

Homeostatic Adaptation of Optimal Population Codes under Metabolic Stress

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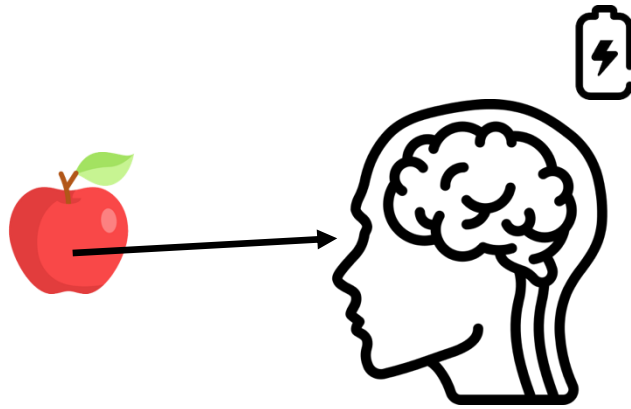
ICLR, 2026

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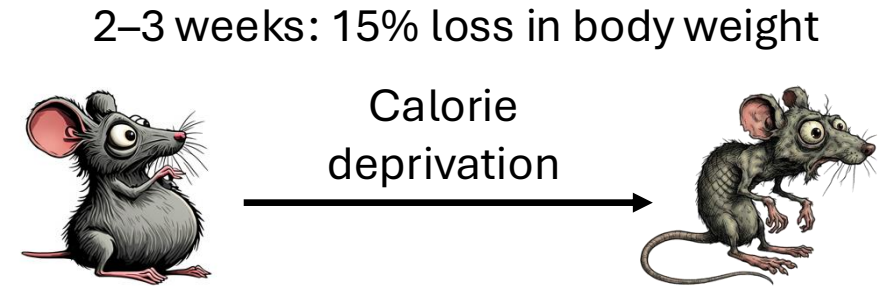
*How does caloric deprivation affect
neural population codes?*

Motivation



Processing visual input requires a considerable amount of energy.

What happens to processing when energy is limited?



In neocortex (L2/3)

- ~29% reduction in ATP usage
- Firing rates are maintained
- Flattened tuning curves



[Padamsey et al., 2022]

Is there any normative model that can describe this adaptation to metabolic stress?

Homeostatic Constrained Optimization

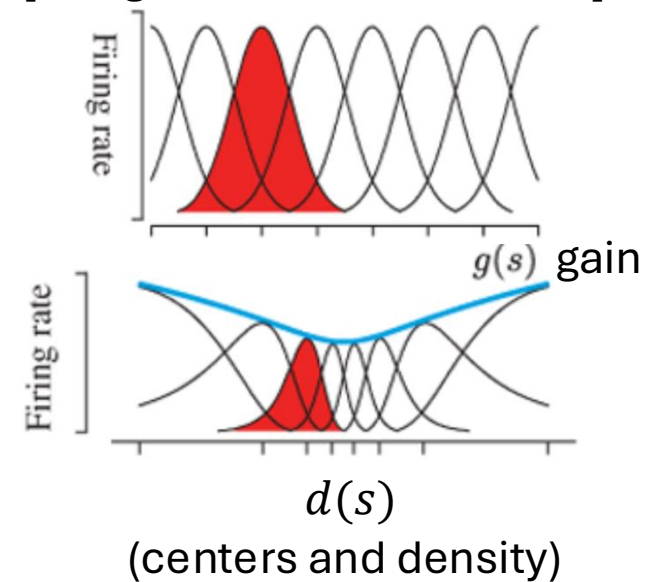
$$\operatorname{argmax}_{h_1(\cdot), \dots, h_N(\cdot)} \int p(s) f(FI(s; E)) ds, \quad \text{s.t.} \int p(s) h_n(s) ds = R_n, \quad \forall n$$

Optimize tuning curves to **transmit information efficiently** w.r.t. **energy** while **maintaining FR**.

Accurate but not explainable due to requirement of numerical solvers

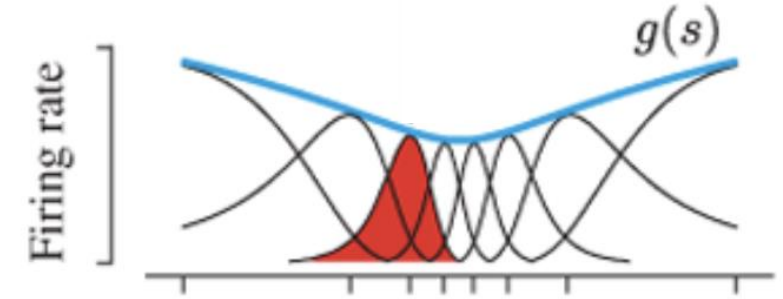
Need simplifying assumptions:
Gaussian population, gain and density

[Ganguli & Simoncelli, 2010]

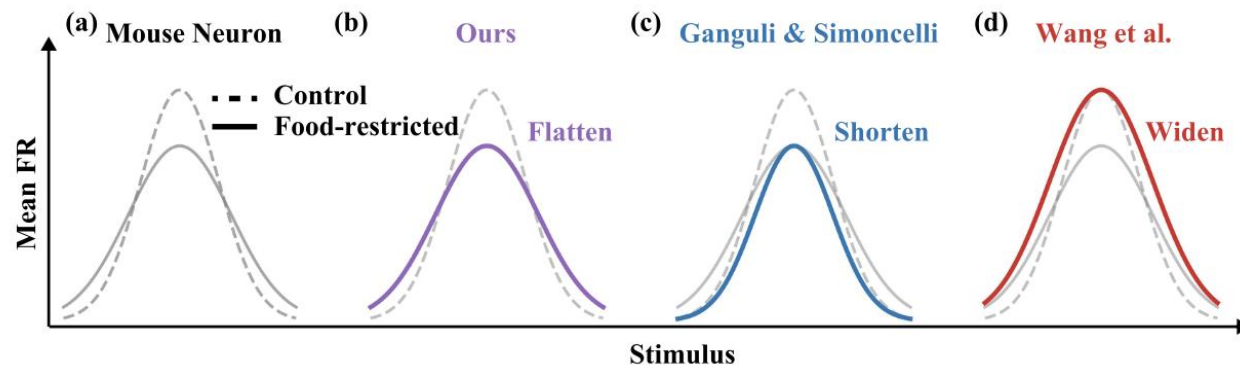


infomax: $f(x) = \log(x) \rightarrow$ L0 norm
discrimax: $f(x) = -x^{-1} \rightarrow$ L2 norm

Comparison to Prior Work

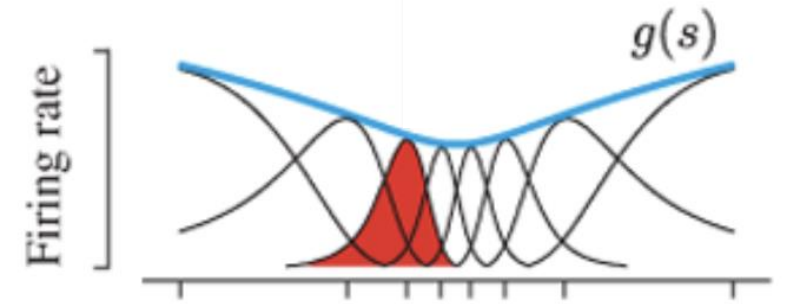


Works	Constraints	Constraints under Tiling	Noise	Low Energy
Ganguli & Simoncelli	mean FR population size	$\int p(s)g(s)ds = R$ $\int d(s)ds = N$	Poisson	$R \downarrow$, shortens (N fixed)
Wang et al.	max FR total FI states	$g(s) \propto 1$ $\int \sqrt{g(s)d(s)^2}ds = C$	Poisson or Gaussian	$C \downarrow$, widens (g fixed)
Ours	energy approx. homeostasis	$\int p(s)g(s)^\alpha ds = \bar{E}$ $\frac{g(s)}{d(s)} = \frac{R(s)}{p(s)}$	Energy-dependent Dispersed Poisson	$E \downarrow$, flattens ($R(s)$ fixed)



Wrong constraints \rightarrow Wrong adaptation!

Proposed Model



- Two novel constraints: homeostasis, energy budget
- Noise-energy trade-off gives 3 new parameters (E , α , η) that we ground in biophys sim

$$\operatorname{argmax}_{g(\cdot), d(\cdot)} \int p(s) f(\eta_{\kappa}(E)^{-1} g(s) d(s)^2) ds,$$

Approximate information in gain and density (**Append. A**)

$$\text{s.t. } p(s) \frac{g(s)}{d(s)} = R(s)$$

Approximate FR homeostasis (**Prop. C.1**)

$$\int p(s) g(s)^{\alpha} ds = E, \quad \text{where } \alpha \geq 1. \quad \text{Energy limit}$$

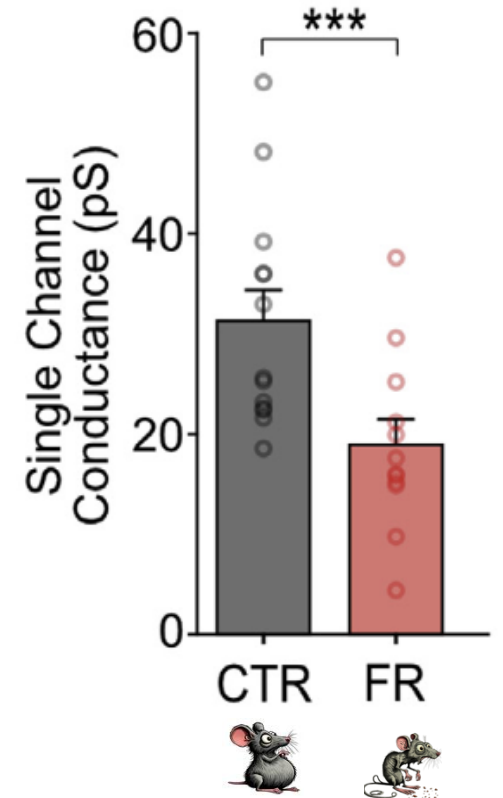
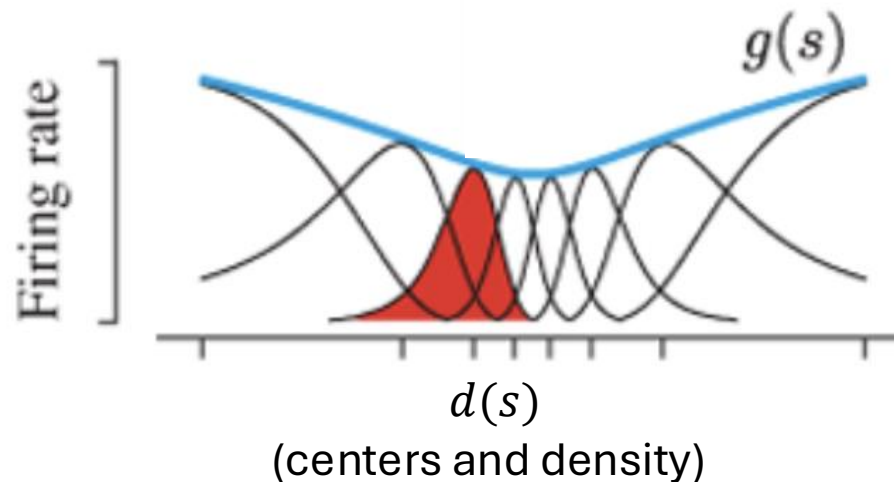
Model Details --- energy limit

$$\int p(s)g(s)^\alpha ds = E, \quad \text{where } \alpha \geq 1.$$

Expected ATP in natural scenes

Flattened TCs are less excitable, have lower peaks

DOF



Analytical Solution

$$\operatorname{argmax}_{g(\cdot), d(\cdot)} \int p(s) f(\eta_\kappa(E)^{-1} g(s) d(s)^2) ds,$$

$$\text{s.t. } p(s) \frac{g(s)}{d(s)} = R(s)$$

$$\int p(s) g(s)^\alpha ds = E, \quad \text{where } \alpha \geq 1.$$

infomax: $f(x) = \log(x) \rightarrow$ L0 norm

discrimax: $f(x) = -x^{-1} \rightarrow$ L2 norm

	infomax	discrimax	L_p error, $p = -2\beta$
Optimized function	$f(x) = \log x$	$f(x) = -x^{-1}$	$f(x) = -x^\beta, \beta < \frac{\alpha}{3}$
Density (tuning width) ⁻¹ $d(s)$	$E^{\frac{1}{\alpha}} R(s)^{-1} p(s)$	$\propto E^{\frac{1}{\alpha}} R(s)^{\frac{-\alpha-1}{\alpha+3}} p(s)^{\frac{\alpha+1}{\alpha+3}}$	$\propto E^{\frac{1}{\alpha}} R(s)^{\frac{\alpha-\beta}{3\beta-\alpha}} p(s)^{\frac{\beta-\alpha}{3\beta-\alpha}}$
Gain $g(s)$	$E^{\frac{1}{\alpha}}$	$\propto E^{\frac{1}{\alpha}} R(s)^{\frac{2}{\alpha+3}} p(s)^{\frac{-2}{\alpha+3}}$	$\propto E^{\frac{1}{\alpha}} R(s)^{\frac{2\beta}{3\beta-\alpha}} p(s)^{\frac{-2\beta}{3\beta-\alpha}}$
Fisher information $FI(s)$	$\propto \frac{E^{\frac{3}{\alpha}} p(s)^2}{\eta_\kappa(E) R(s)^2}$	$\propto \frac{E^{\frac{3}{\alpha}} p(s)^{\frac{2\alpha}{\alpha+3}}}{\eta_\kappa(E) R(s)^{\frac{2\alpha}{\alpha+3}}}$	$\propto \frac{E^{\frac{3}{\alpha}} p(s)^{\frac{-2\alpha}{3\beta-\alpha}}}{\eta_\kappa(E) R(s)^{\frac{2\alpha}{\alpha-3\beta}}}$
Discriminability bound $\delta_{\min}(s)$	$\propto E^{\frac{-3}{2\alpha}} p(s)^{-1}$	$\propto E^{\frac{-3}{2\alpha}} p(s)^{\frac{-\alpha}{\alpha+3}}$	$\propto E^{\frac{-3}{2\alpha}} p(s)^{\frac{\alpha}{3\beta-\alpha}}$

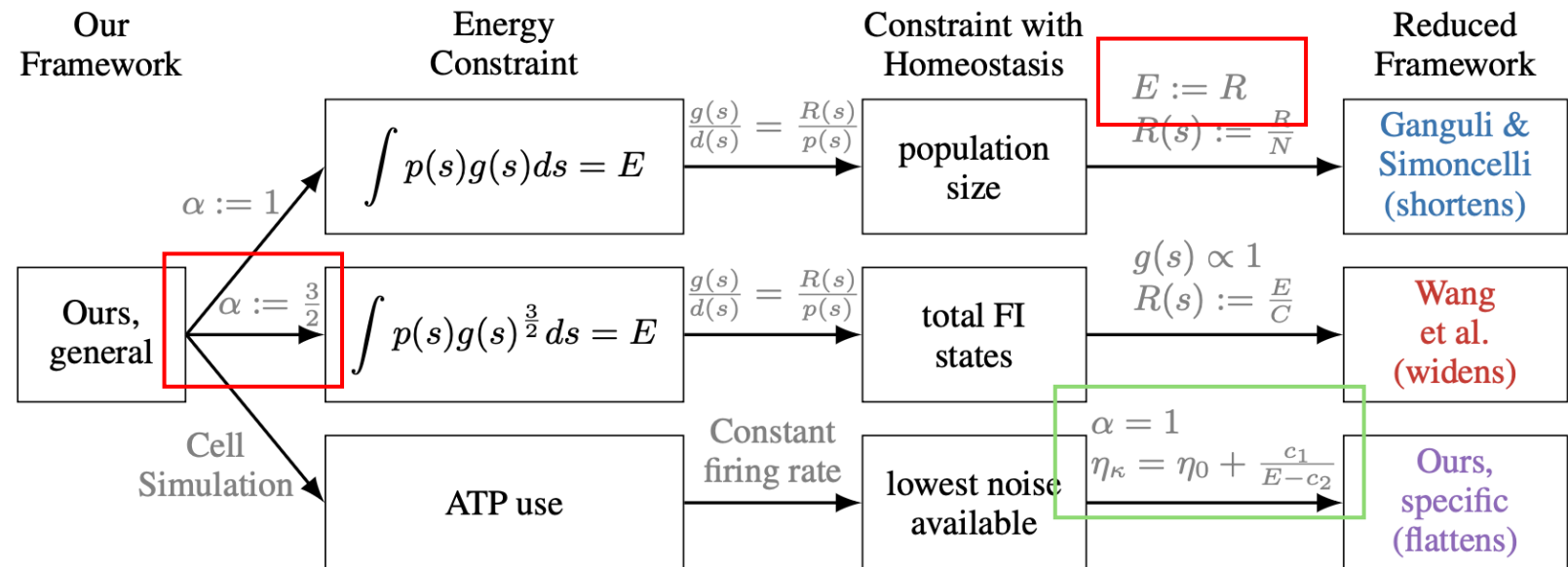
E, α, η_κ can be found by simulation (**Sec. 4**), but first...!

Relation to Existing Models

Works	Constraints	Constraints under Tiling	Noise	Low Energy
Ganguli & Simoncelli	mean FR population size	$\int p(s)g(s)ds = R$ $\int d(s)ds = N$	Poisson	$R \downarrow$, shortens (N fixed)
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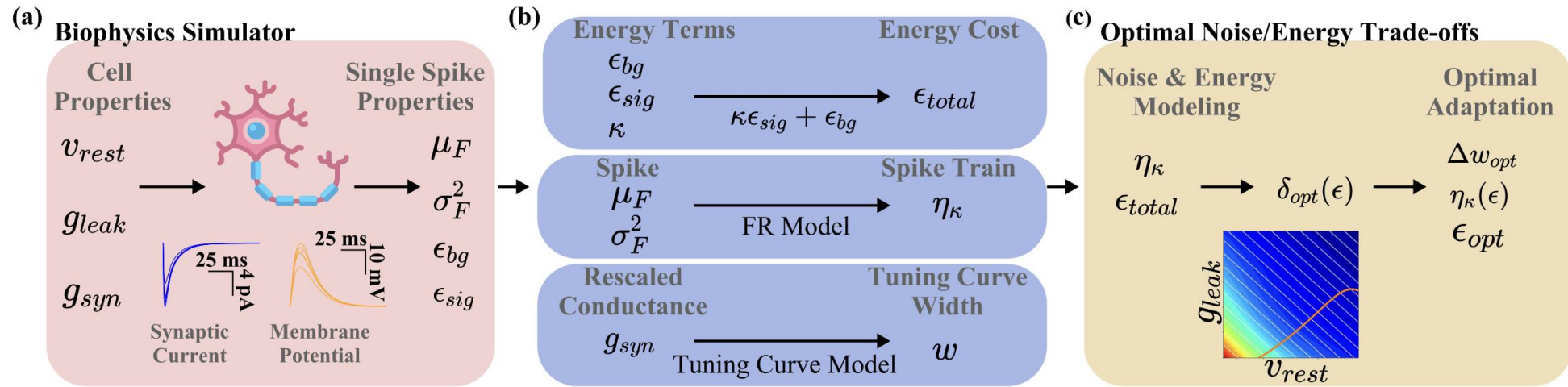
We introduce novel constraints!

We can generalize to other methods although using novel constraints!



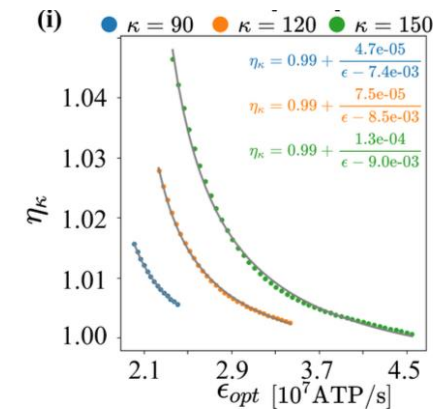
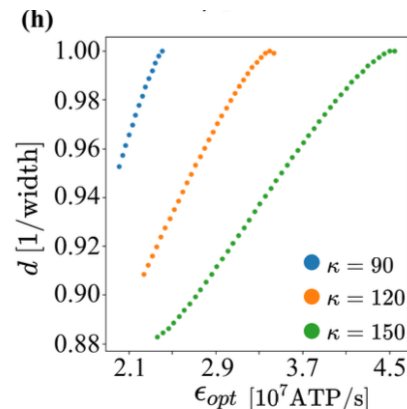
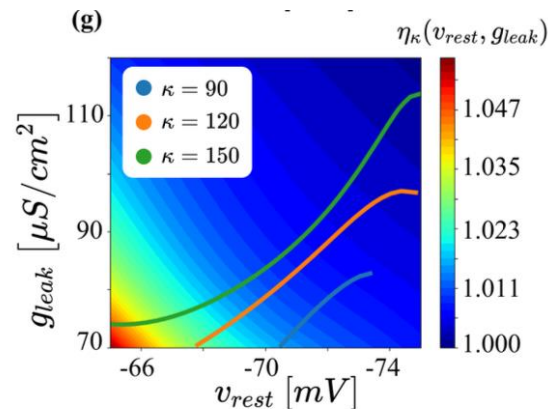
How to set the parameters (α and η_κ)?

- We find the noise/energy trade-off curve by simulation single-compartment neuron model (NEURON package).



How to set the parameters (α and η_κ)?

- We find
 - The noise-energy trade-off curves depend on the input signal activity
 - The linear relation between density and energy $\Rightarrow \alpha = 1$
 - $\eta_\kappa = \eta_0 + c_1/(\epsilon - c_1)$



Compare to Experimental Data

Simulation-driven parameters

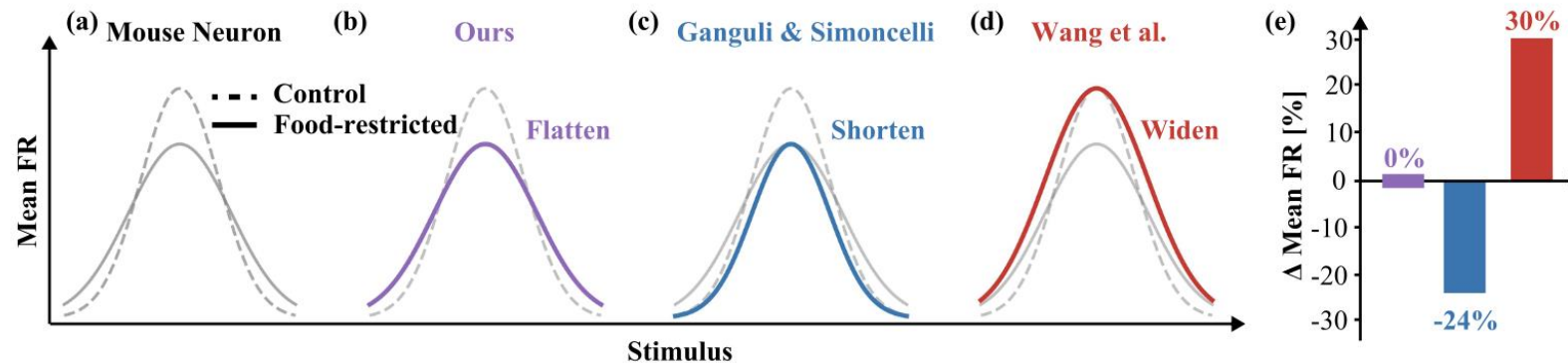
$$\alpha = 1$$

$$\eta_{\kappa} = \eta_0 + c_1 / (\epsilon - c_1)$$



Analytical solution

	infomax
Optimized function	$f(x) = \log x$
Density (tuning width) ⁻¹ $d(s)$	$E^{\frac{1}{\alpha}} R(s)^{-1} p(s)$
Gain $g(s)$	$E^{\frac{1}{\alpha}}$



$$\operatorname{argmax}_{g(\cdot), d(\cdot)} \int p(s) f(\eta_{\kappa}(E)^{-1} g(s) d(s)^2) ds,$$

$$\text{s.t. } p(s) \frac{g(s)}{d(s)} = R(s)$$

$$\int p(s) g(s)^{\alpha} ds = E, \quad \text{where } \alpha \geq 1.$$

Conclusion

- We introduce **two mathematically-tractable constraints** that more accurately characterize real neurons' response to metabolic stress.
- Our framework **generalizes previous models**, and can recover disparate results from the literature
- **Biophysical simulation** grounds key parameters of our model.

