

Domain Expansion: A Latent Space Construction Framework for Multi-Task Learning



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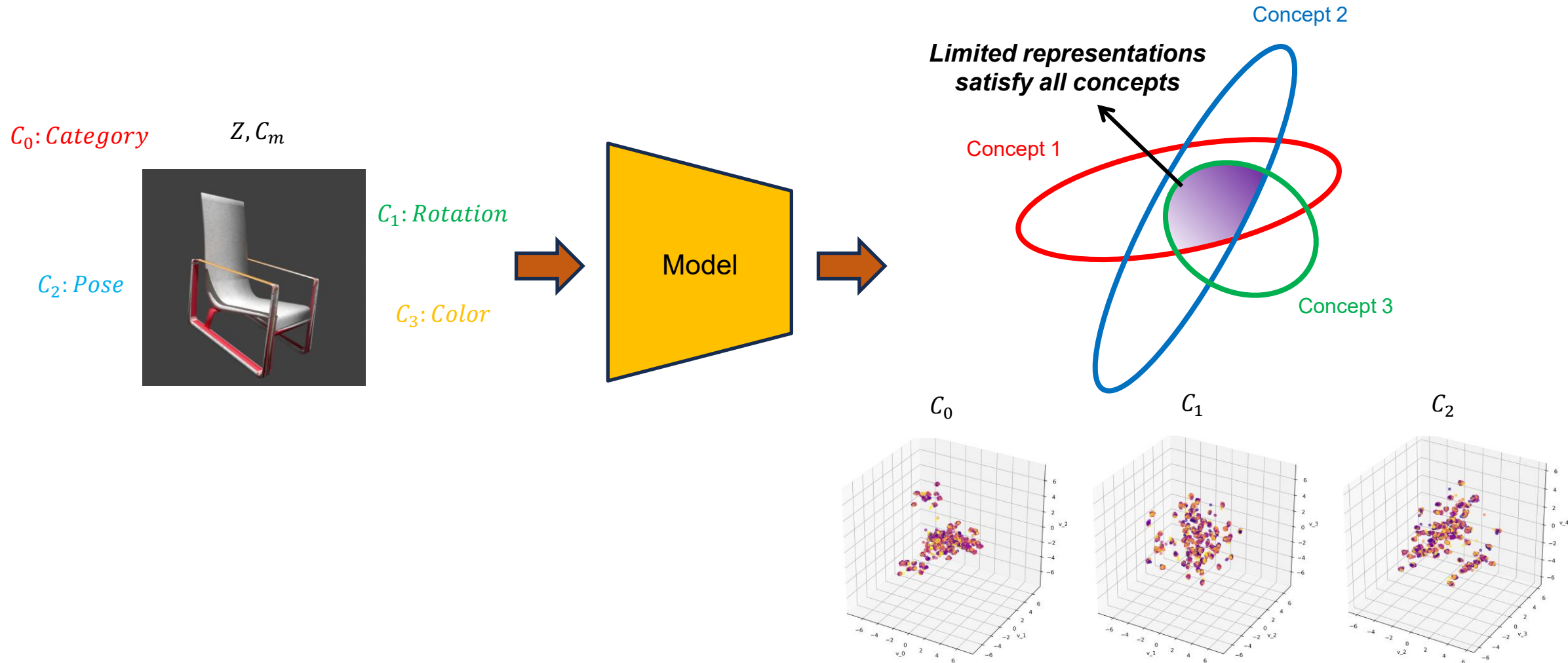
Duo Lu



Yezhou Yang

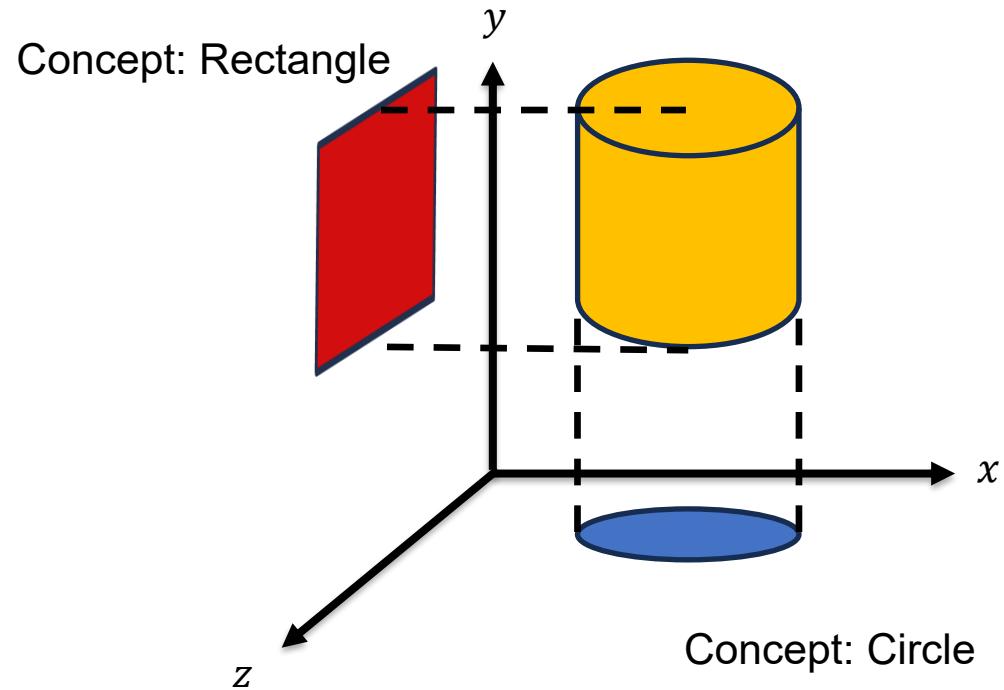


Latent representation collapse



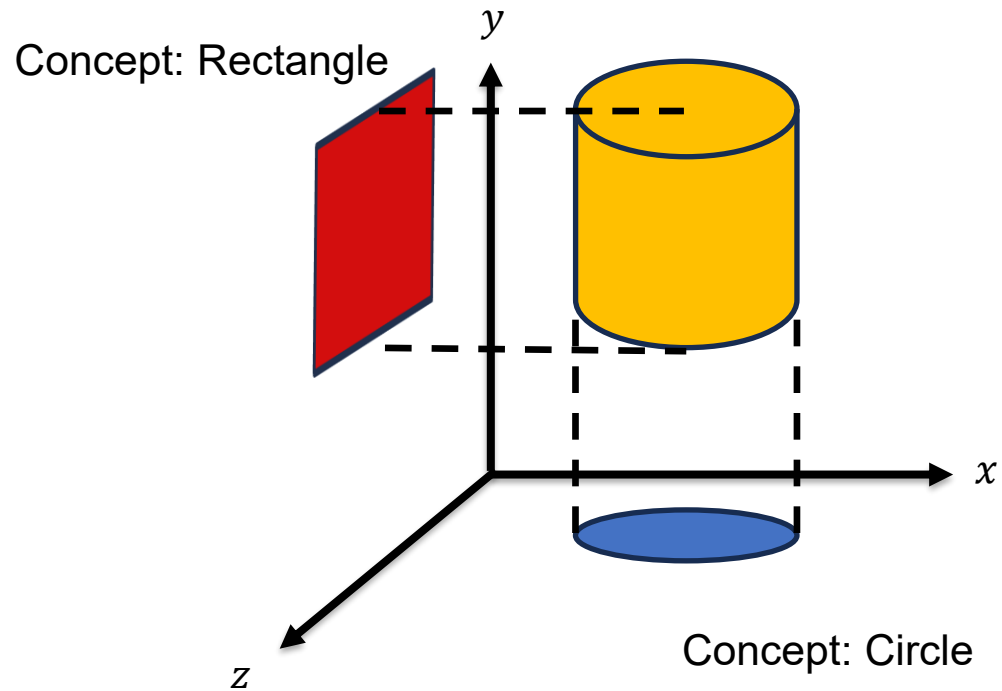
Orthogonal pooling

(a) Anamorphic Shape

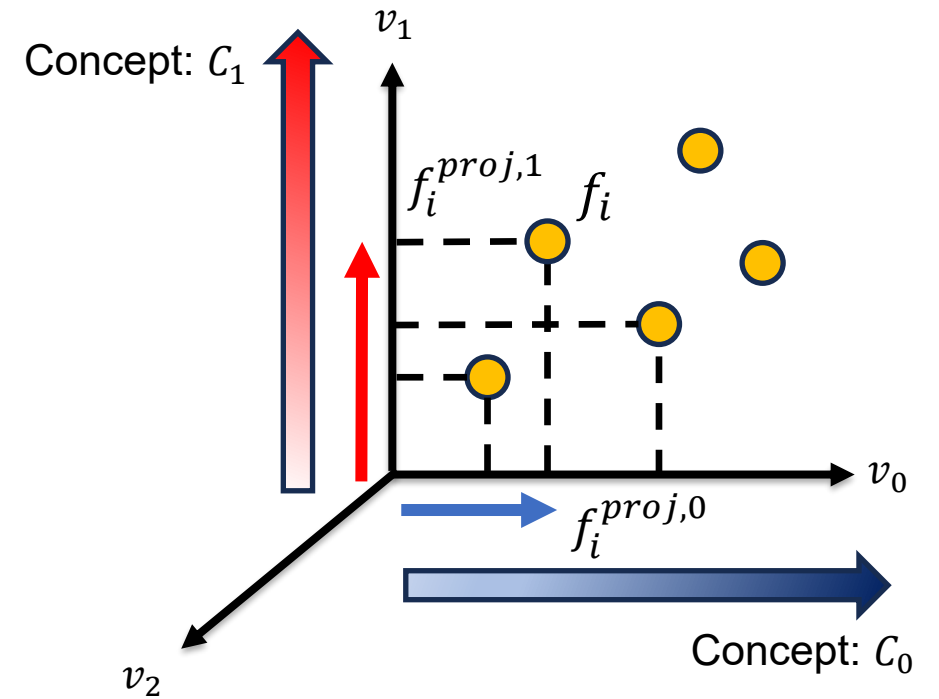


Orthogonal pooling

(a) Anamorphic Shape

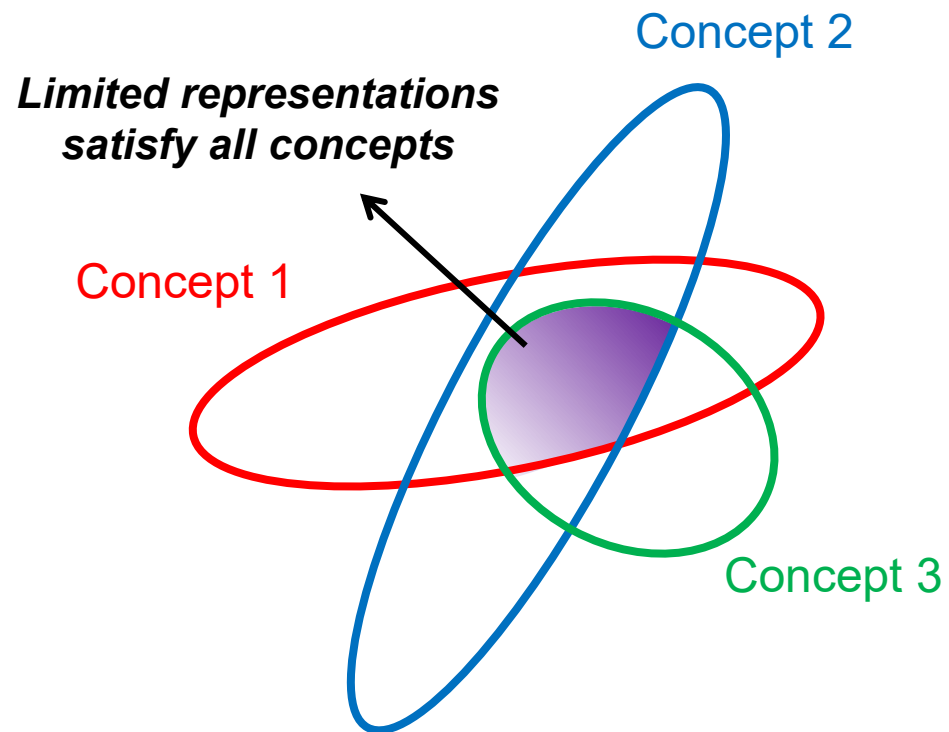


(b) Domain Expansion



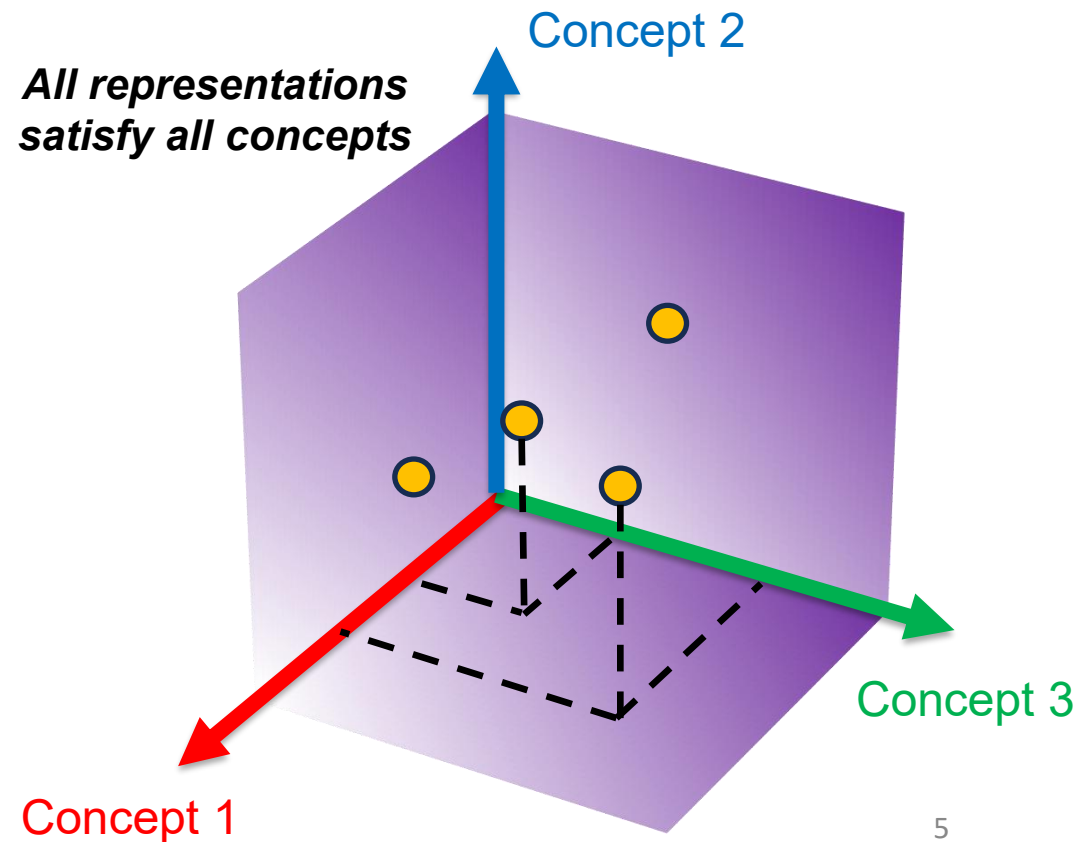
(a) Latent representation collapse

$$\mathcal{Z} \xrightarrow{\text{Enc}} \mathcal{F} \xrightarrow{\text{Dec}} \mathcal{C}$$

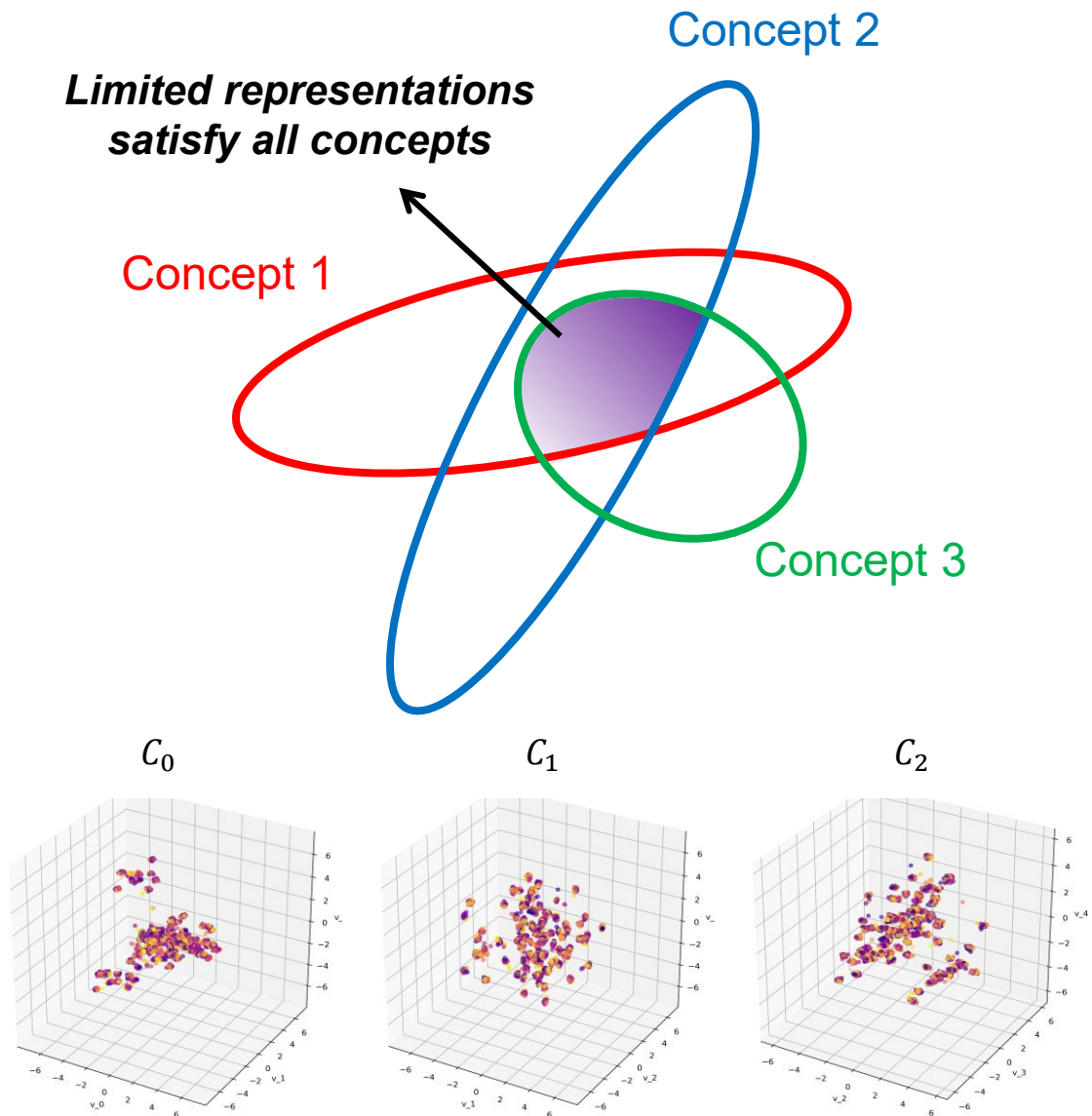


(b) Domain Expansion (ours)

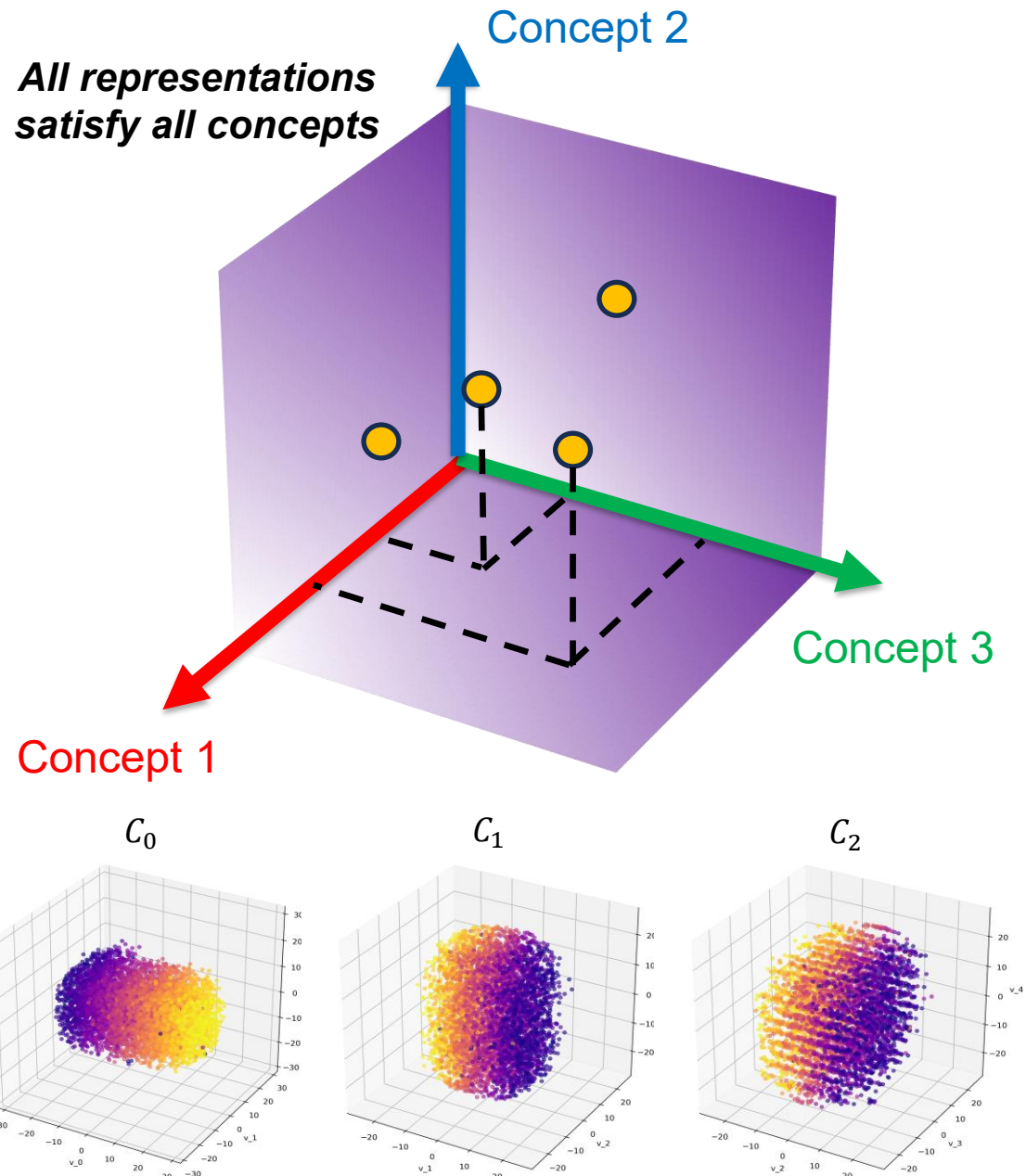
$$\mathcal{Z} \xrightarrow{\text{Enc}} \mathcal{F} \xrightarrow{\text{Orthogonal Pooling}} \begin{cases} \mathcal{F}_0^{\text{proj}} \xrightarrow{\text{Dec}_0} \mathcal{C}_0 \\ \mathcal{F}_1^{\text{proj}} \xrightarrow{\text{Dec}_1} \mathcal{C}_1 \\ \vdots \\ \mathcal{F}_{M-1}^{\text{proj}} \xrightarrow{\text{Dec}_{M-1}} \mathcal{C}_{M-1} \end{cases}$$



(a) Latent representation collapse

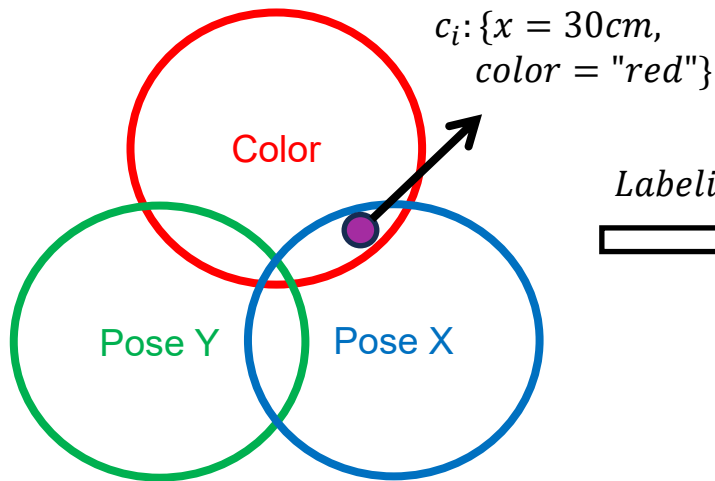


(b) Domain Expansion (ours)

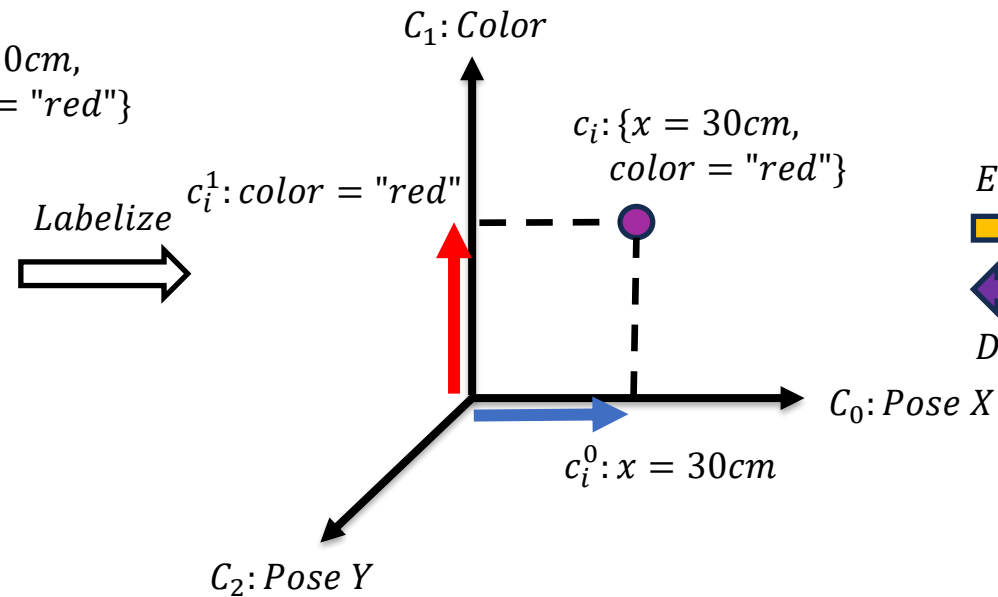


Properties and operators

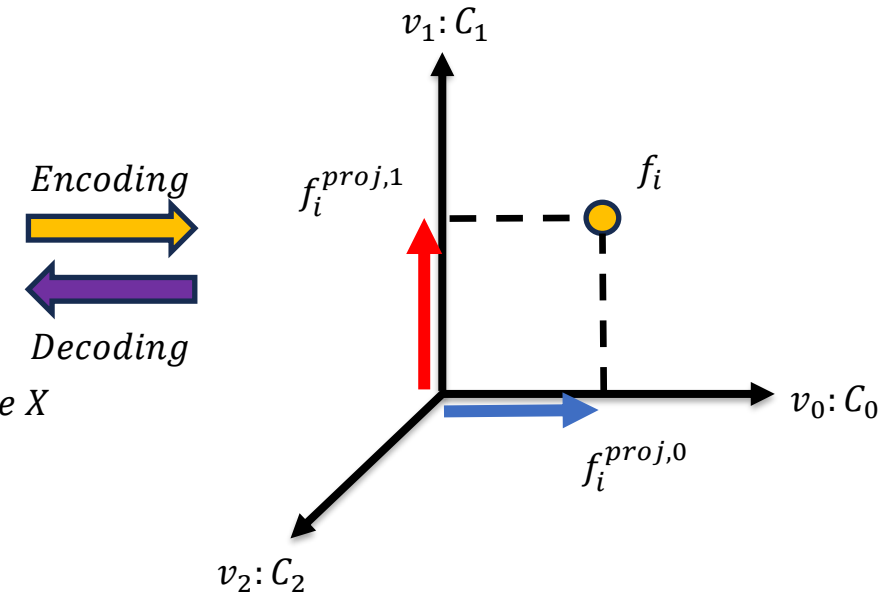
Abstract Concept Space



Numerical Concept Space



Latent Space

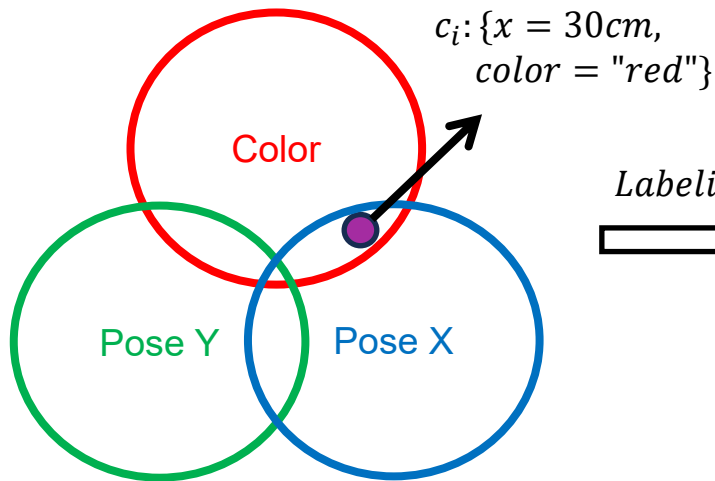


- Property 1: Multi-concept Encoding.

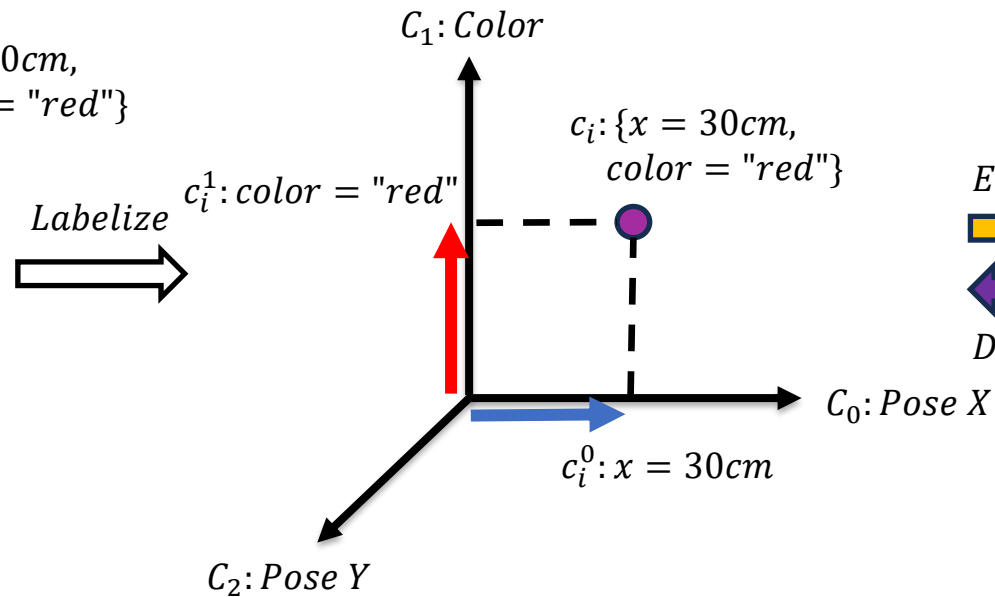
$$f_i \xrightarrow{\text{Pooling}} \{f_i^{\text{proj},0}, \dots, f_i^{\text{proj},M-1}\} \xrightarrow{\text{Dec}} \{c_i^0, \dots, c_i^{M-1}\} \rightarrow c_i.$$

Properties and operators

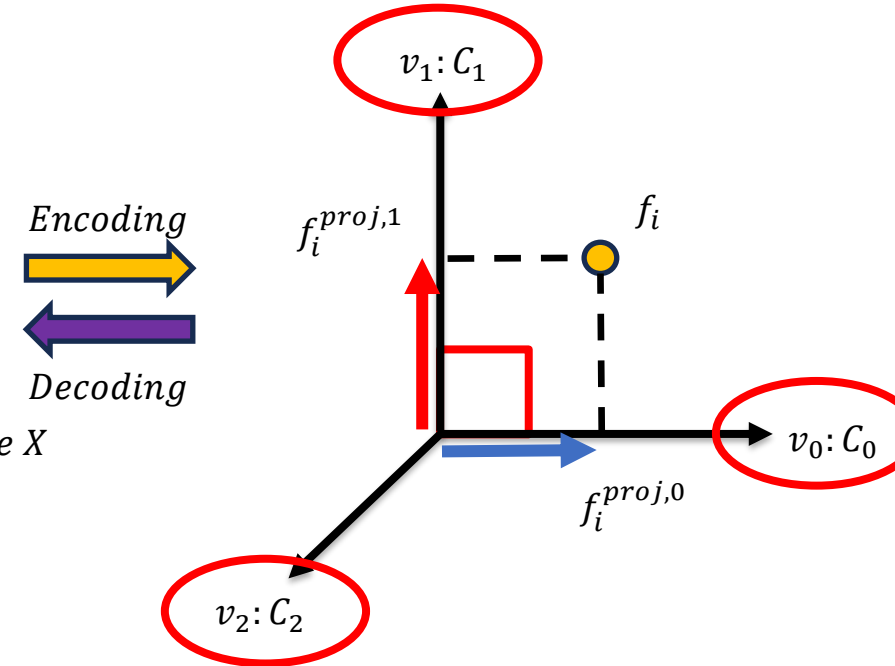
Abstract Concept Space



Numerical Concept Space



Latent Space

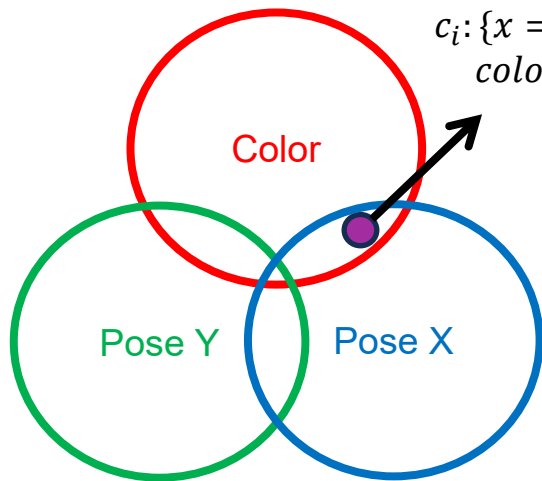


- Property 2: Orthogonality of Target Concepts.

$$\mathcal{F}_0^{\text{proj}} \perp \mathcal{F}_1^{\text{proj}} \perp \dots \perp \mathcal{F}_{M-1}^{\text{proj}} \implies \mathcal{C}_0 \perp \mathcal{C}_1 \perp \dots \perp \mathcal{C}_{M-1}.$$

Properties and operators

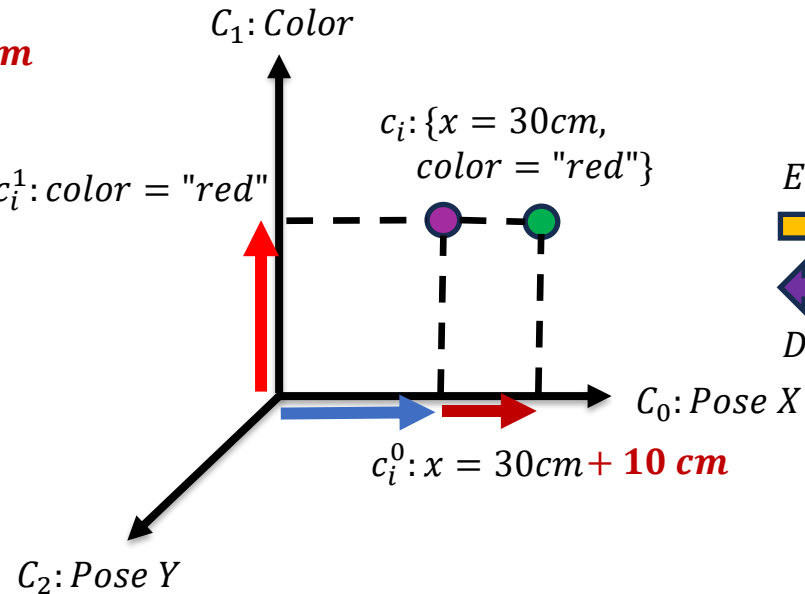
Abstract Concept Space



$c_i: \{x = 30\text{cm}, +10\text{ cm}$
color = "red"

Labelize

Numerical Concept Space

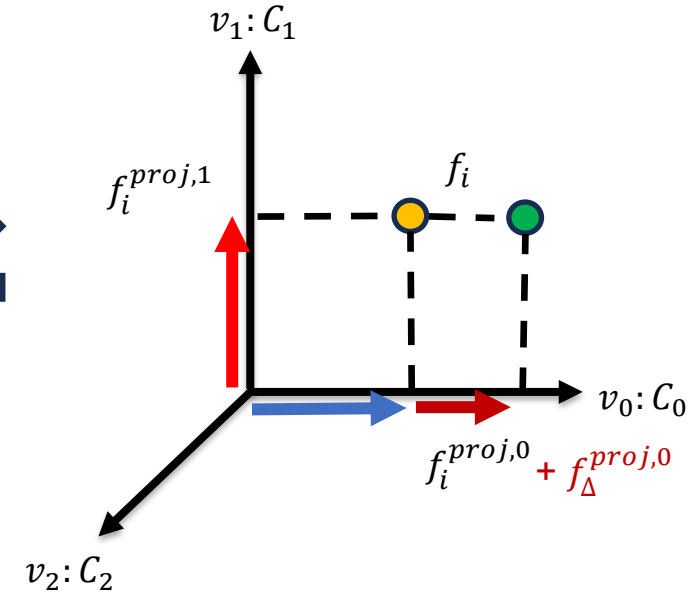


$c_i: \{x = 30\text{cm},$
color = "red"

Encoding

Decoding

Latent Space



$f_i^{proj,1}$

f_i

$f_i^{proj,0} + f_{\Delta}^{proj,0}$

- Operator 1: Concept-Specific Adjustment (\oplus^m) and (\ominus^m).

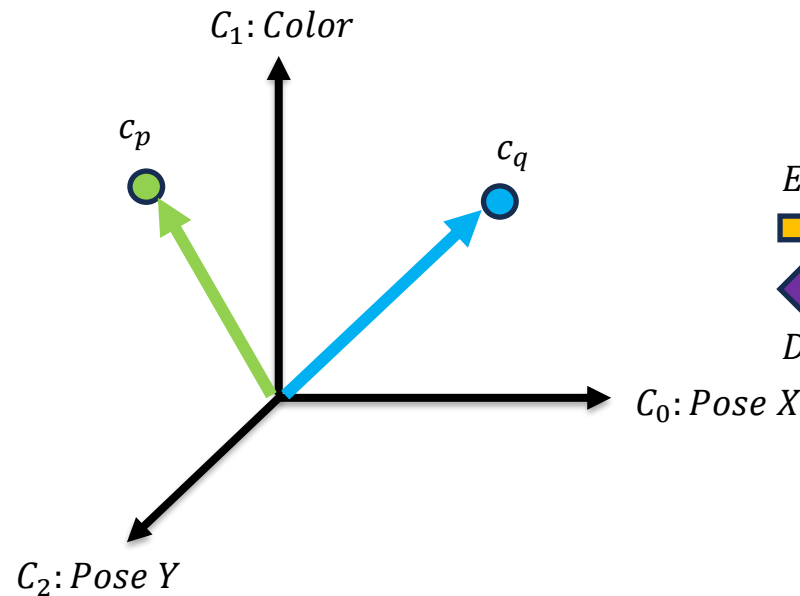
$$c_i \oplus^m c_{\Delta}^m \rightarrow \{c_i^0, \dots, \{c_i^m \oplus^m c_{\Delta}^m\}, \dots, c_i^{M-1}\}$$

$$\xrightarrow{\text{Dec}^{-1}} \{f_i^{proj,0}, \dots, \{f_i^{proj,m} + f_{\Delta}^{proj,m}\}, \dots, f_i^{proj,M-1}\}$$

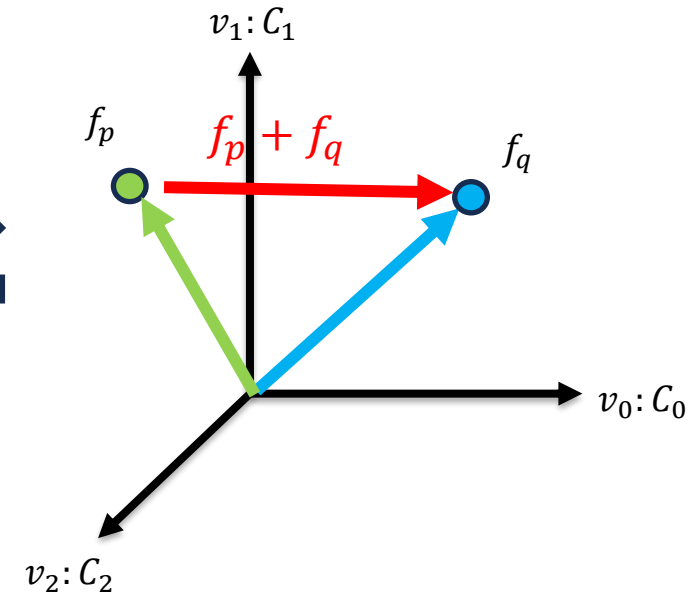
$$\xrightarrow{\text{Reconst}} f_i + f_{\Delta}^{proj,m}.$$

Properties and operators

Numerical Concept Space



Latent Space



Encoding



Decoding

- Operator 2: Concept Composition (\oplus) and (\ominus).

$$\boxed{c_p} \oplus \boxed{c_q} \rightarrow \left\{ \left\{ c_p^0 \oplus^0 c_q^0 \right\}, \left\{ c_p^1 \oplus^1 c_q^1 \right\}, \dots, \left\{ c_p^{M-1} \oplus^{M-1} c_q^{M-1} \right\} \right\}$$

$$\xrightarrow{Dec^{-1}} \left\{ \left\{ f_p^{proj,0} + f_q^{proj,0} \right\}, \left\{ f_p^{proj,1} + f_q^{proj,1} \right\}, \dots, \left\{ f_p^{proj,M-1} + f_q^{proj,M-1} \right\} \right\}$$

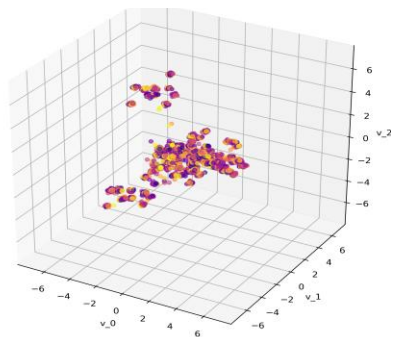
$$\xrightarrow{Reconst} \boxed{f_p + f_q}$$

Table 1: Comprehensive comparison of representation quality, predictive performance, and concept composition. Arrows indicate whether higher (\uparrow) or lower (\downarrow) values are better.

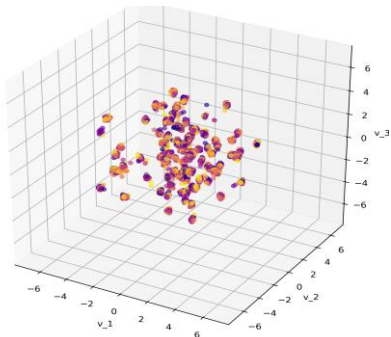
Objective Set	Method	Representation & Predictive Performance										Concept Comp.
		Spearman \uparrow			V-score \uparrow		MAE $^\circ$ \downarrow			Acc. \uparrow		Sim. \uparrow
		az	el	rot	cat	id	az	el	rot	cat	id	\oplus and \ominus
Objective Set 1	baseline	0.41	0.34	0.35	0.16	0.14	0.12	0.09	0.09	0.28	0.37	0.22
	FAMO	0.49	0.41	0.42	0.00	0.00	0.12	0.09	0.09	0.19	0.18	0.28
	Nash-MTL	0.38	0.41	0.42	0.00	0.00	0.11	0.09	0.09	0.17	0.13	0.28
	IMTL	0.31	0.16	0.16	0.39	0.28	0.14	0.11	0.12	0.92	0.79	0.14
	Ours	0.95	0.87	0.85	0.99	0.91	0.08	0.08	0.09	0.99	0.97	0.95
Objective Set 2	baseline	0.01	0.01	0.01	0.99	0.00	0.77	0.38	0.38	0.99	0.99	0.42
	FAMO	0.28	0.23	0.22	0.99	0.00	0.19	0.14	0.13	0.99	0.99	0.28
	Nash-MTL	0.45	0.39	0.39	0.15	0.00	0.12	0.08	0.09	0.99	0.99	0.35
	IMTL	0.39	0.18	0.16	0.99	0.00	0.15	0.11	0.13	0.99	0.99	0.28
	Ours	0.95	0.87	0.85	0.98	0.96	0.07	0.08	0.09	0.98	0.94	0.93

Baseline

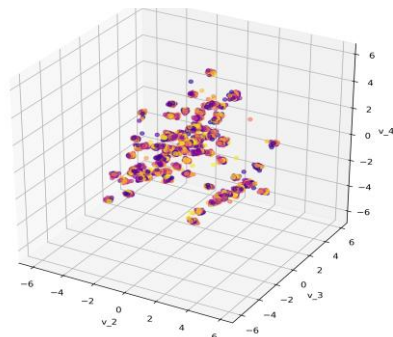
C_0 : azimuth



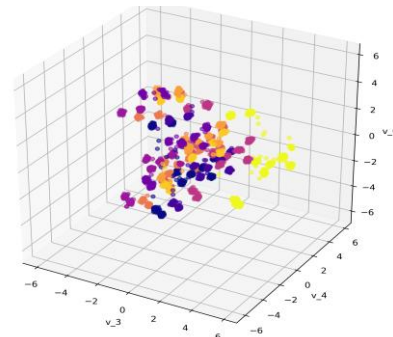
C_1 : elevation



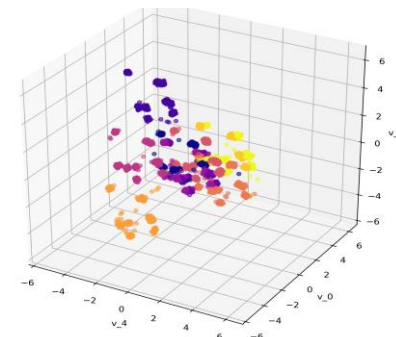
C_2 : rotation



C_3 : category

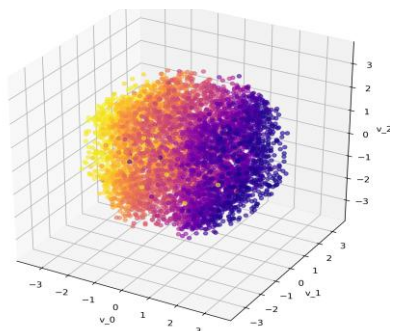


C_4 : id

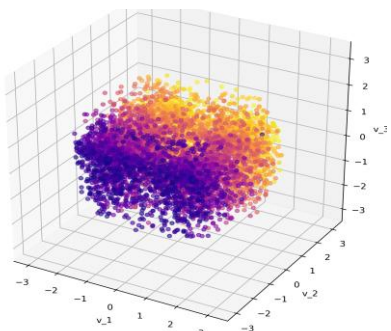


Nash-MTL

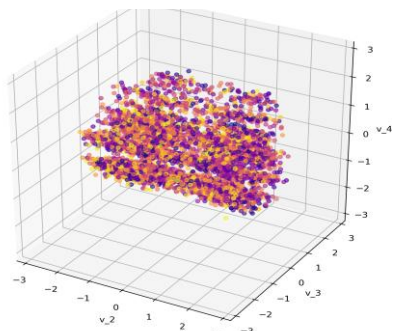
C_0 : azimuth



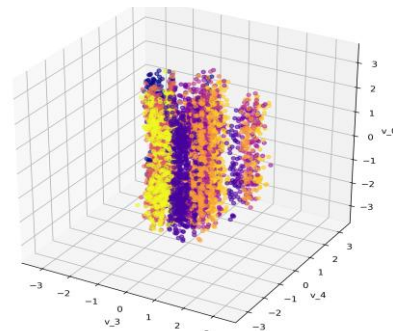
C_1 : elevation



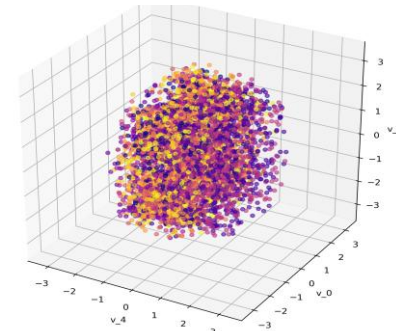
C_2 : rotation



C_3 : category

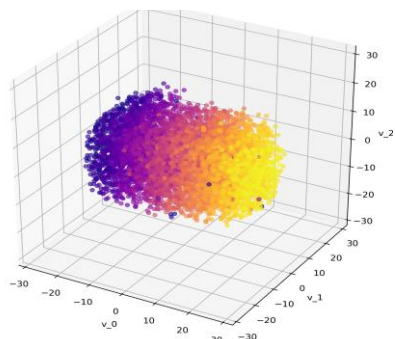


C_4 : id

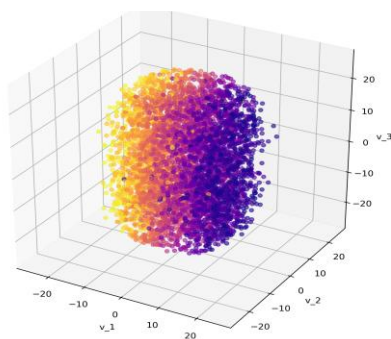


Domain Expansion (ours)

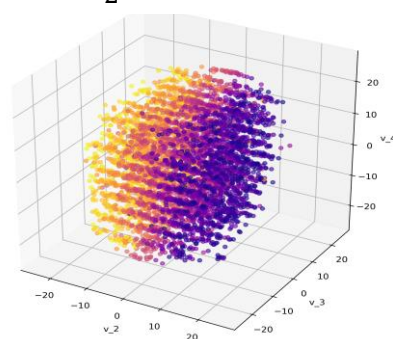
C_0 : azimuth



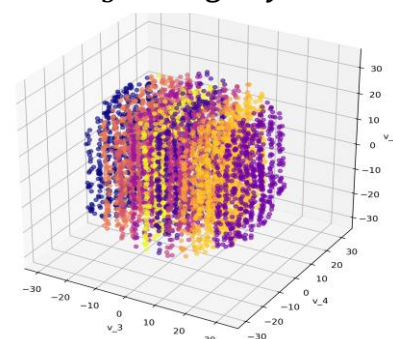
C_1 : elevation



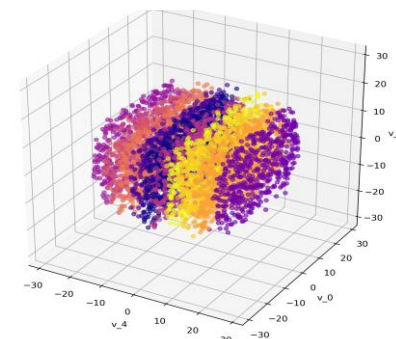
C_2 : rotation

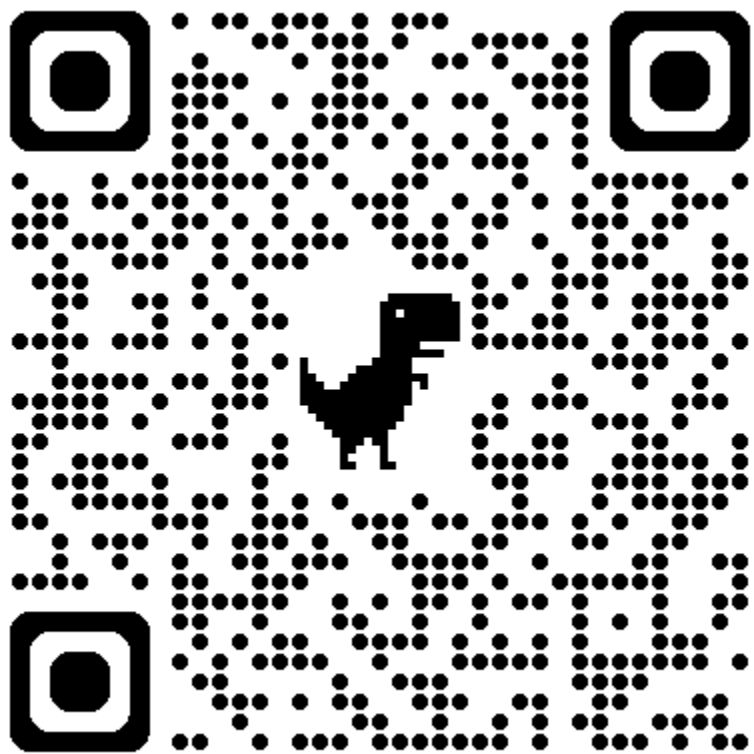


C_3 : category



C_4 : id





<https://huang-chiyao.github.io/DE/>