

# Measuring the Intrinsic Dimension of Earth Representations

Arjun Rao, Marc Rußwurm, Konstantin Klemmer, Esther Rolf



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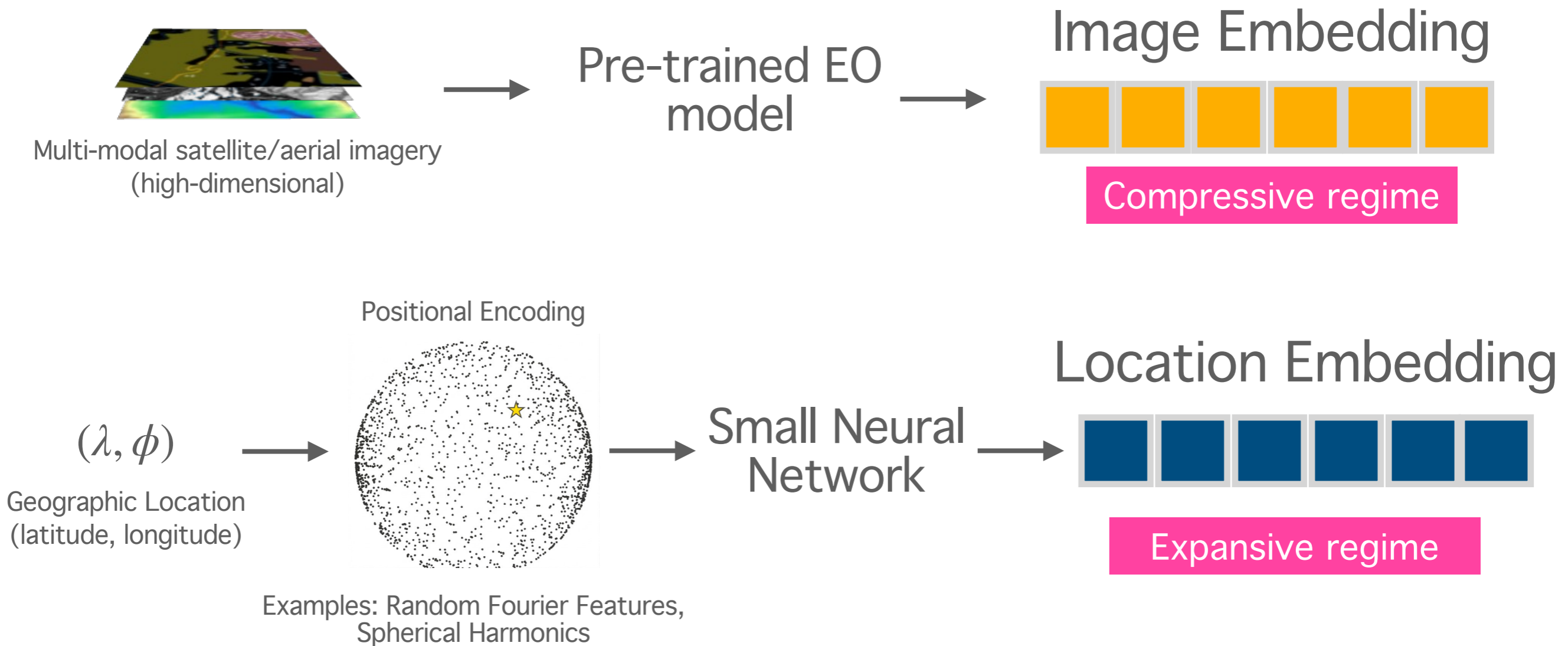


**ICLR**  
International Conference On  
Learning Representations

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## Image Embedding



Compressive regime

How many available dimensions of an Earth representation are actually used?

How does the information content encoded by Earth representations vary **spatially**?

## Location Embedding



Expansive regime

Can we design a **task-agnostic evaluation metric** to assess embedding quality?



The **Intrinsic Dimension (ID)** captures the minimum number of degrees of freedom required to describe data locally.

Earth models map  $x = (\lambda, \phi)$  into high-dimensional embeddings  $z = f(x) \in \mathbb{R}^D$ , where  $D$  is the fixed ambient dimension. Estimate the ID  $d(z) \leq D$



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## Distance-Based Estimators

Infer  $d(x)$  from how quickly the Euclidean distances  $R_1(z) \leq \dots \leq R_k(z)$  to the  $k$ -nearest neighbors of  $z$  grow.

The Maximum Likelihood Estimator (MLE) estimator:

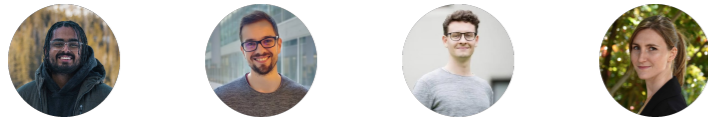
$$d(x) = \left[ \frac{1}{k-1} \sum_{j=1}^{k-1} \ln \frac{R_k(z)}{R_j(z)} \right]^{-1}$$

## Angle-Based Estimators

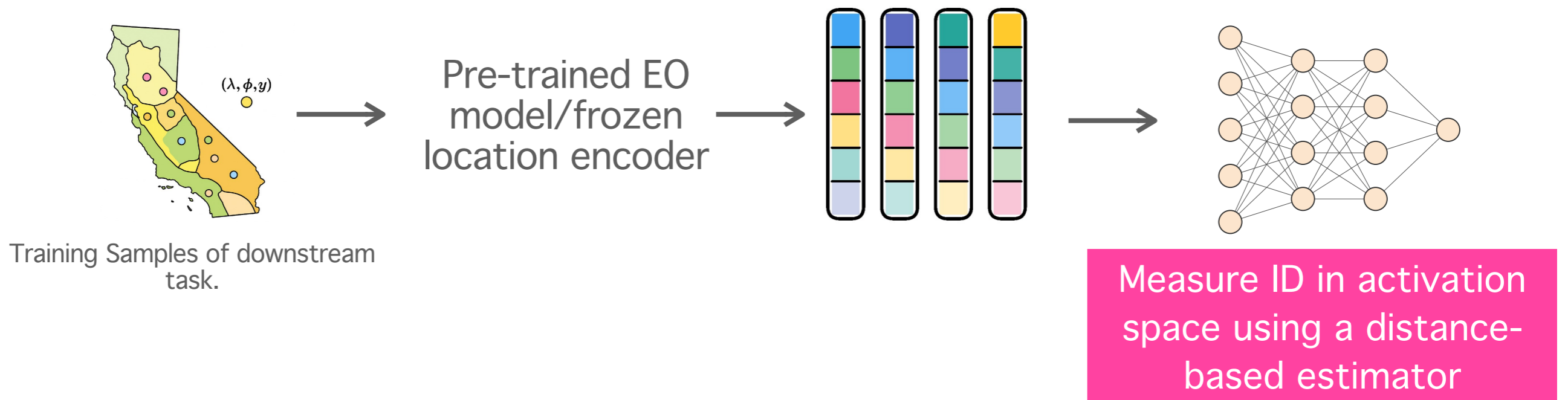
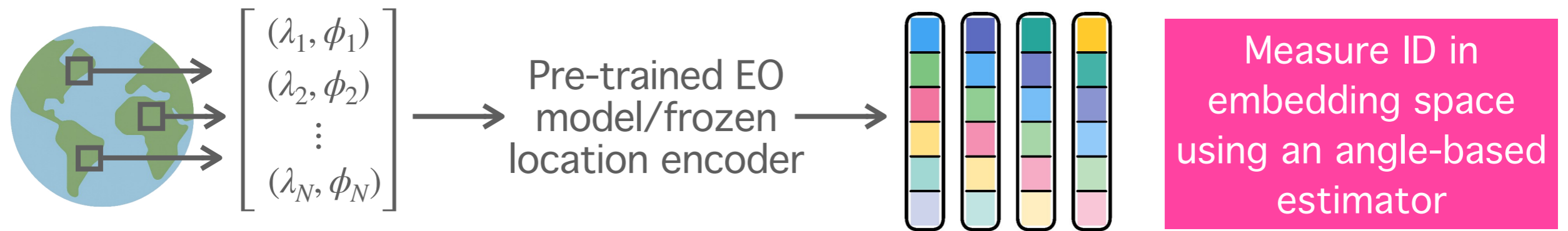
Infer dimensionality from the angular spread of the embeddings  $z = f(x)$  rather than their radial distances

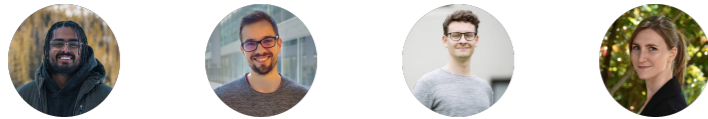
FisherS estimator calculates the empirical inseparability frequency  $\bar{p}(\alpha)$  at a given cosine similarity margin  $\alpha$

$$d(x) = \frac{W \left( -\frac{\ln(1-\alpha^2)}{2\pi\bar{p}(\alpha)^2\alpha^2(1-\alpha^2)} \right)}{-\ln(1-\alpha^2)}$$



**Key Idea:** Using a specific estimator-type on a specific stage of Earth representations can uncover previously unknown properties!



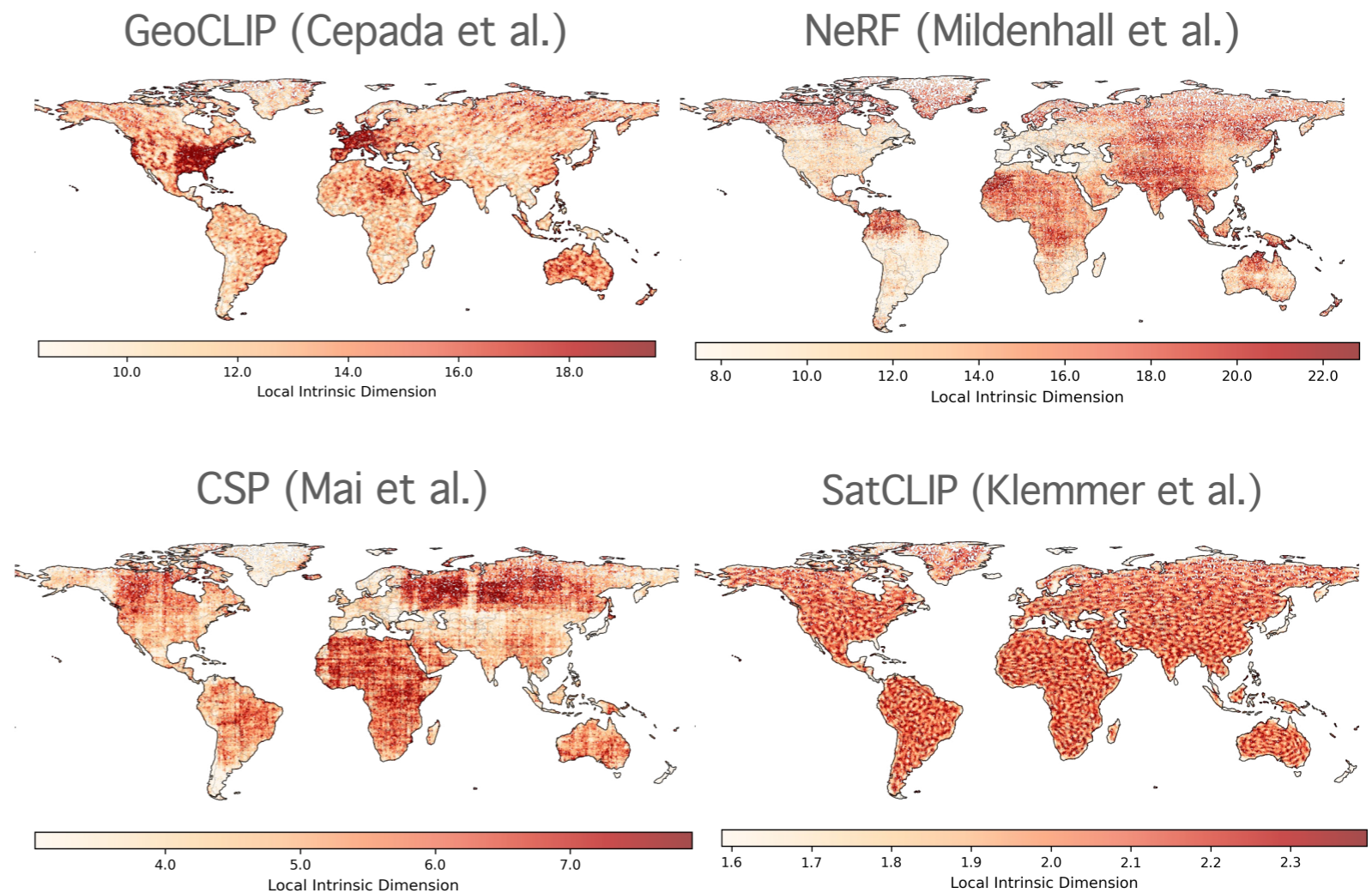


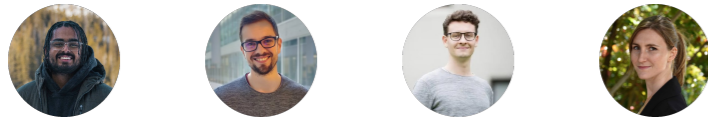
**Finding 1:** ID estimates of Earth representations are an order of magnitude lower than their ambient embedding size\*

**Finding 2:** When plotted spatially, ID reveals spatial artifacts.

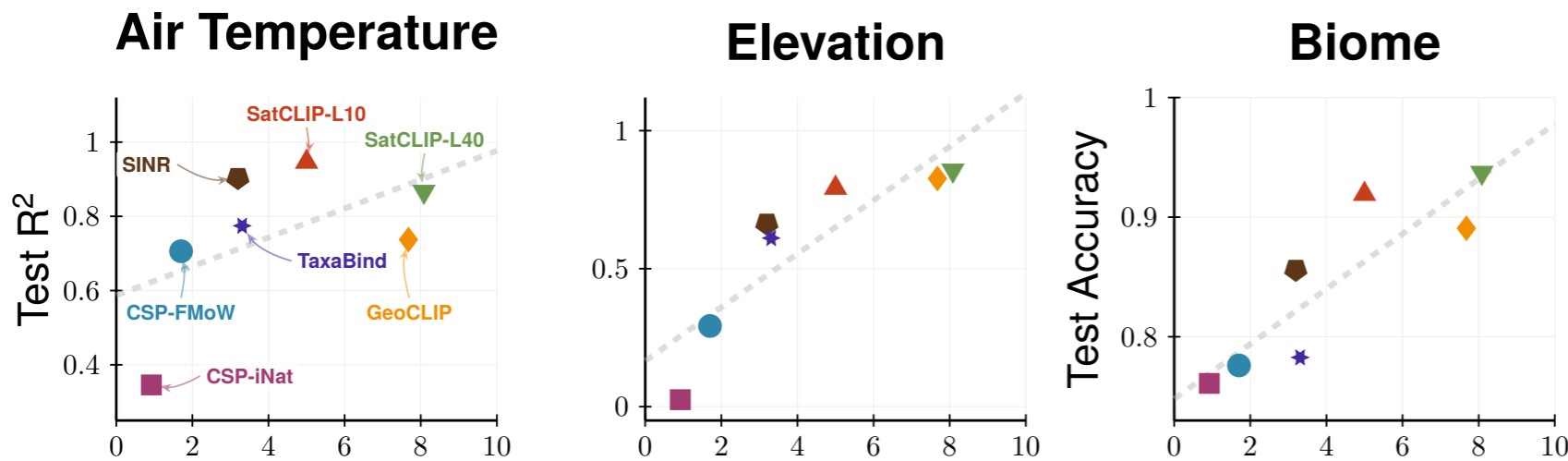
\* But still  $> 2!$  (ID of  $S^2$ )

Model	$D$	FisherS	MLE	MOM	TLE
<b>Location encoders (Land sampling, 100k points)</b>					
SatCLIP-L10	256	5.00	1.96	2.02	2.16
SatCLIP-L40	256	<b>8.08</b>	2.03	2.39	2.32
<b>GeoCLIP</b>	512	7.68	<b>11.21</b>	<b>13.02</b>	<b>11.53</b>
CSP-fMoW	256	1.70	5.18	5.23	6.25
CSP-iNat	256	0.92	3.37	4.64	4.14
SINR	256	3.19	2.19	3.36	2.74
TaxaBind-Loc	512	3.33	9.44	11.56	10.30
<b>Image encoders (S2-100K)</b>					
RCF	512	1.64	6.32	5.23	7.10
<b>CROMA</b>	768	<b>9.79</b>	19.57	17.00	20.30
DOFA	768	3.32	15.58	13.78	16.20
ResNet18	512	6.32	16.14	12.27	16.80
ResNet50	2048	6.42	16.27	13.18	17.00
ResNet152	2048	7.60	<b>20.72</b>	17.50	<b>21.50</b>
ViT-Small	384	3.33	18.53	15.80	19.20
AlphaEarth	64	4.67	7.80	9.48	8.61
TaxaBind-Sat (RGB)	512	3.39	10.04	14.98	12.80
ScaleMAE (RGB)	1024	2.96	10.16	8.90	11.00
ResNet18 (RGB)	512	0.92	10.85	8.70	11.70
ResNet50 (RGB)	2048	0.92	9.92	8.10	10.80
DINOv3-Sat (RGB)	4096	3.87	13.03	<b>19.52</b>	15.95



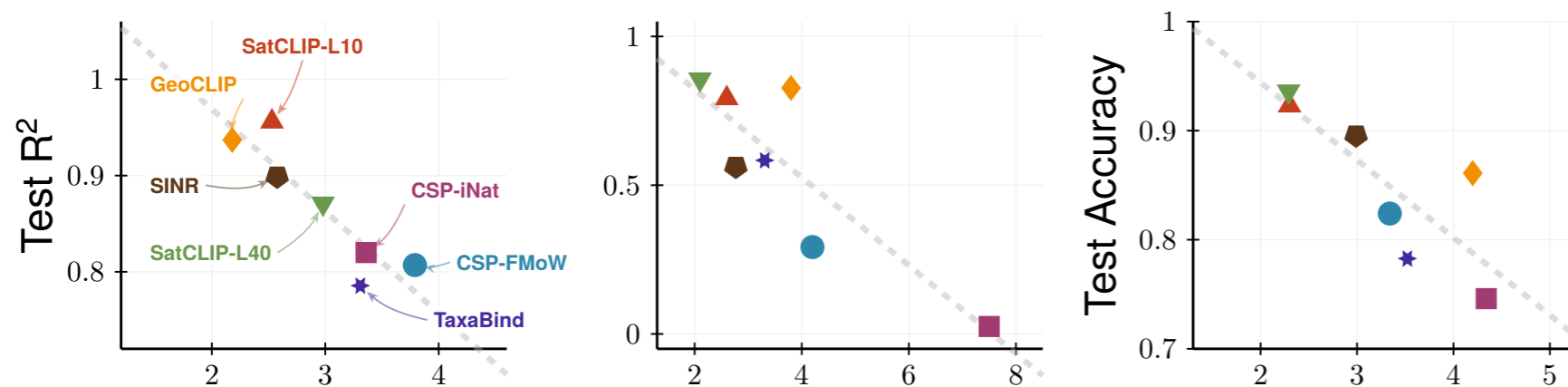


# Finding 3: ID estimates of Earth representations informs how representative and task-aligned the embedding is.



Earth embeddings with a **higher** global FisherS ID in embedding space record higher task performance!

FisherS Global ID in **embedding space** measures **representativeness**



Earth embeddings with a **lower** global TwoNN ID in activation space record higher task performance!

TwoNN ID in **activation space** measures **task-alignment**



# Implications

- 1 Intrinsic dimension (ID) serves as an architecture- and task-agnostic metric to quantify the information content encoded in geographic INRs.
- 2 Global ID enables label-free model selection at the pre-training stage.
- 3 Local ID maps expose underlying spatial artifacts and geographic biases, which can be used to guide targeted data collection and region-aware fine-tuning.



Paper



Code

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