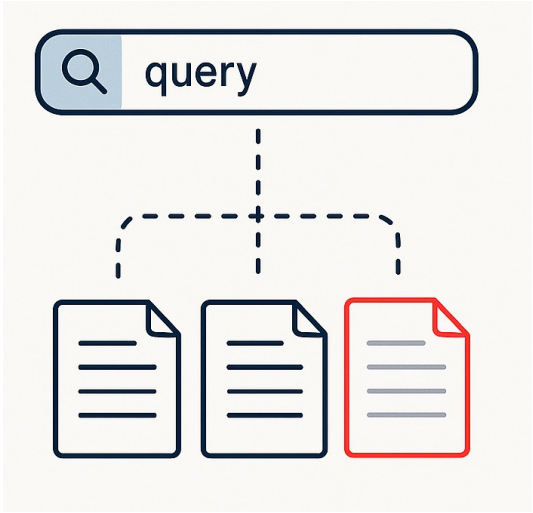
Graph-based Nearest Neighbors with Dynamic Updates via Random Walks

Nina Mishra, Yonatan Naamad, Tal Wagner, Lichen Zhang

Approximate Nearest Neighbor Search (ANN)



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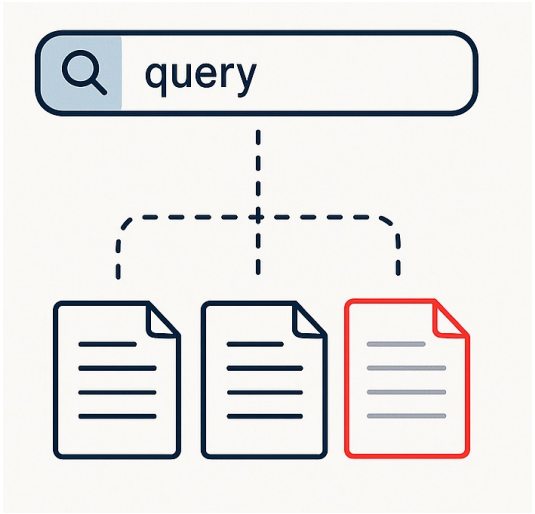
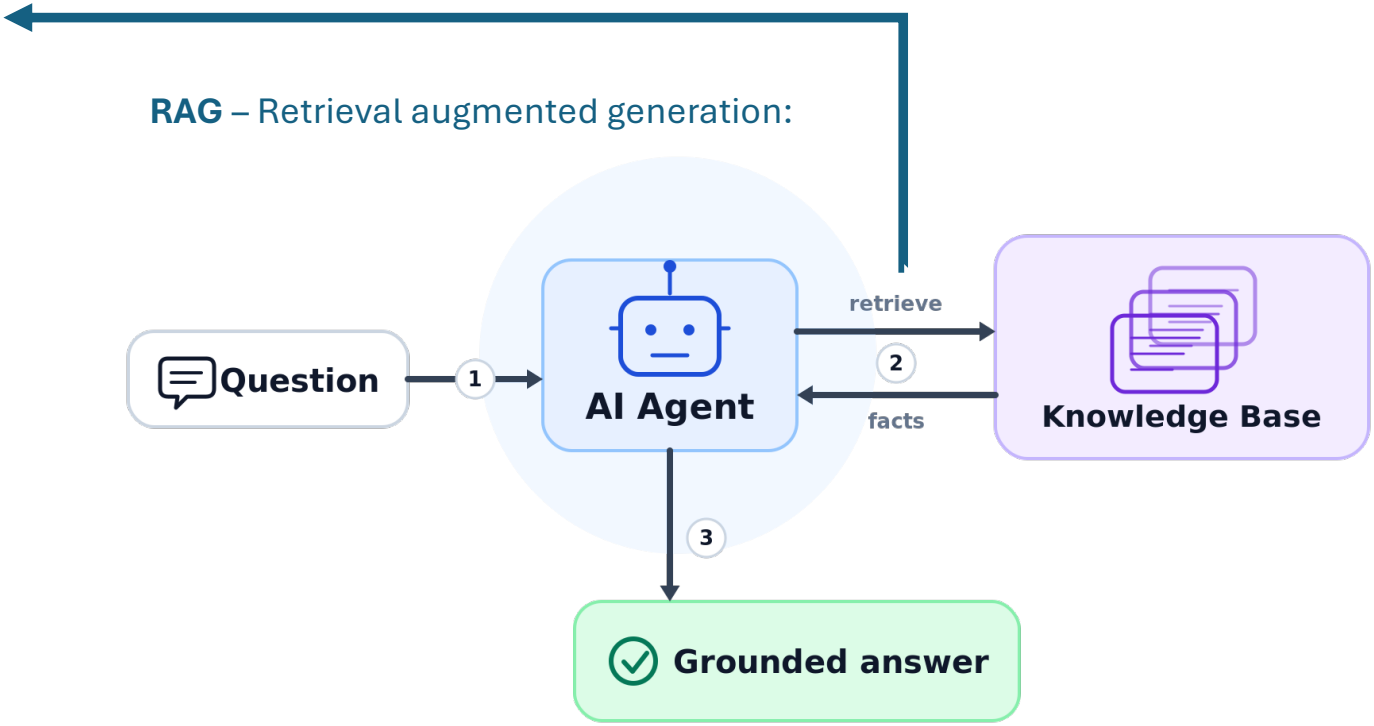
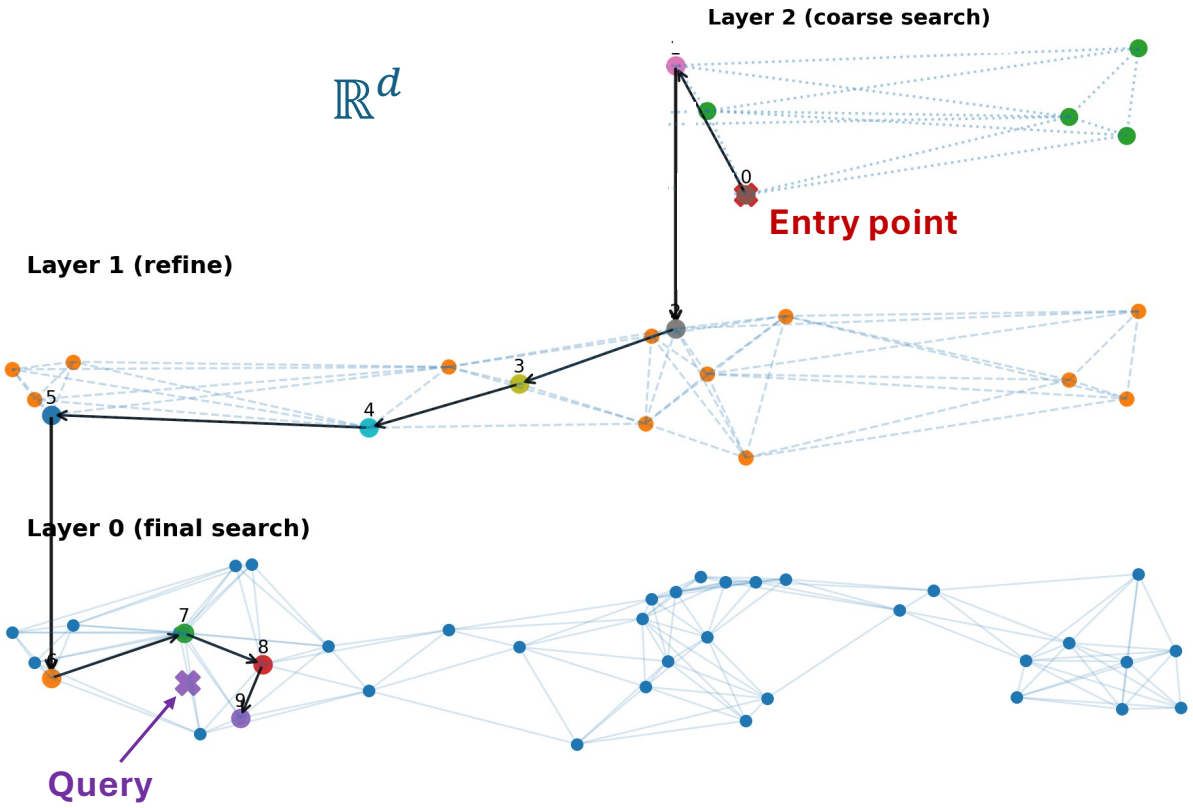




Graph Search Methods in ANN



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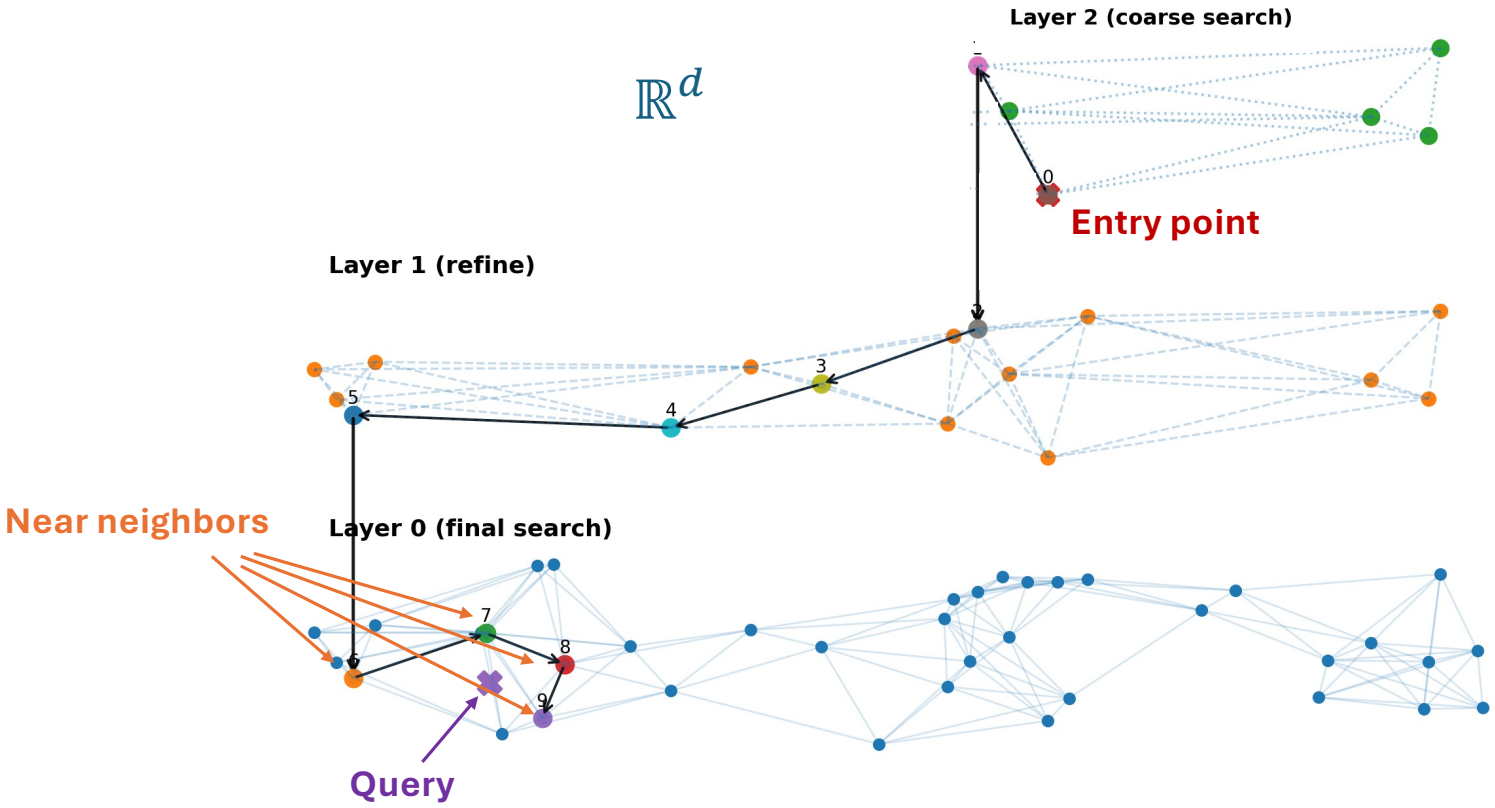

ANN with Dynamic Updates

- **Challenge:** Data is **dynamic**

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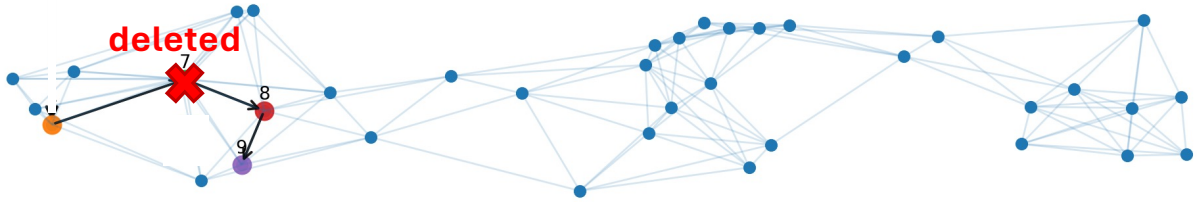
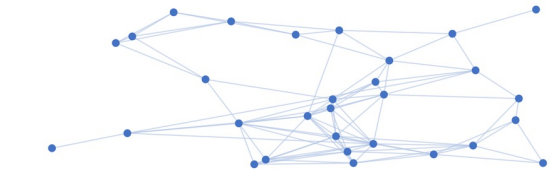
- **How to update the navigation graph?**
  - Insertions are easier
  - **Deletions are harder**



Vanilla Solution 1: Periodic Rebuilds



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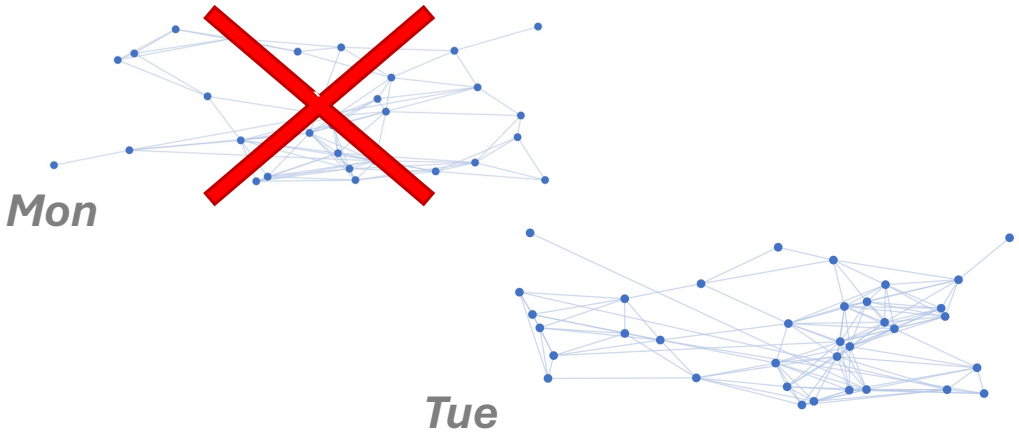

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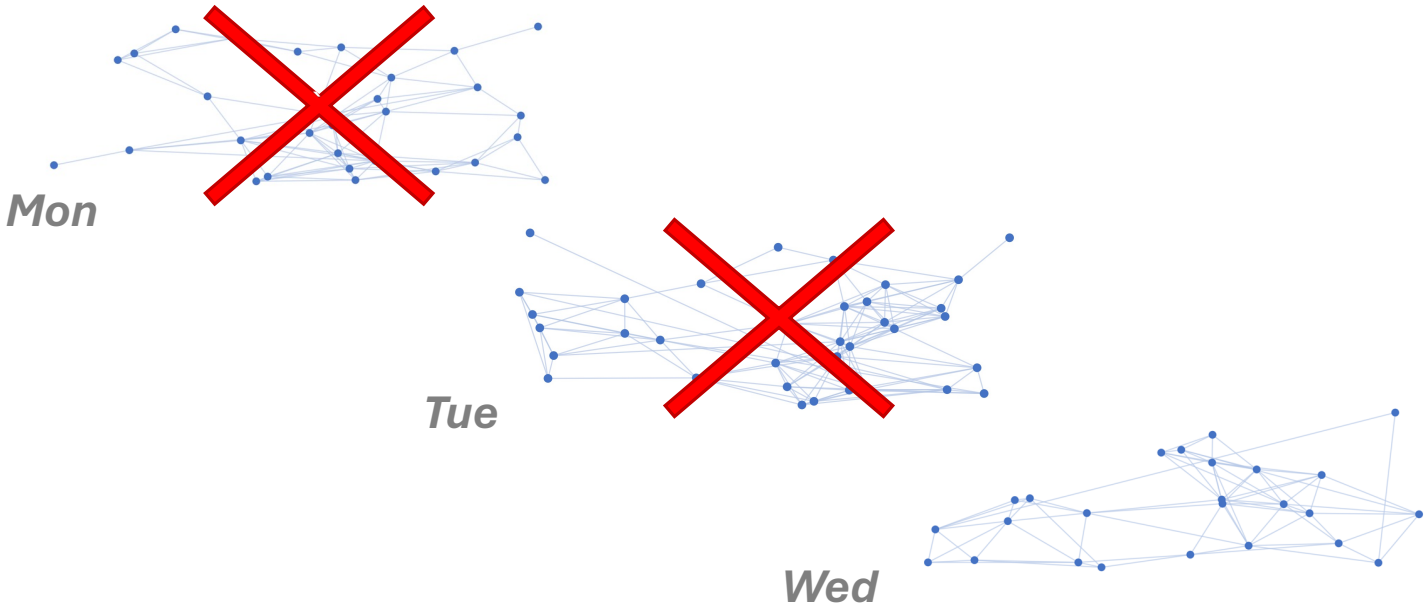

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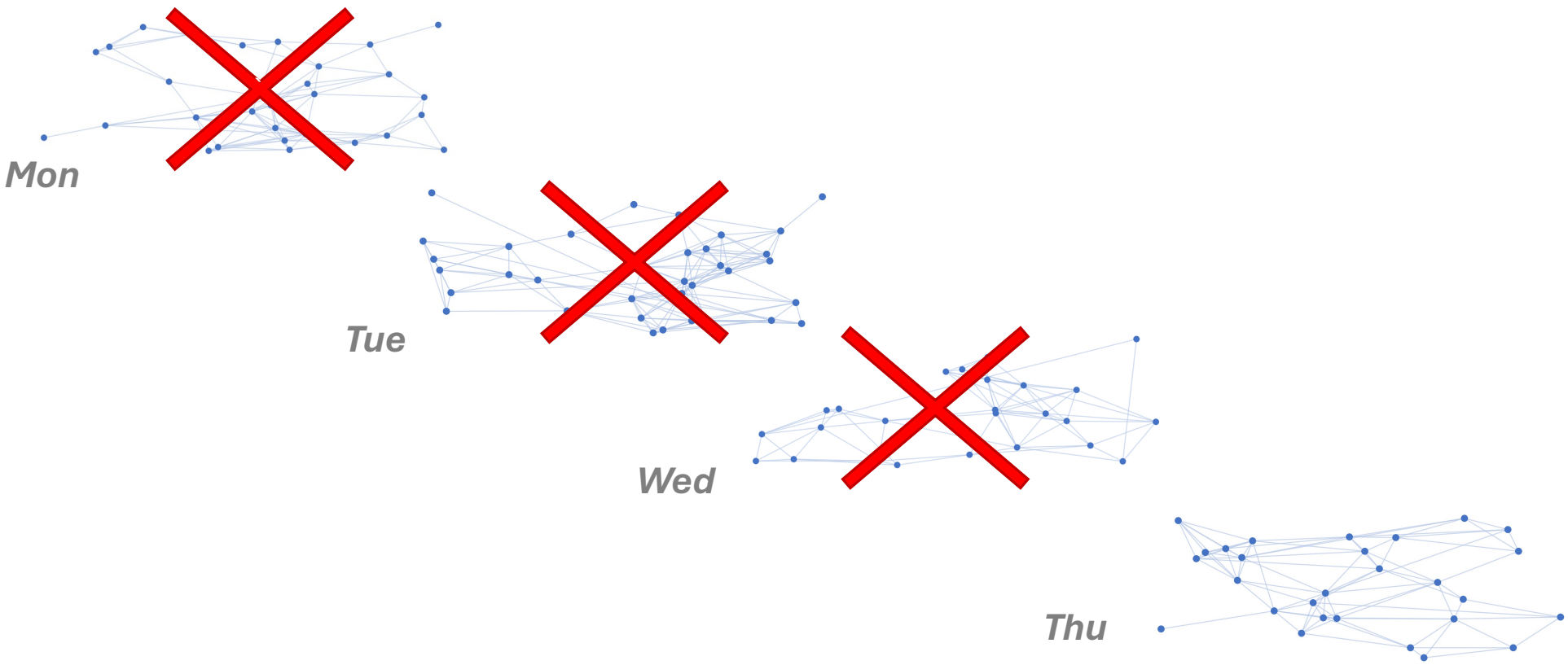
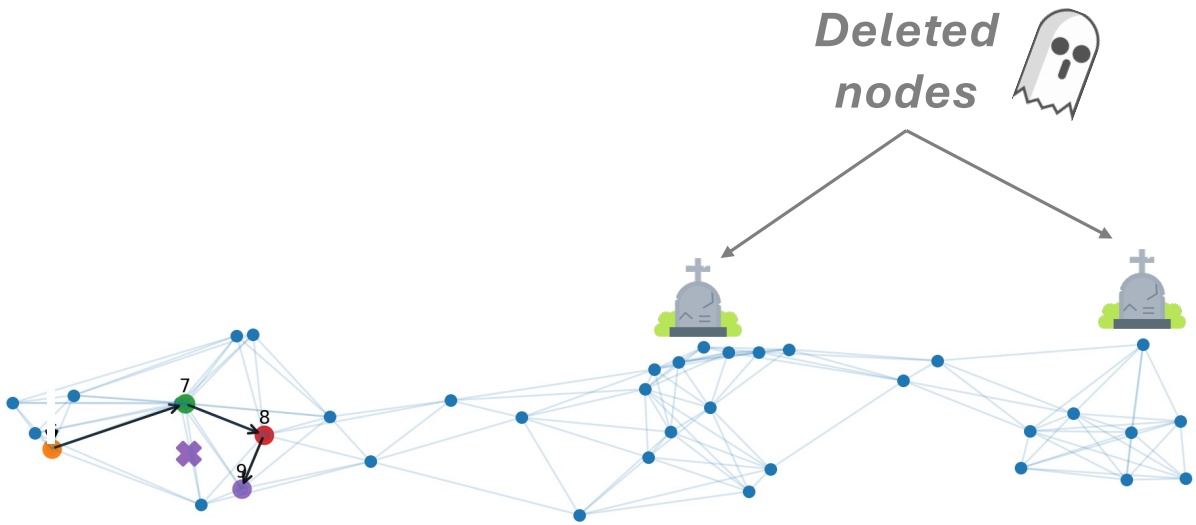


Vanilla Solution 2: Tombstones



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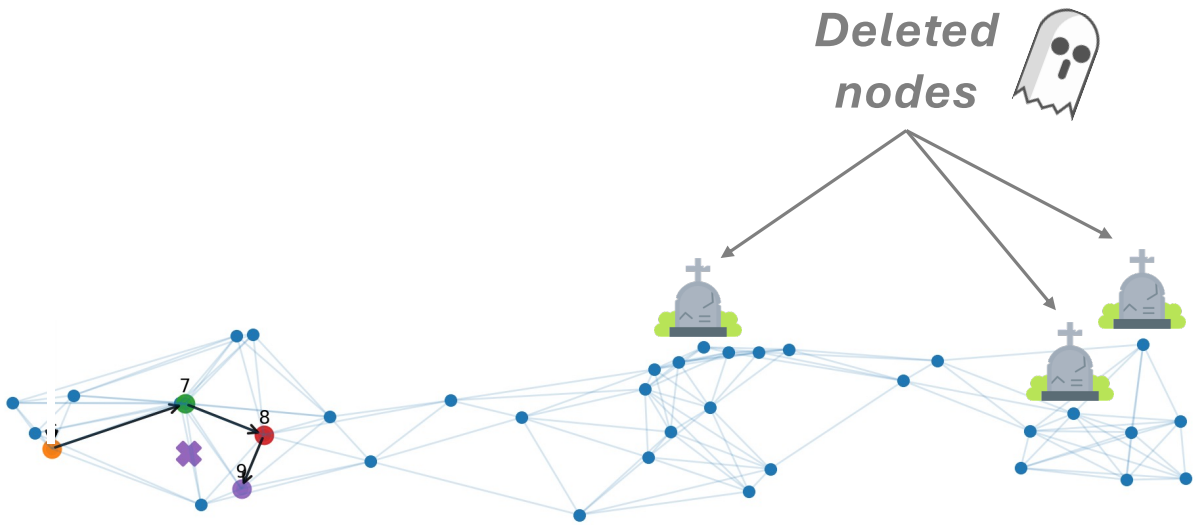

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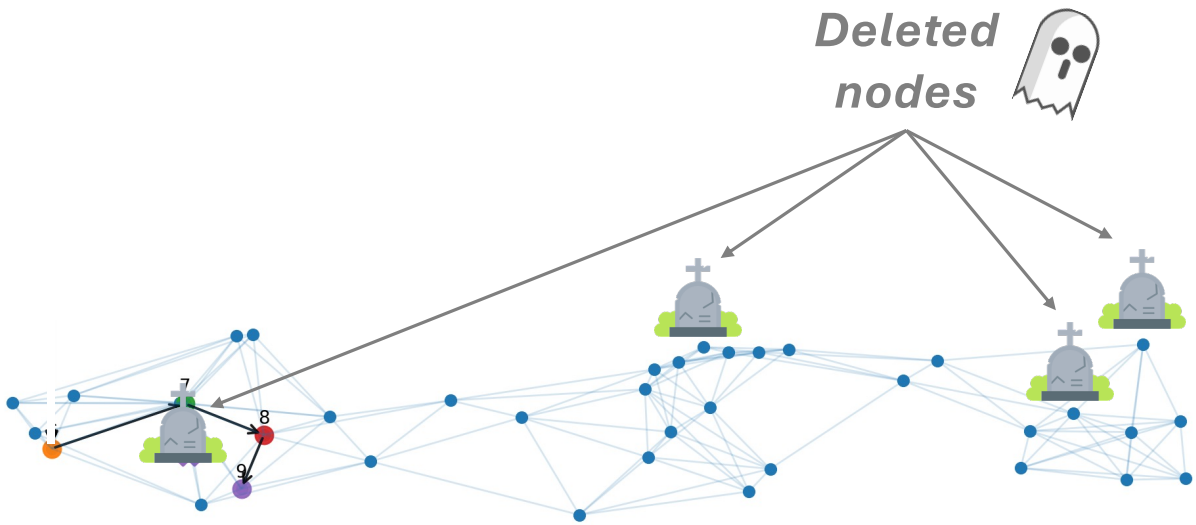

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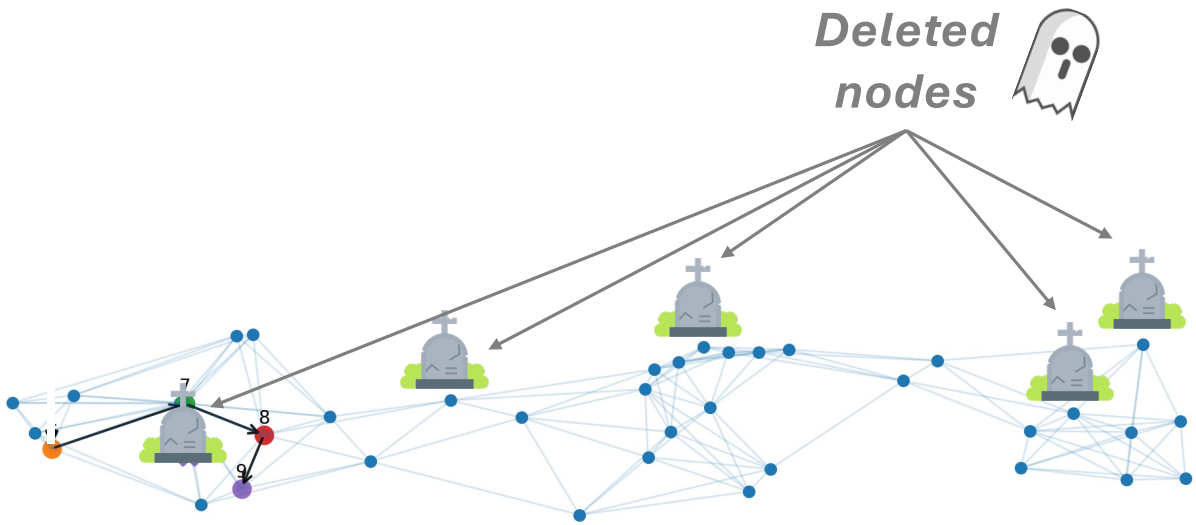

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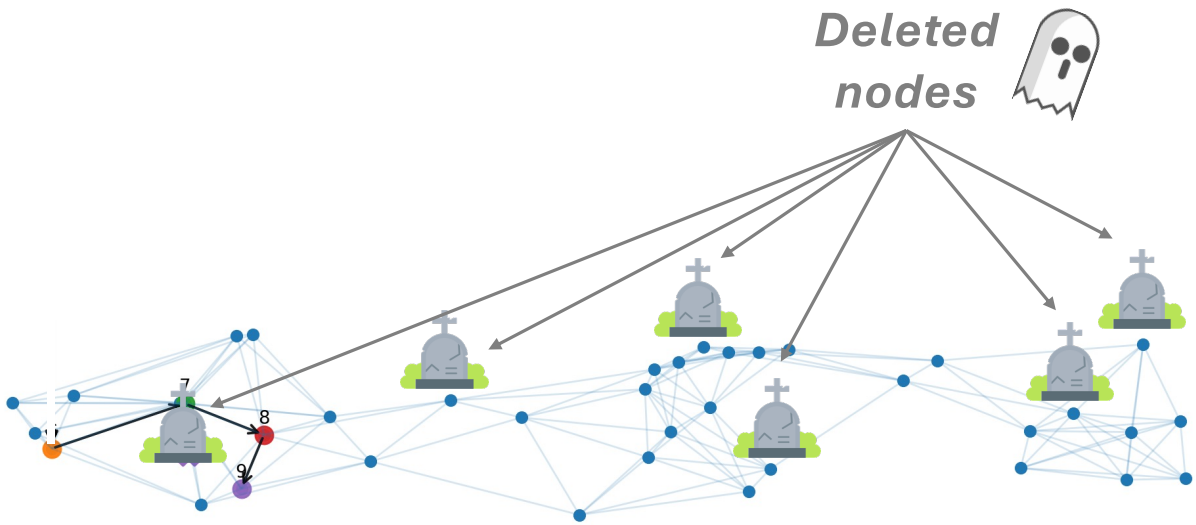

Our Approach: Local Patching

- **But how?**

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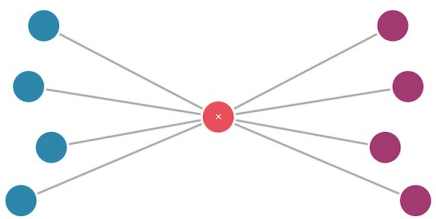

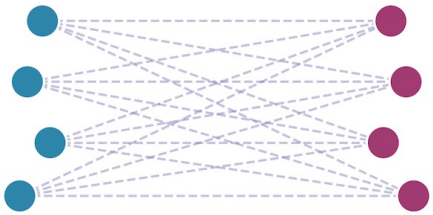

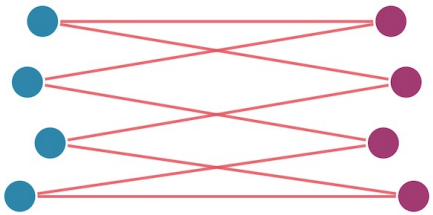

Our formulation: “Softmax Walk”

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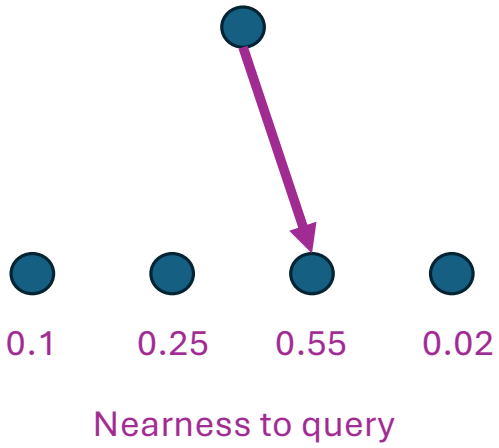
Greedy walk (vanilla graph-search):
Deterministic walk



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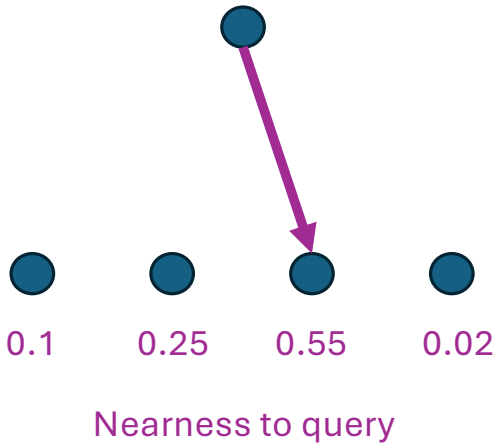
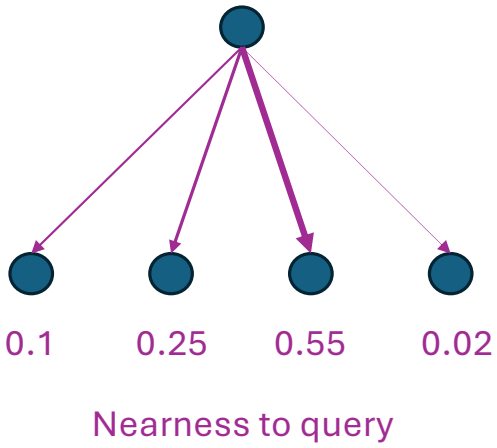


Softmax walk (our conceptual analog):
Random walk



Softmax Walk: Why?

- Access to random walk theory on graphs
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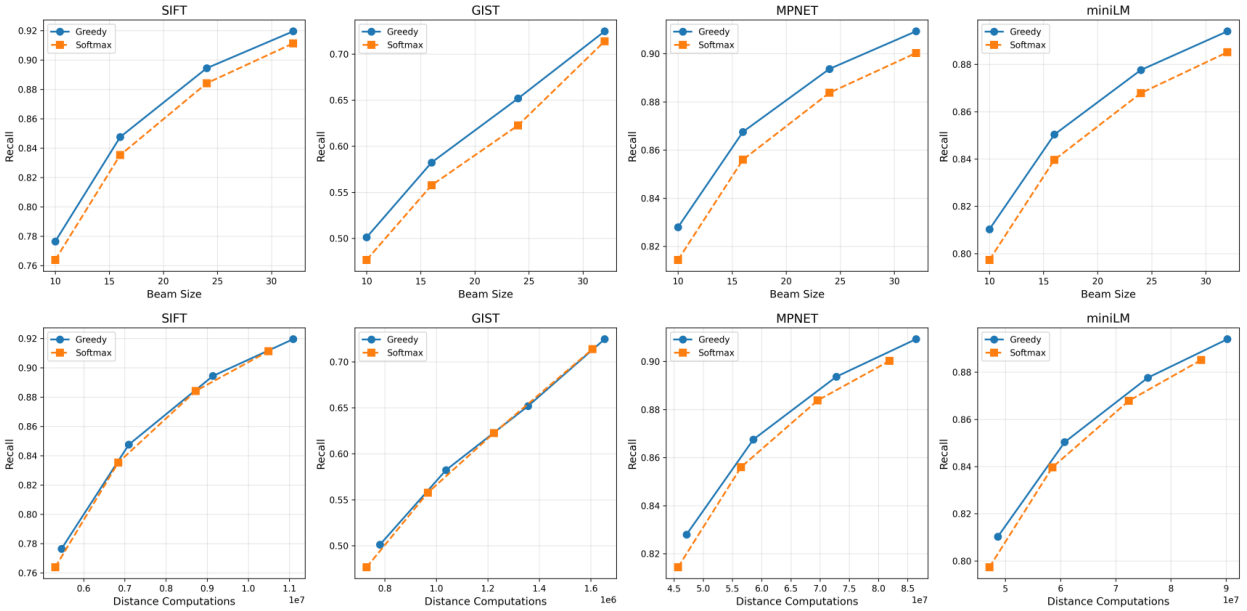
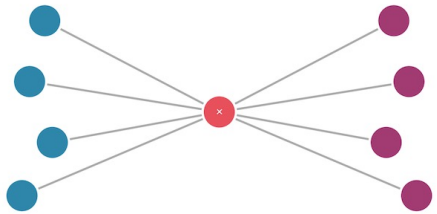


SPatch

SPatch

1. Delete node



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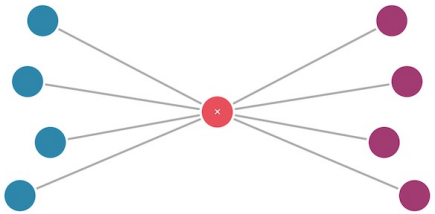
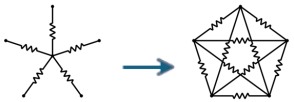
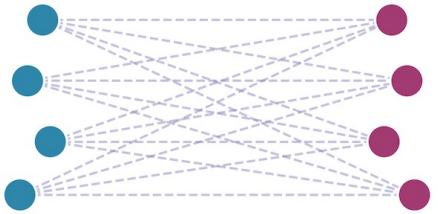


2. Star-mesh transform





- Restores random walk connectivity
- Makes neighborhood too dense

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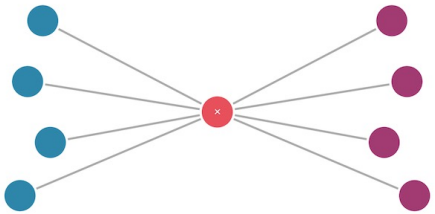
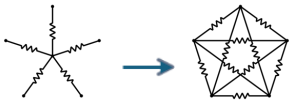


2. Star-mesh transform 
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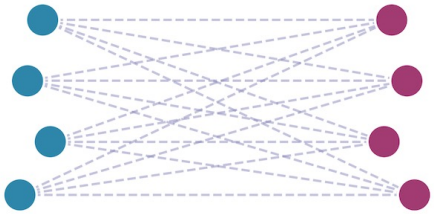
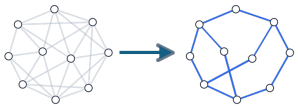


3. Spectral sparsification 
   - ➢ Approximately preserves restored connectivity
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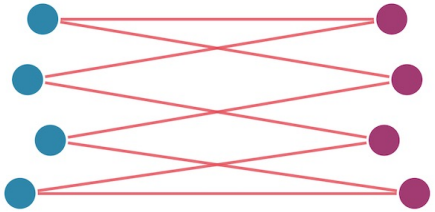

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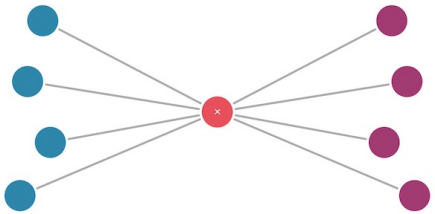
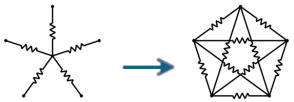


Spectral graph theory

2. Star-mesh transform 
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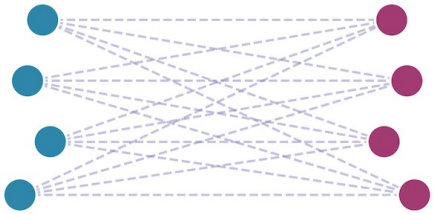
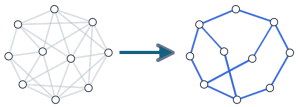


3. Spectral sparsification 
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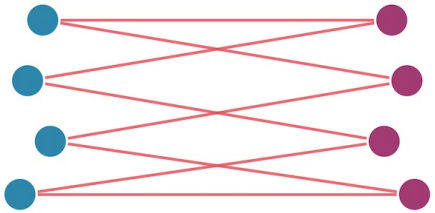
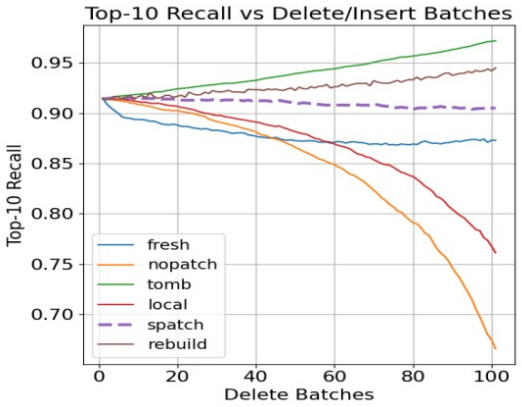
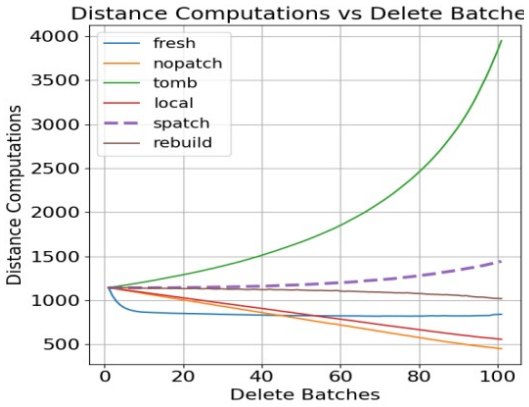


Experiments

Method	Recall	Speed	Del Time	Space
Tombstone	✓	×	✓	×
No patch	×	✓	✓	✓
Local	×	✓	✓	✓
FreshDiskANN	✓	✓	×	✓
Global	✓	✓	×	✓
`SPatch` (ours)	✓	✓	✓	✓

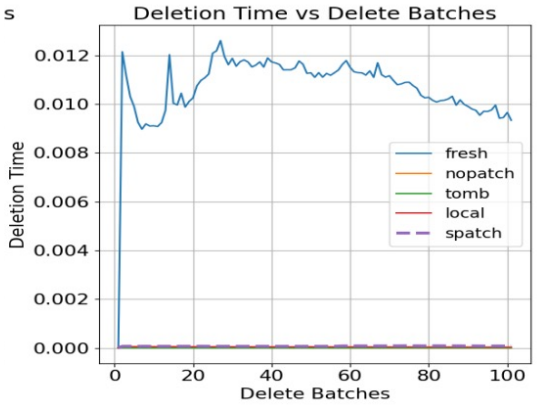
Accuracy

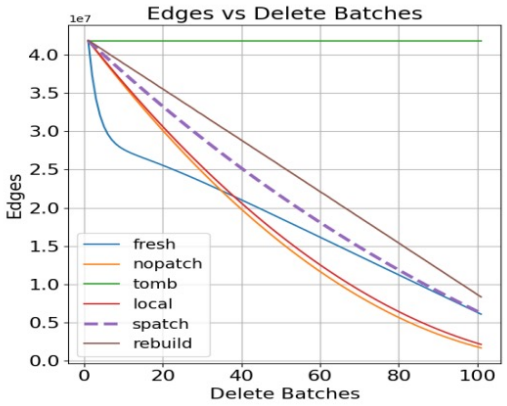




Query time

Deletion time





Memory usage

Conclusion

- **SPatch** is a new deletion algorithm for graph-based nearest neighbor search indices like HNSW
  - Mathematically grounded in spectral graph theory
  - Based on a “softmax walk” formulation
  - Efficient and accurate empirically

- **Thank you**