On the trade-off between expressivity & privacy in graph representation learning

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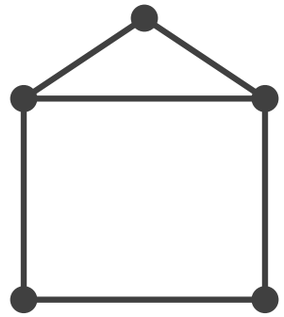
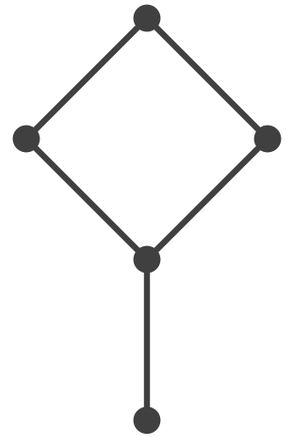
ICLR

What's the deal

In graph ML we want *good* representations $\varphi(G)$ of a graph G

G_1 

G_2 

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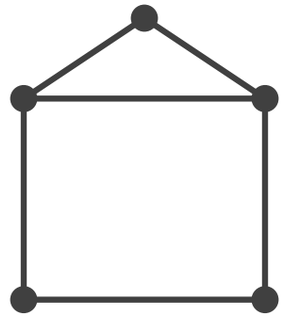
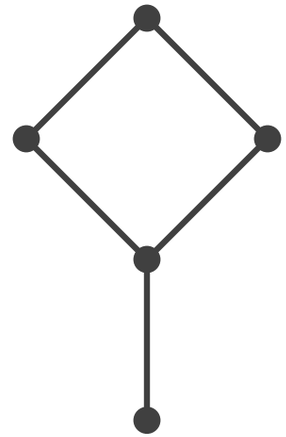
G_1 

G_2 

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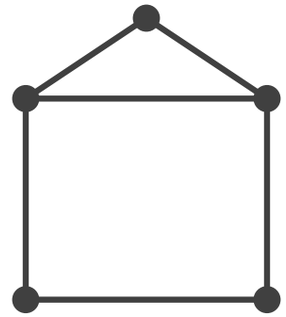
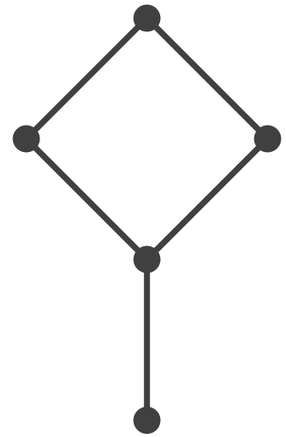
G_1 

G_2 

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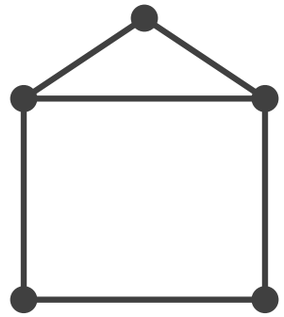
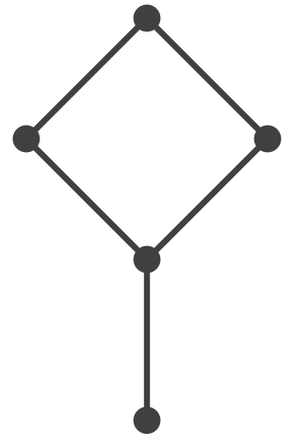
G_1 

G_2 

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Tamara

$$\varphi(G_1) = \varphi(G_2) \iff G_1 \simeq G_2$$

Same representations iff the graphs are isomorphic

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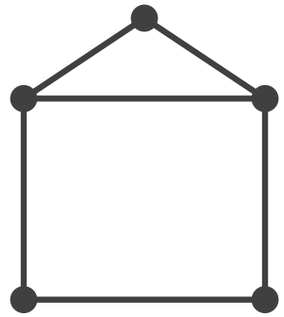
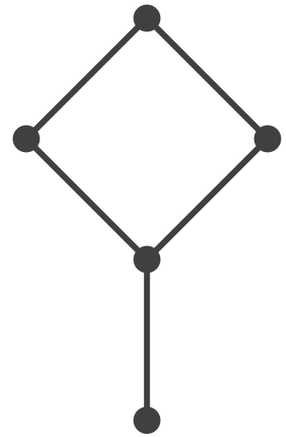
G_1 

G_2 

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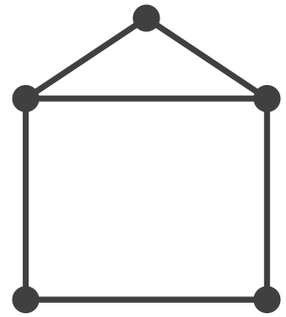
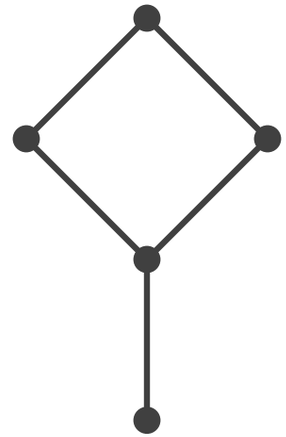
G_1 

G_2 

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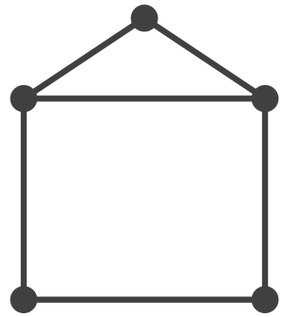
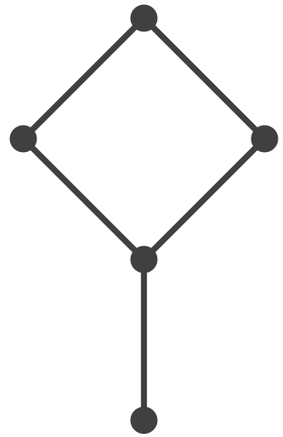
G_1 

G_2 

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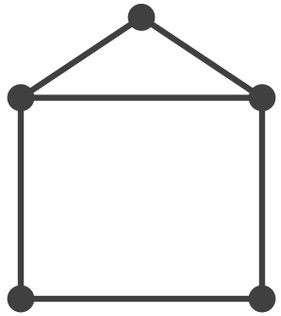
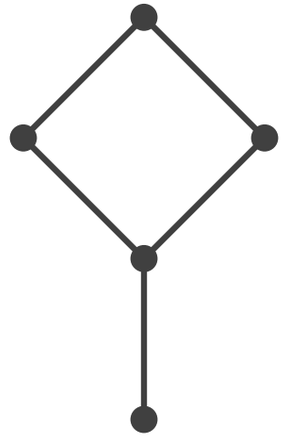
Patrick

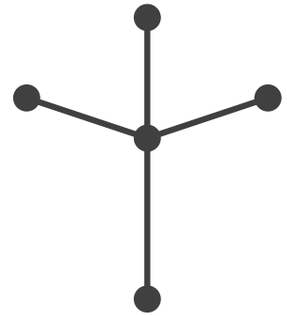
$$d_{\text{edge}}(G_1, G_2) = 1 \implies \varphi(G_1) \approx \varphi(G_2)$$

Similar representations for edge-adjacent graphs

Why not both?

G_1 

G_2 

G_3 

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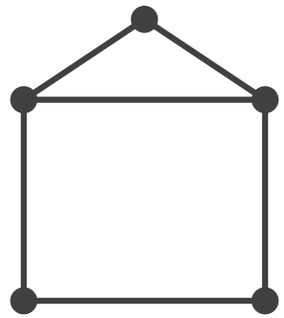
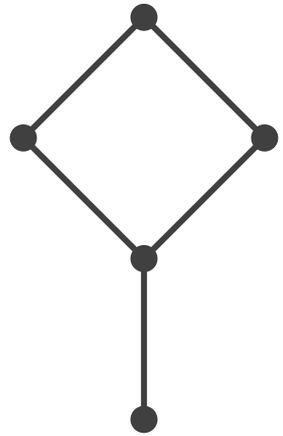
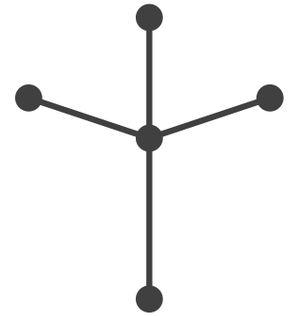
G_1 

G_2 

G_3 

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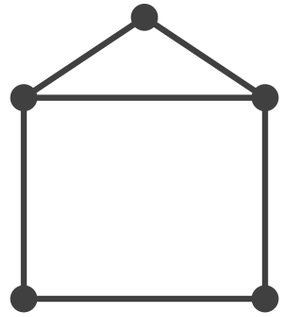
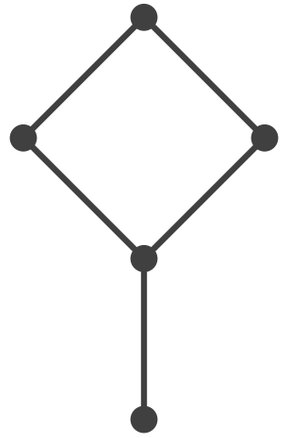
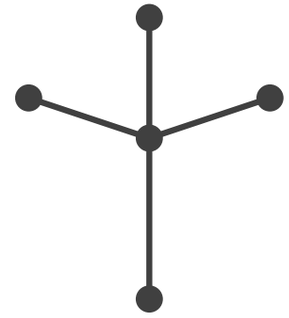
G_1 

G_2 

G_3 

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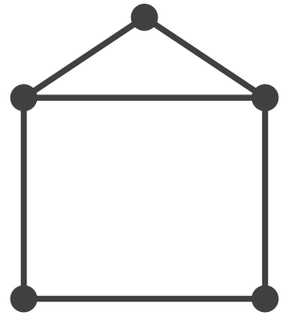
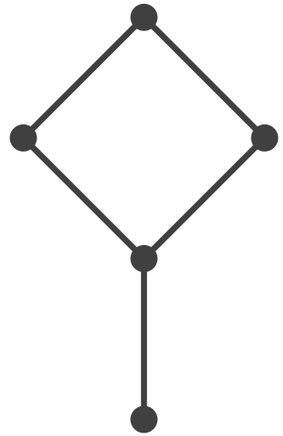
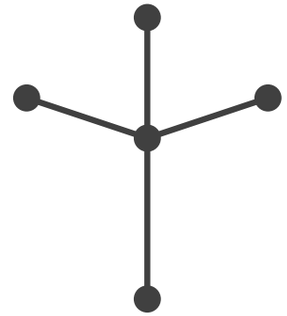
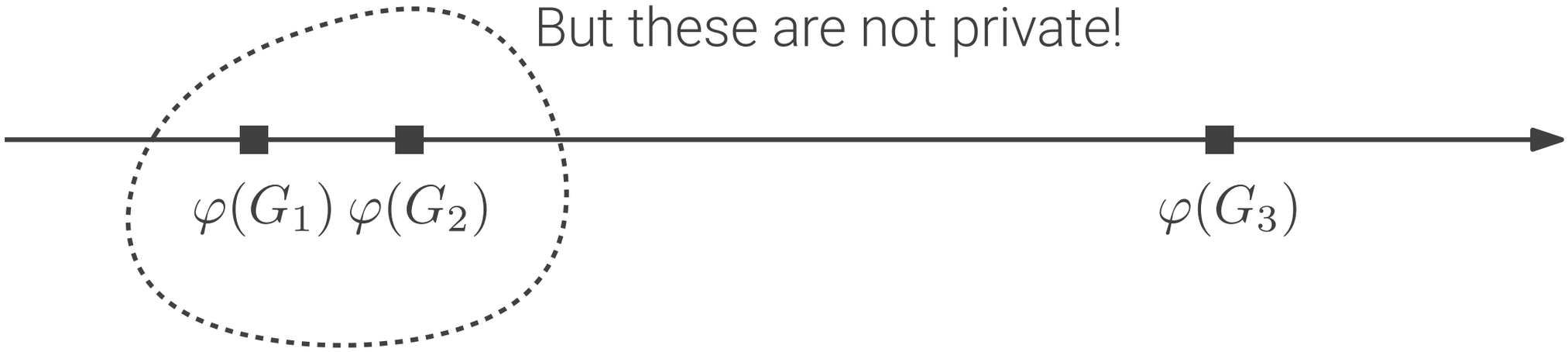
G_1 

G_2 

G_3 



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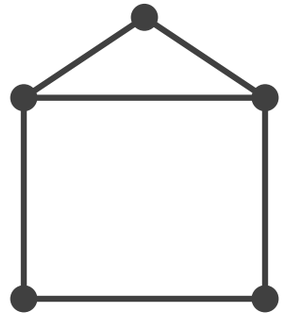
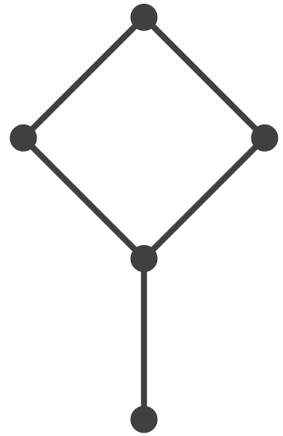
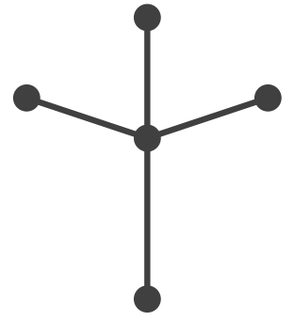
G_1 

G_2 

G_3 

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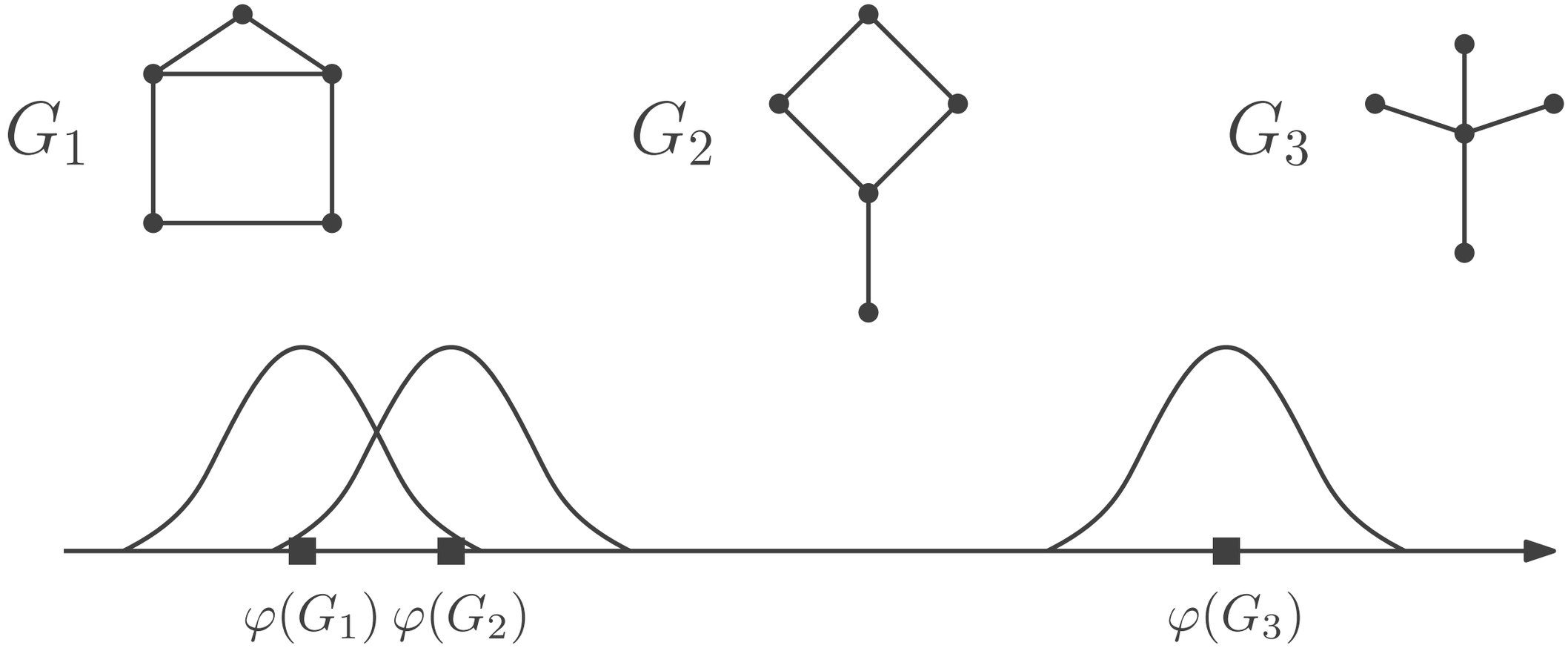

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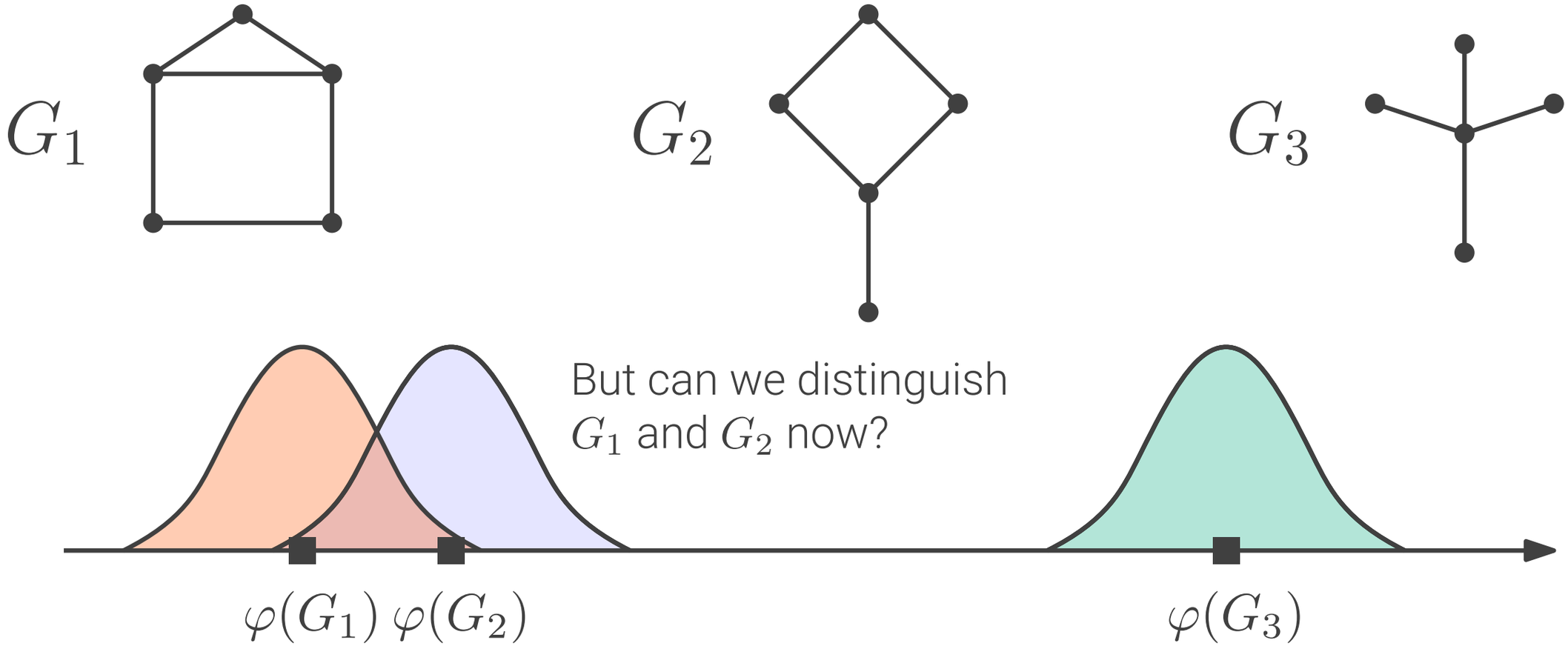

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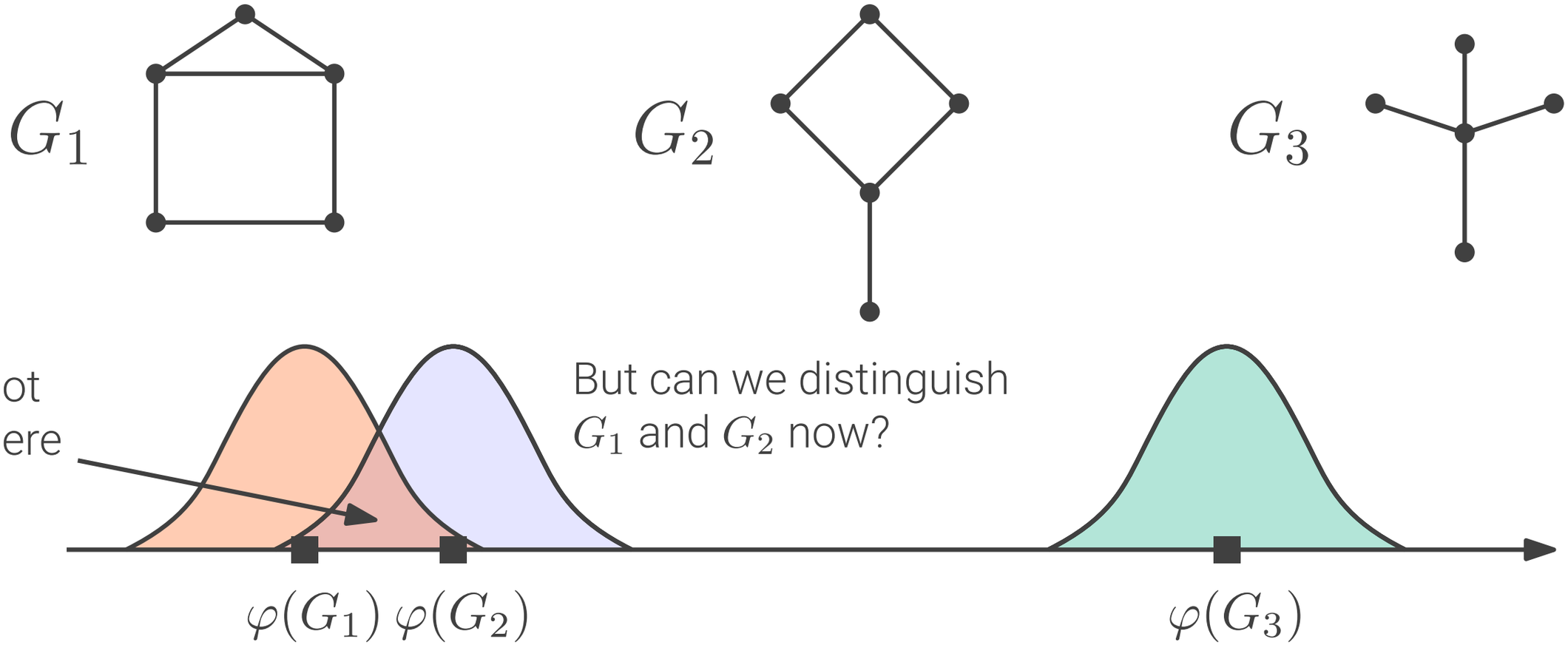

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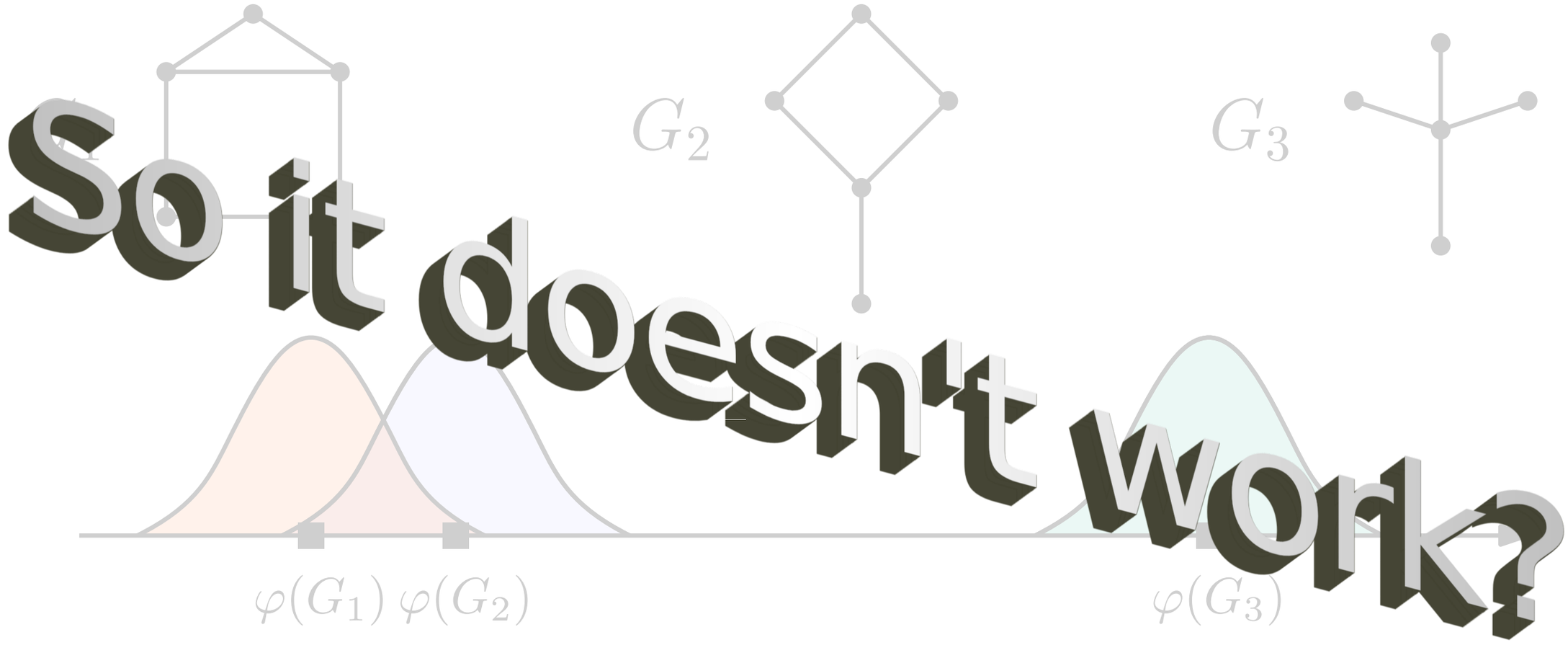

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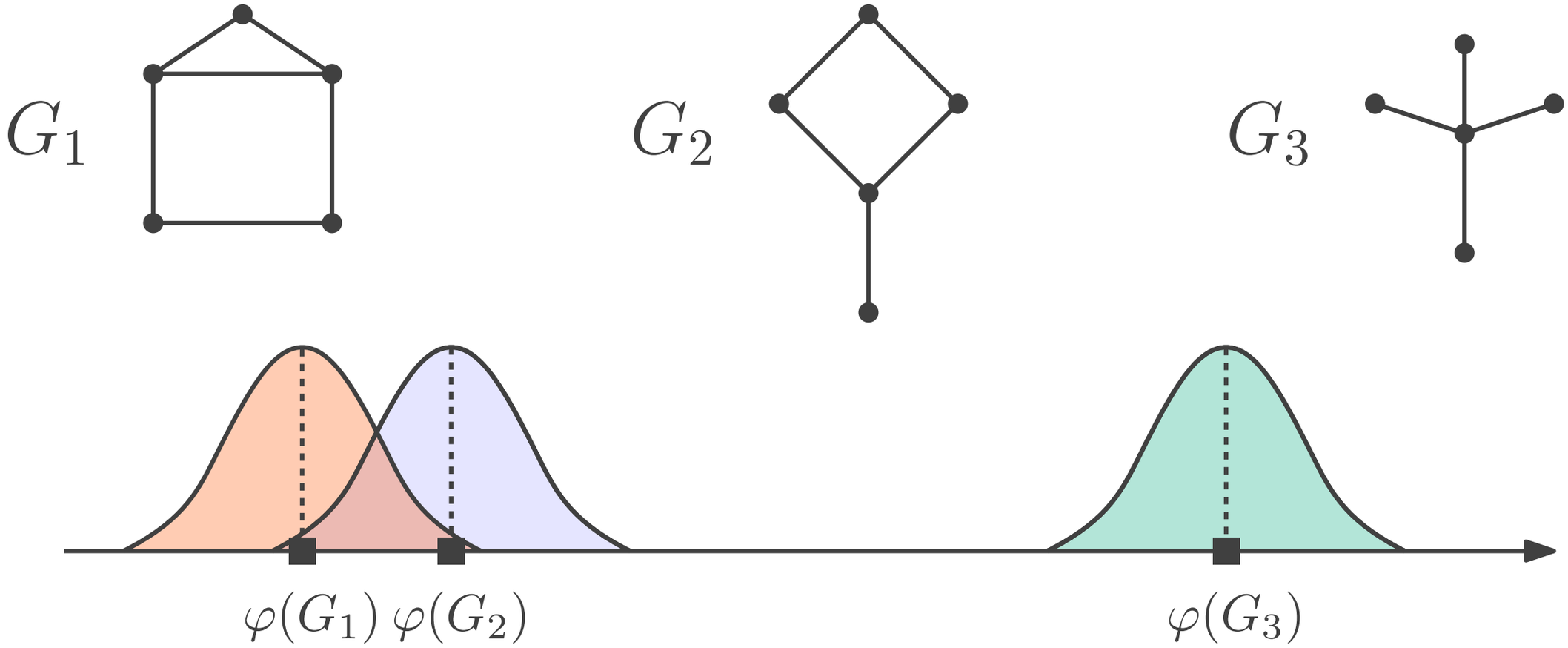

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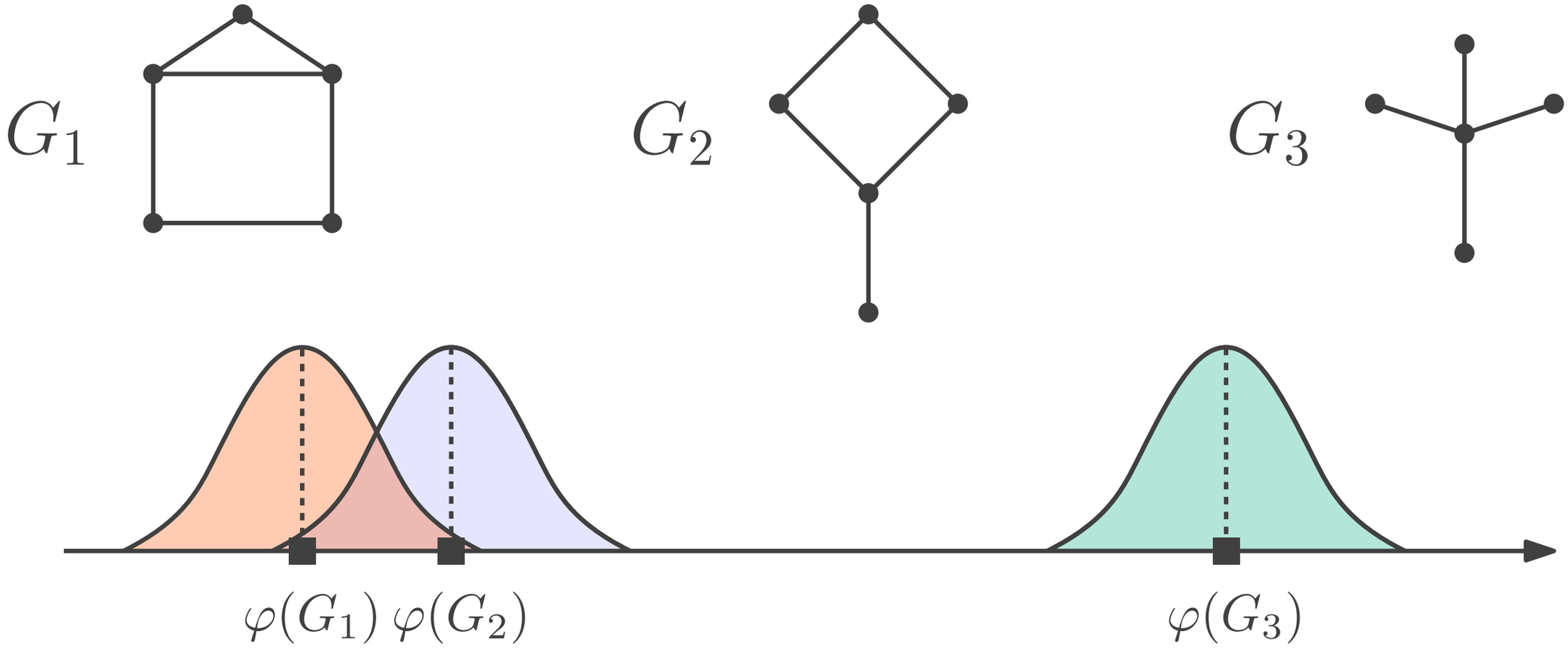

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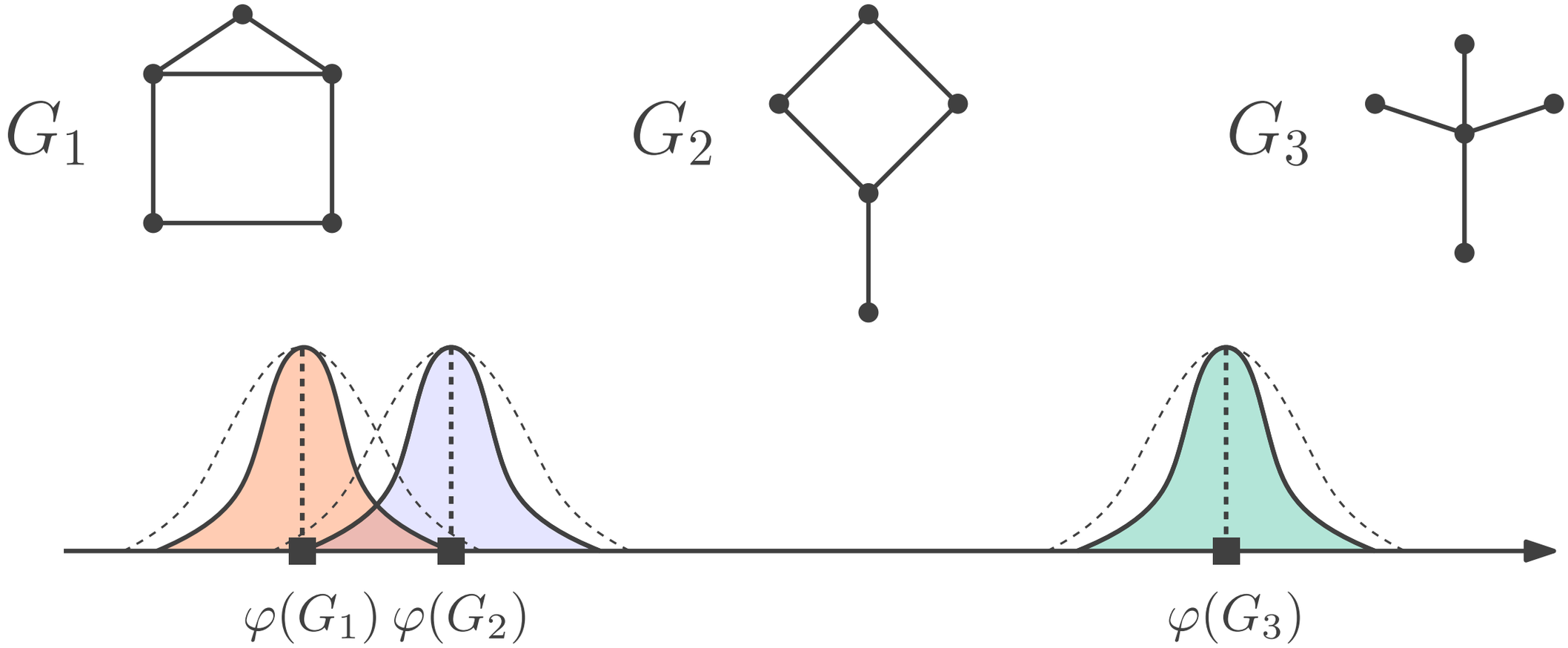

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We consider expressivity **in expectation** and a **local** notion of differential privacy.

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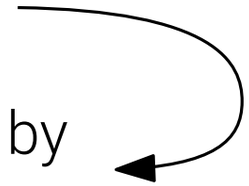

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determined by *pattern class*

Result 2. **More expressive** pattern classes often require **more noise** to be added to guarantee privacy.

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Pattern graphs $(F_1, \ldots, F_n)$

$\varphi(G) =$

F_1	F_2	F_3		F_n
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How much noise?

We add **small-ish** noise that depends on a **local** notion of privacy

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expressive in expectation

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TLDR: We sample **noisy homomorphism densities** to obtain **provably** expressive and private graph representations

By kinda relaxing privacy a bit and looking at a *local* notion of it

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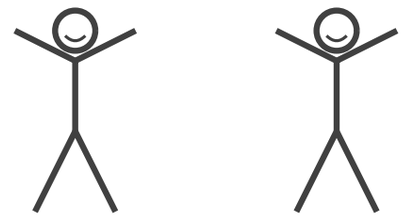
We also run **experiments** and they work!

THANKS:)



Tamara Patrick

Both moderately happy