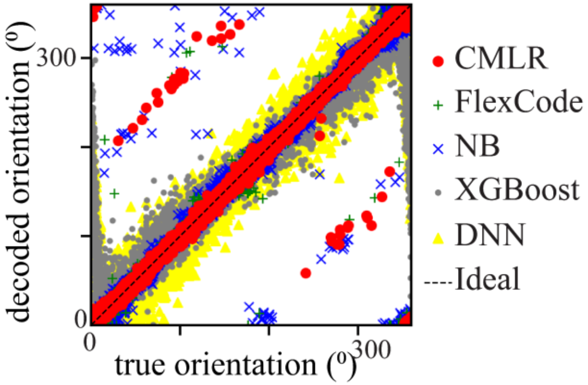
Error vs decoding resolution



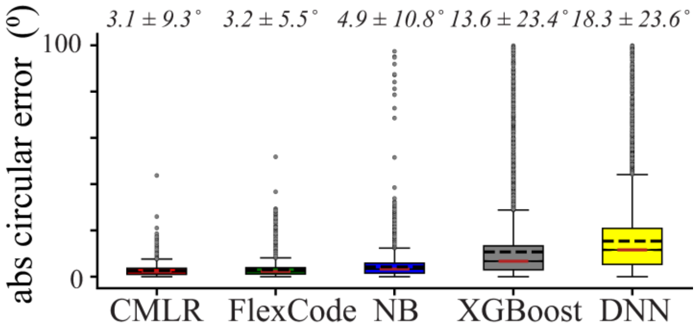
Neural correlations boosts performance

Comparing with all baselines

True vs decoded



Error comparisons



Consistent gains over strong baselines

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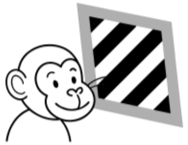
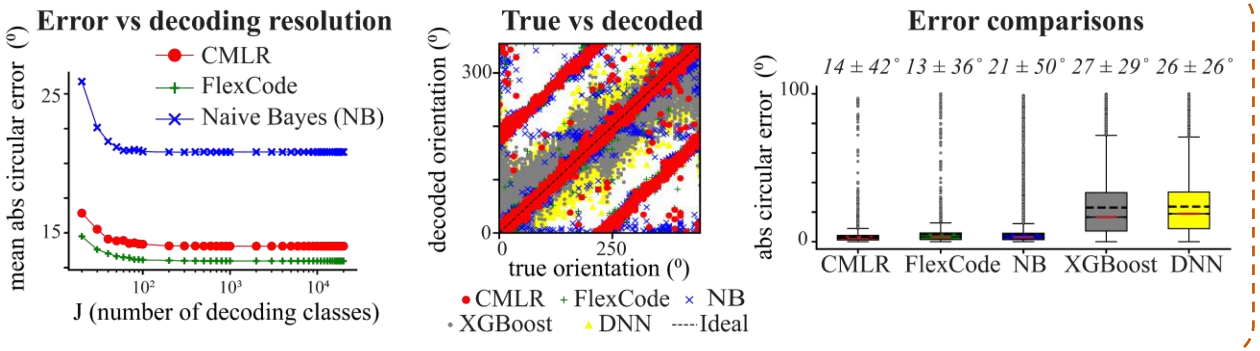
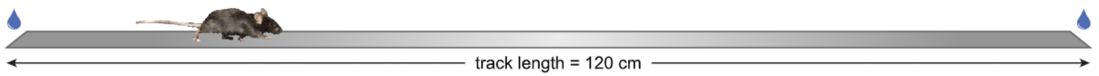
CMLR decoding applications…

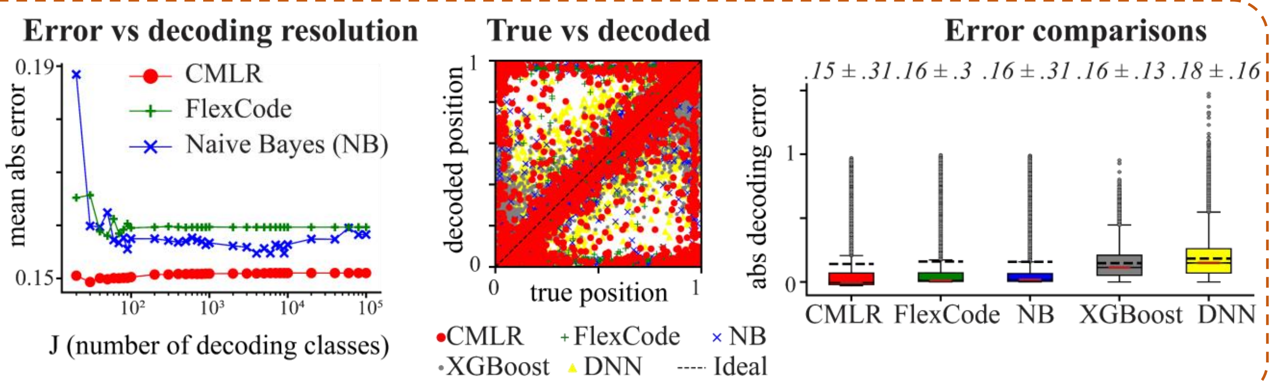
- **Macaque V1:**
  decoding **discrete** grating orientation



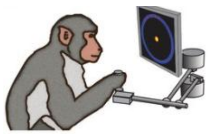


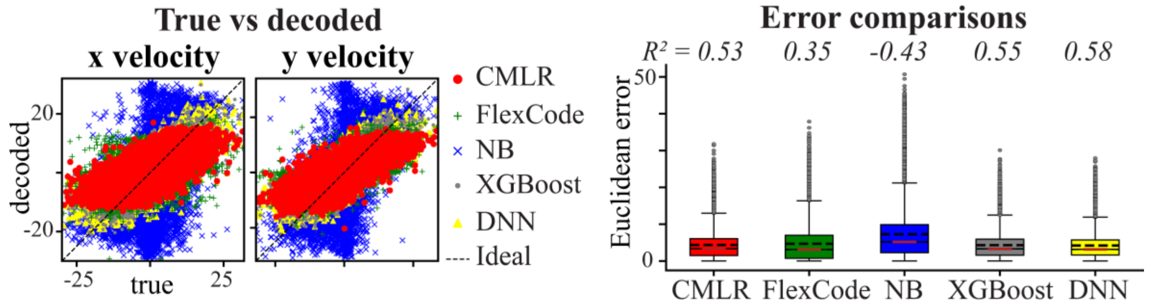
- **Hippocampus CA1:**
  decoding **position** during navigation





- **Motor Cortex:**
  decoding **2D velocity** in a reaching task





Conclusions

- **Continuous Multinomial Logistic Regression (CMLR):**
  extends MLR to continuous decoding variables
- **Gaussian process tuning functions:**
  smooth and interpretable neuron-specific weights
- **Flexible conditional density estimation (CDE):**
  captures circular, multimodal, and asymmetric distributions
- **Strong empirical performance:**
  outperforms Naive Bayes, XGBoost, DNNs, and FlexCode across datasets

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Data sources:

- Stringer et al., 2021 (mouse V1 datasets)
- Graf et al., 2011 (macaque V1 datasets)
- Hazon et al., 2022 (mouse CA1 datasets)
- Glaser et al., 2018 (monkey MC dataset)