

Learning Exposure Mapping Function for Inferring Heterogeneous Peer Effects

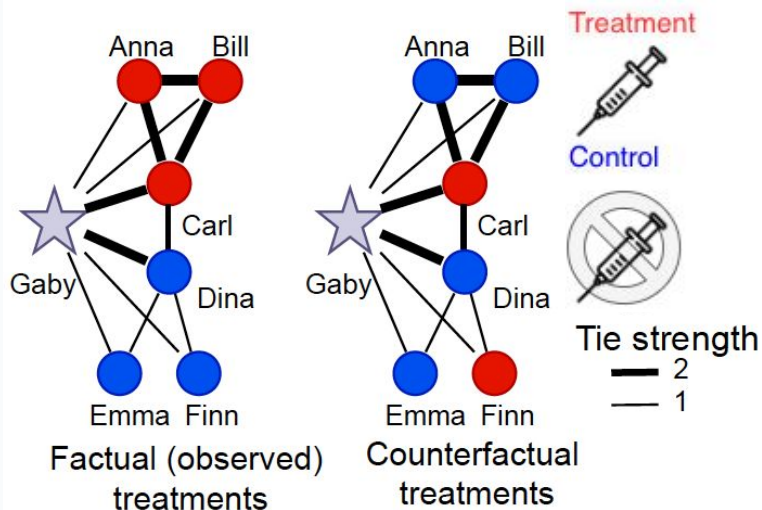
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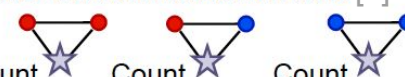
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Peer Effects and Exposure Mapping Function

- Peer effect is the difference in counterfactual outcomes for two peer exposure values (composite peer treatments)
- Exposure mapping function maps peer treatments and relevant contexts to peer exposure representation



| Example exposure mapping functions | Peer exposure value | |
|--|---------------------|----------------|
| | Factual | Counterfactual |
| Binary (at least a peer treated) [1] | 1 | 1 |
| Fraction of treated peers [2,3] | 3/6 | 2/6 |
| Linear threshold (40%) [4] | 1 | 0 |
| Weighted fraction (tie strengths) [5] | 4/8 | 3/8 |
| Weighted fraction (attribute similarity: female) [6] | 1/3 | 0/3 |
| Local structure: Causal network motifs [7] E.g., Closed triads:  Count Count Count | < 3, 1, 2 > | < 0, 4, 2 > |

[1] Bargagli-Stoffi et al. Heterogeneous Treatment and Spillover Effects under Clustered Network Interference. Arxiv 2020.

[2] Jiang and Sun. Estimating Causal Effects on Networked Observational Data via Representation Learning. CIKM 2022.

[3] Chen et al. Doubly Robust Causal Effect Estimation under Networked Interference via Targeted Learning. ICML 2024.

[4] Tran and Zheleva. Heterogeneous Peer Effects in the Linear Threshold Model. AAAI 2022.

[5] Forastiere et al. Identification and estimation of treatment and interference effects in observational studies on networks. JASA 2021.

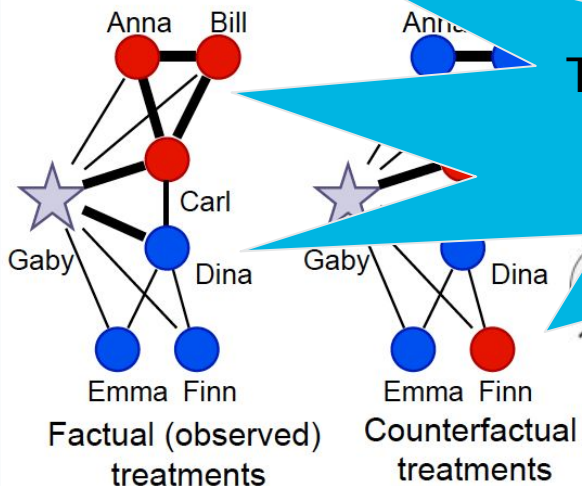
[6] Zhao et al. Learning Individual Treatment Effects under Heterogeneous Interference in Networks. TKDD 2024.

[7] Yuan et al. Causal Network Motifs: Identifying Heterogeneous Spillover Effects in A/B Tests. WWW 2021.

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- Exposure mapping function maps peer treatments and relevant network contexts to peer exposure representation

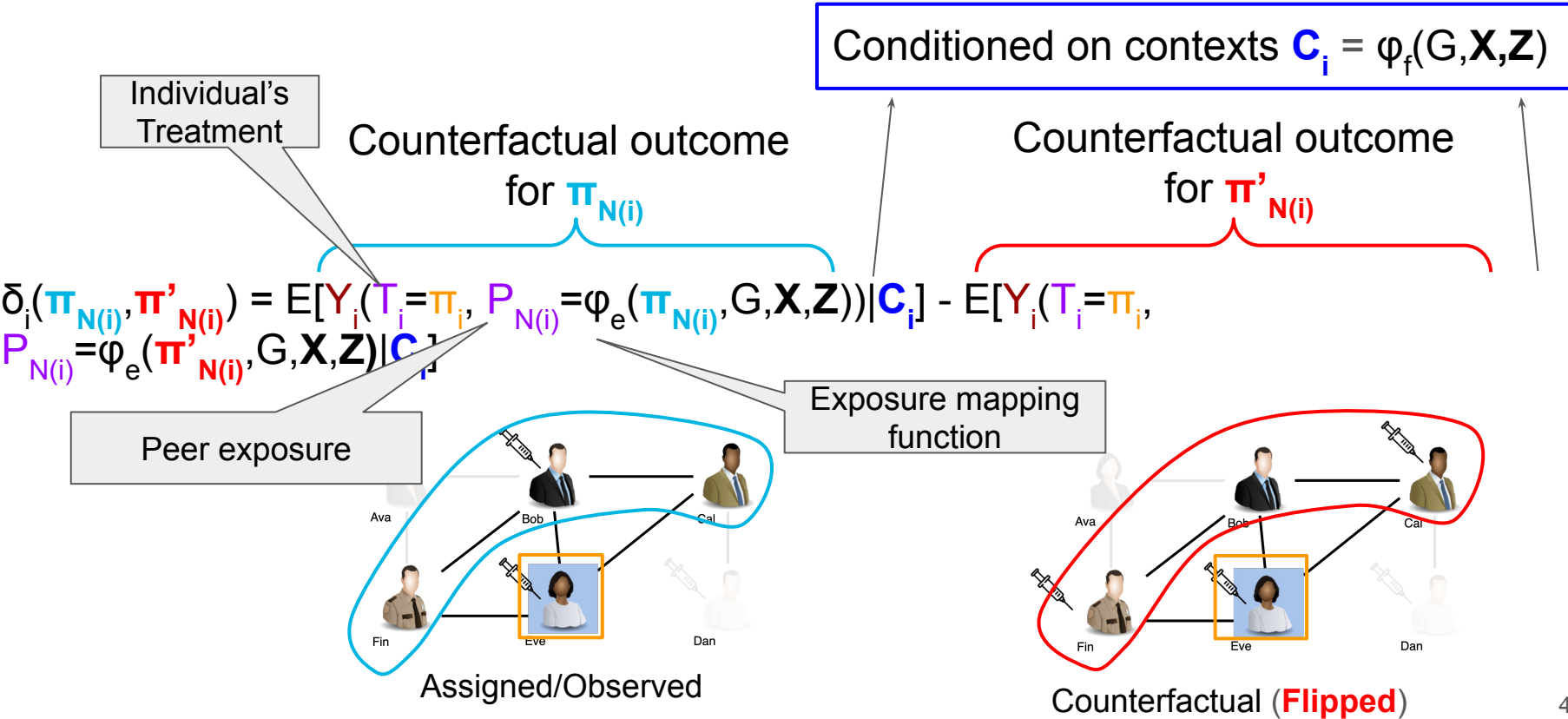
True exposure mapping function is typically unknown and can be misspecified



| Contexts | Peer exposure value | |
|--|---------------------|----------------|
| | Factual | Counterfactual |
| | 1 | 1 |
| | 3/6 | 2/6 |
| | . | 0 |
| | 4/8 | 3/8 |
| | 1/3 | 0/3 |
| Local structural causal network motifs [7] | | |
| g., Closed triads: | Count | Count |
| | < 3, 1, 2 > | < 0, 4, 2 > |

Our goal: Learn the exposure mapping function to capture underlying peer influence mechanisms for heterogeneous peer effect estimation.

Heterogeneous Peer Effect (HPE) Estimation

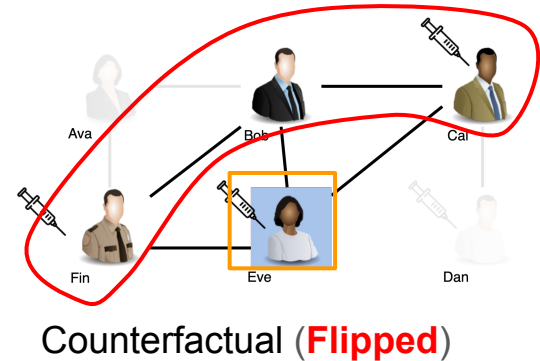
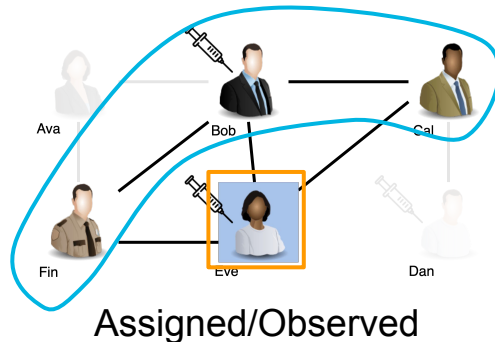


Heterogeneous Peer Effect (HPE) Estimation

- Assuming standard causal inference assumptions (*unconfoundedness, consistency, and positivity*), HPE can be estimated

$$C_i = \varphi_f(G, \mathbf{X}, \mathbf{Z})$$

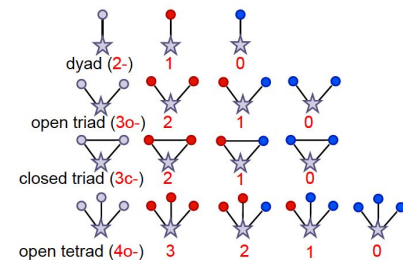
$$\delta_i(\boldsymbol{\pi}_{N(i)}, \boldsymbol{\pi}'_{N(i)}) = E[Y_i | T_i = \boldsymbol{\pi}_i, P_{N(i)} = \varphi_e(\boldsymbol{\pi}_{N(i)}, G, \mathbf{X}, \mathbf{Z}), C_i] - E[Y_i | T_i = \boldsymbol{\pi}_i, P_{N(i)} = \varphi_e(\boldsymbol{\pi}'_{N(i)}, G, \mathbf{X}, \mathbf{Z}), C_i]$$



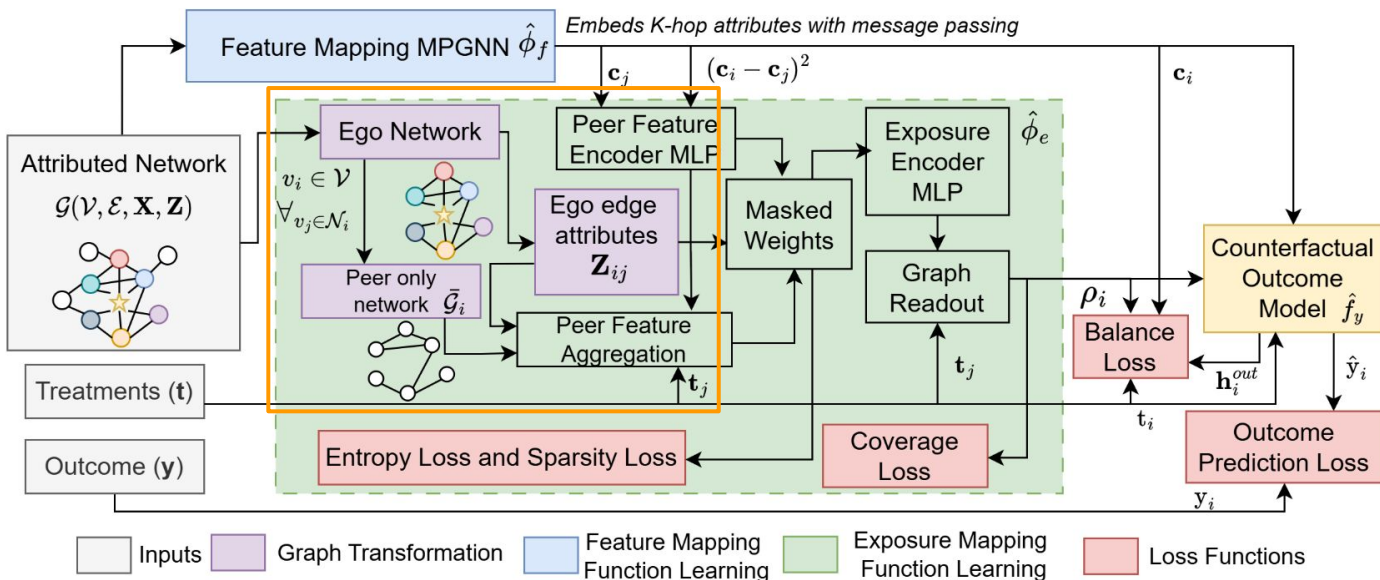
EgoNetGNN Framework

Expressiveness

- Designed to capture attributed substructures



Causal network motifs
(Yuan et al., 2021)

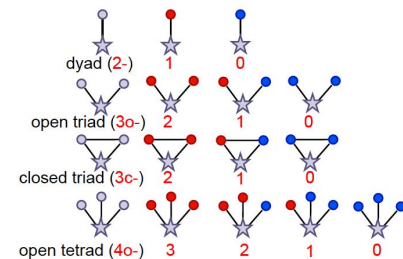


An overview of the proposed EgoNetGNN model to learn exposure mapping function for causal effect estimation in networks under heterogeneous peer influence

EgoNetGNN Framework

Expressiveness

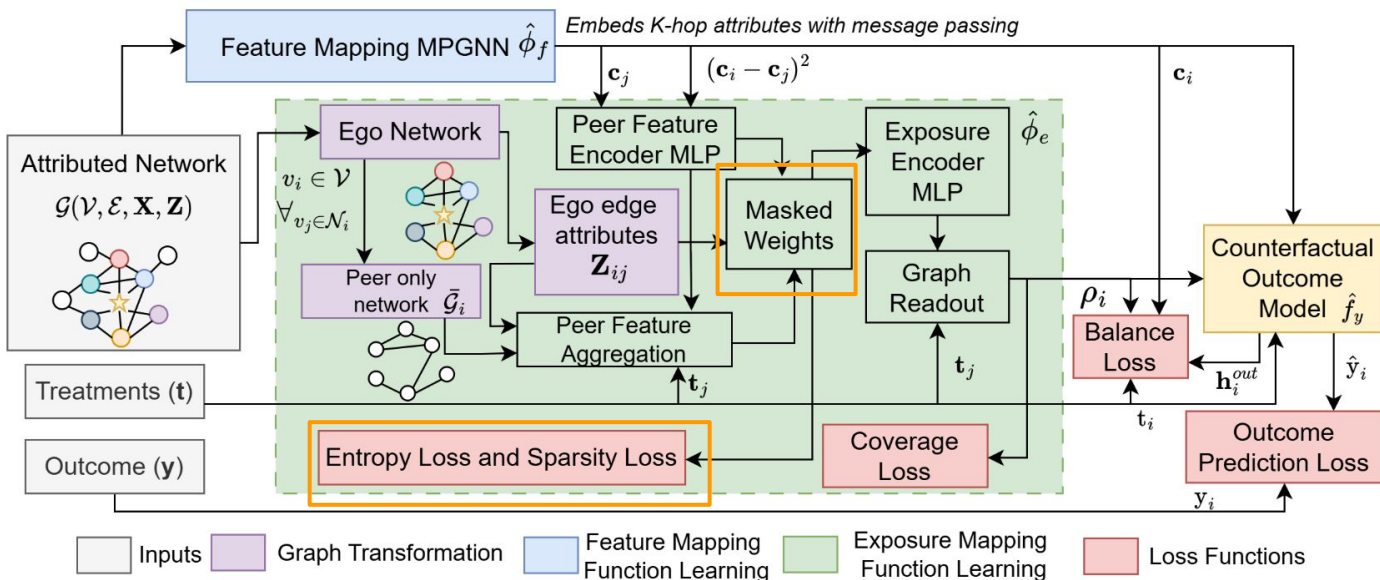
- Designed to capture attributed substructures



Causal network motifs

Invariance to irrelevant contexts

- Masked weights and related losses

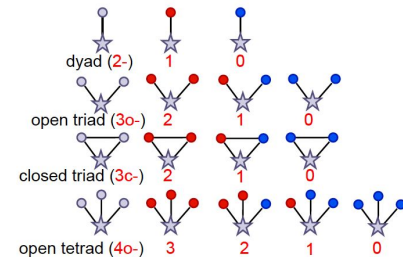


An overview of the proposed EgoNetGNN model to learn exposure mapping function for causal effect estimation in networks under heterogeneous peer influence

EgoNetGNN Framework

Expressiveness

- Designed to capture attributed substructures



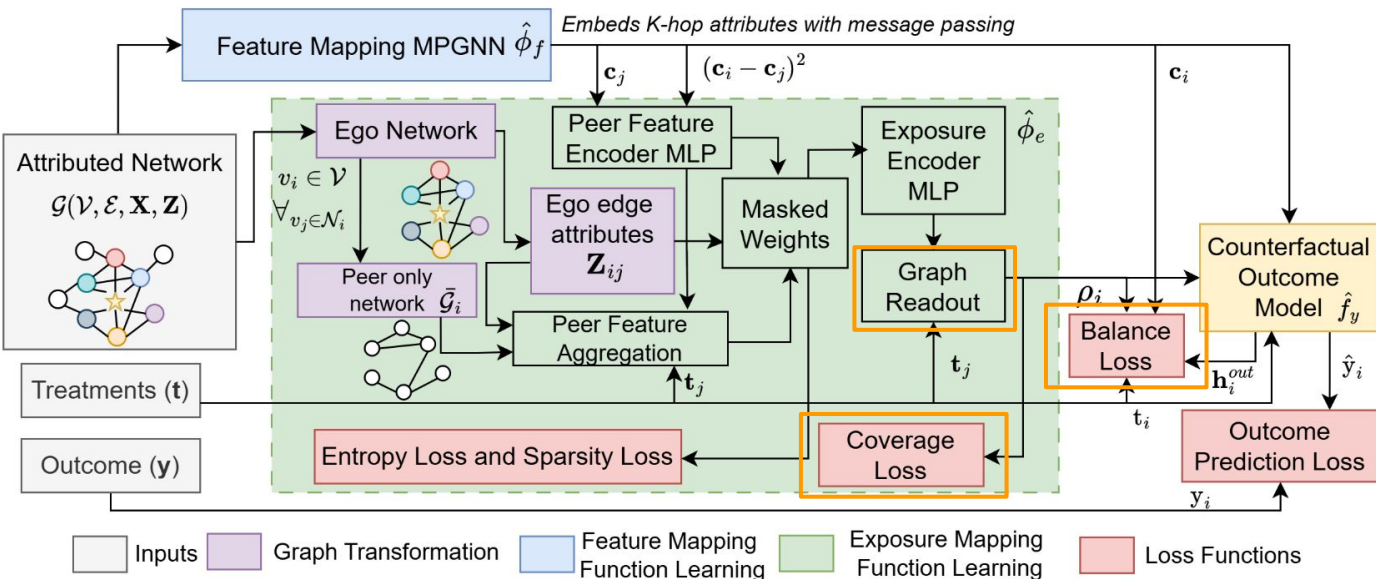
Causal network motifs

Invariance to irrelevant contexts

- Masked weights and related losses

Robustness in causal effect estimation

- Exposure mapping function learning
- Coverage loss
- Balance (and reconstruction) loss



An overview of the proposed EgoNetGNN model to learn exposure mapping function for causal effect estimation in networks under heterogeneous peer influence

Mechanisms:

- Clustering coefficients and Connected components among treated peers
- Weighted fraction of treated peers based on mutual connections and attribute similarity

$$\epsilon_{PEHE} = \sqrt{\frac{1}{n} \sum_i (\delta_i(\boldsymbol{\pi}_{-i}, \boldsymbol{\pi}'_{-i}) - \hat{\delta}_i(\boldsymbol{\pi}_{-i}, \boldsymbol{\pi}'_{-i}))^2}$$

Results: Robustness of EgoNetGNN

| Mechanisms | Ours-TARNet | Ours-CFR ⁺ | GNN-Motifs | INE-TARNet | 1GNN-HSIC | DWR | AEMNet | TNet | NetEst | CauGramer |
|-------------|-----------------------------|-----------------------------|----------------------|-----------------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| Clus. Coef. | <u>2.13</u> _{±1.9} | 0.95 _{±0.5} | 2.39 _{±1.2} | 2.35 _{±0.7} | 6.21 _{±3.7} | 7.49 _{±4.6} | 7.53 _{±6.0} | 9.52 _{±10.5} | 4.53 _{±1.5} | 6.16 _{±2.1} |
| Con. Comp. | 1.47 _{±0.9} | <u>1.50</u> _{±0.7} | 4.98 _{±1.6} | 4.78 _{±1.1} | 6.78 _{±1.9} | 7.68 _{±1.6} | 11.27 _{±9.0} | 9.98 _{±8.3} | 8.56 _{±0.7} | 7.07 _{±1.2} |
| Mut. Con. | 2.86 _{±1.3} | 2.24 _{±1.6} | 2.81 _{±1.3} | <u>2.50</u> _{±0.9} | 10.30 _{±6.0} | 8.72 _{±2.8} | 13.33 _{±9.0} | 11.17 _{±8.5} | 5.34 _{±1.3} | 5.18 _{±2.0} |
| Attr. Sim. | 3.95 _{±2.7} | <u>3.65</u> _{±2.4} | 4.64 _{±2.1} | 3.59 _{±1.8} | 15.25 _{±4.7} | 17.96 _{±3.7} | 14.10 _{±5.0} | 14.60 _{±5.0} | 11.71 _{±2.2} | 14.45 _{±5.7} |

| Mechanisms | Ours-TARNet | Ours-CFR ⁺ | GNN-Motifs | INE-TARNet | 1GNN-HSIC | DWR | AEMNet | TNet | NetEst | CauGramer |
|-------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|-------------------------------|
| Clus. Coef. | 7.38 _{±3.0} | 3.71 _{±4.2} | <u>5.05</u> _{±1.9} | 5.64 _{±0.8} | 13.36 _{±8.9} | 9.95 _{±2.8} | 7.45 _{±1.9} | 19.12 _{±19.3} | 10.57 _{±3.4} | 22.50 _{±35.0} |
| Con. Comp. | <u>1.64</u> _{±1.1} | 1.19 _{±1.3} | 2.34 _{±0.7} | 2.38 _{±0.9} | 19.16 _{±28.3} | 3.32 _{±1.0} | 11.59 _{±10.5} | 7.86 _{±7.6} | 3.77 _{±1.3} | 22.57 _{±20.4} |
| Mut. Con. | 2.72 _{±1.7} | 2.20 _{±1.8} | 2.64 _{±1.1} | <u>2.41</u> _{±1.1} | 3.22 _{±1.3} | 3.67 _{±1.6} | 5.72 _{±4.1} | 14.63 _{±24.3} | 3.19 _{±1.4} | 31.27 _{±36.5} |
| Attr. Sim. | <u>8.27</u> _{±7.8} | 6.97 _{±5.8} | 11.57 _{±12.3} | 10.66 _{±11.3} | 21.12 _{±15.4} | 25.29 _{±14.8} | 13.56 _{±12.1} | 40.51 _{±55.6} | 18.58 _{±11.7} | 13.70 _{±11.9} |

Summary

EgoNetGNN improves the estimation of heterogeneous peer effects compared to state-of-the-art baselines by learning an exposure mapping function that is

- Expressive
- Invariant to irrelevant features
- Bounded and balanced representation with substantial coverage

Future Directions

Representation Learning: Expressive GNNs and Causal Inference

Network Science: Explainability to understand underlying influence mechanisms