Mathesis:

Towards Formal Theorem Proving from Natural Languages

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github.com/Huawei-AI4Math/Mathesis

Problem & Motivation

The Gap

- Real-world math problems are in natural language
- ATPs (DeepSeek-Prover, Kimina, Goedel) require formal statements in Lean 4
- Manual formalization demands deep expertise and significant effort
- Autoformalization errors lead to misleading proofs or unprovable statements
- Semantically equivalent formalizations can vary significantly in proof difficulty

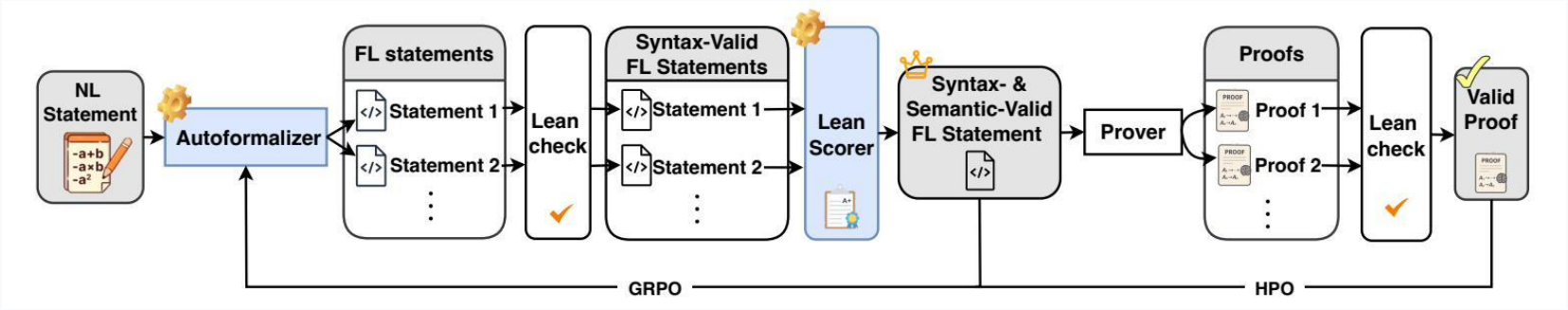
Informal Statement: Let S_n denote the sum of the first n terms of the sequence a_n. Given that $\frac{2S_n}{n} + n = 2a_n + 1$, prove that $\{a_n\}$ is an arithmetic sequence.

Two Formal Statement Cases:

```
theorem case_one (a : ℕ → ℝ)
(ha : ∃ d, ∀ n, a (n + 1) = a n + d) ⇒ Erroneously includes the desired goal in
assumptions
(h : ∀ n, 2 * (∑ i in Finset.range n, a i) / n + n = 2 * a n + 1) :
∃ d, ∀ n, a (n + 1) = a n + d := by sorry

theorem case_two (a : ℕ → ℝ) (S : ℕ → ℝ)
(hS : ∀ (n : ℕ), n ≥ 1, S n = ∑ k in Finset.range n, a k) ⇒
∑ k in Finset.Icc 1 n, a k
(h : ∀ (n : ℕ), n ≥ 1, 2 * S n / (n : ℝ) + (n : ℝ) = 2 * a n + 1) :
∃ (d : ℝ), ∀ (n : ℕ), n ≥ 1, a (n + 1) = a n + d := by sorry
```

Pipeline Overview



1 Autoformalization

NL → Lean 4 candidates
via Mathesis-Autoformalizer
(RL-trained with HPO)

2 Validation

Lean compiler (syntax) +
LeanScorer (semantics)

- Continuous score, not binary
- Decompose the task for fine-grained evaluation

3 Proving

Validated statements →
downstream provers →
machine-verified proofs

Mathesis-Autoformalizer: RL Training

STAGE 1

GRPO + Composite Rewards

Semantic reward R_sem: LLM evaluator judges if meaning is preserved

Syntactic reward R_ver: Lean 4 compiler verifies type validity

Final reward r = R_sem + R_ver (simple but effective)

Optimization GRPO normalizes within-group preferences over G candidates

STAGE 2

DPO for Prover Alignment

Preferences derived from proof success/failure

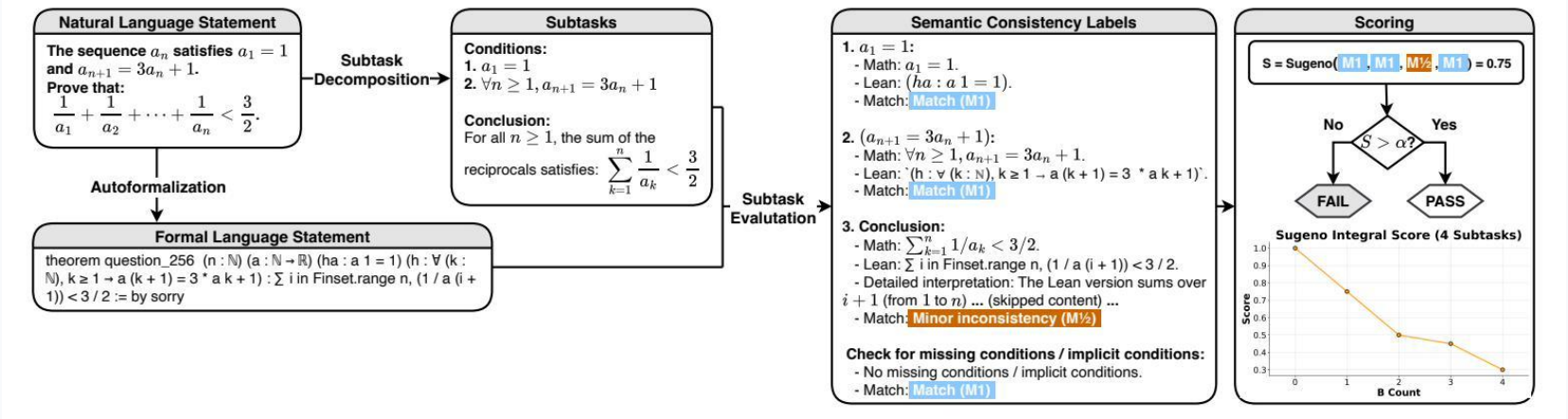
Preferred formalizations leading to Lean-verified proofs

Rejected formalizations where the prover fails

Config β=0.1 KL regularization, single epoch

HPO = GRPO (strong init) → **DPO** (proof-aligned tuning)

LeanScorer: Fine-Grained Semantic Evaluation



Three-Level Labeling

- **M1** Match — exact alignment
- **M½** Minor Inconsistency
- **M0** Major Inconsistency

Aggregated via Sugeno Fuzzy Integral

Evaluation vs Human Annotations

Method	Prec	Rec	F1
LLM-as-a-Judge	73	**100**	0.85
Re-informalization	93	50	0.65
LeanScorer (Ours)	**94**	89	**0.92**

Results: Autoformalization Quality

*LC = Lean Check
LSC = LeanScorer Semantic Check

		MiniF2F-Test		Putnam		Gaokao-Formal	
Model	**k**	**LC**	**LC+LSC**	**LC**	**LC+LSC**	**LC**	**LC+LSC**
API Models							
GPT-4o	6	80	65	24	9	48	28
DeepSeek-V3	6	91	84	22	10	69	56
DeepSeek-R1	6	86	76	58	**37**	81	57
Open-Source 7B							
Herald-AF	6	95	69	64	9	78	27
Kimina-AF	6	100	91	30	10	91	49
Mathesis-HPO	**6**	**100**	**96**	**73**	30	**98**	**71**

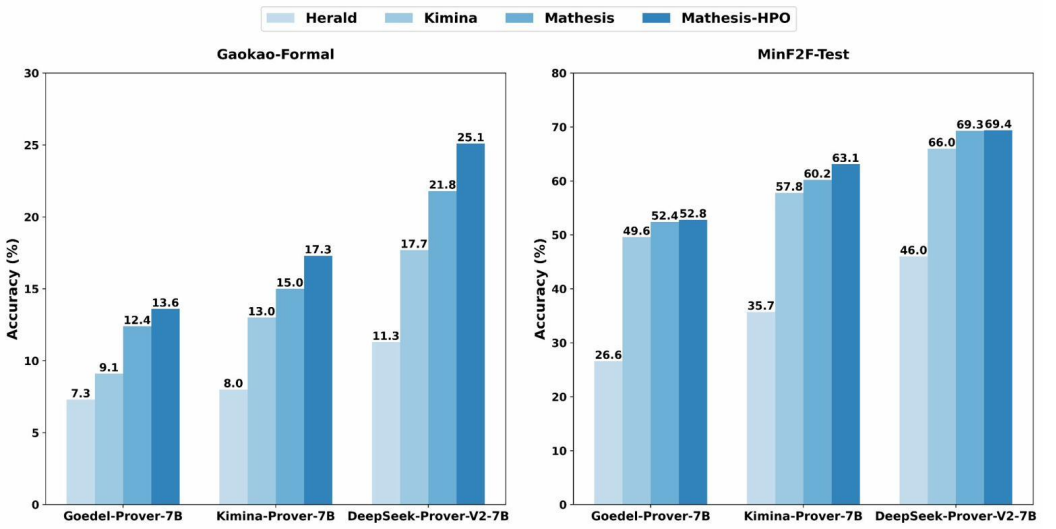
+5 pts
on miniF2F vs Kimina

+20 pts
on Putnam vs Kimina

+22 pts
on Gaokao vs Kimina

7B > API
outperforms GPT-4o, R1, V3

End-to-End Theorem Proving Results



Key Findings

11%-25% Herald → Mathesis-HPO for DSP-V2 on Gaokao

+122%

17%-25% Kimina → Mathesis-HPO for DSP-V2 on Gaokao

+42%

Key Insight

Improving the autoformalizer could yields **larger accuracy gains** than upgrading the prover itself.

Takeaways

01 First systematic study of formal theorem proving from natural language

02 Mathesis-Autoformalizer: first RL-trained autoformalizer with HPO, achieving SOTA

03 LeanScorer: fine-grained semantic evaluation framework (0.92 F1)

04 Gaokao-Formal: new benchmark of 495 complex proof problems

05 Formalization quality is the key bottleneck — more impactful than upgrading the prover

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Thank you!