Zero-shot Human Pose Estimation using Diffusion-based Inverse solvers

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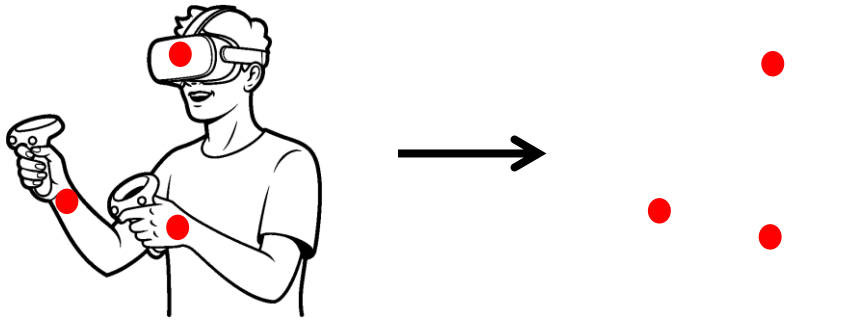
Problem Statement



Real-world use case

Sparse motion measurement

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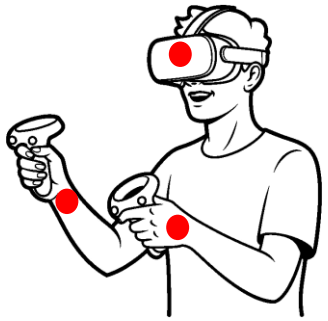

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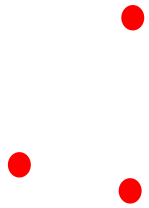
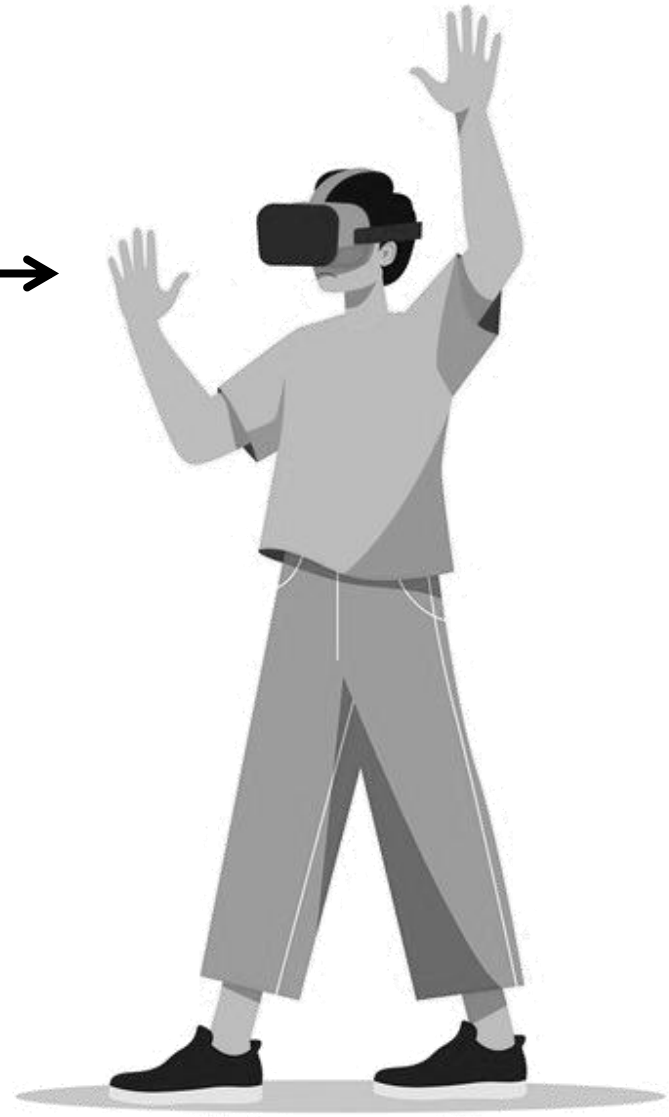


Sparse motion
measurement

InPose
(our algorithm)

Full body
3D pose



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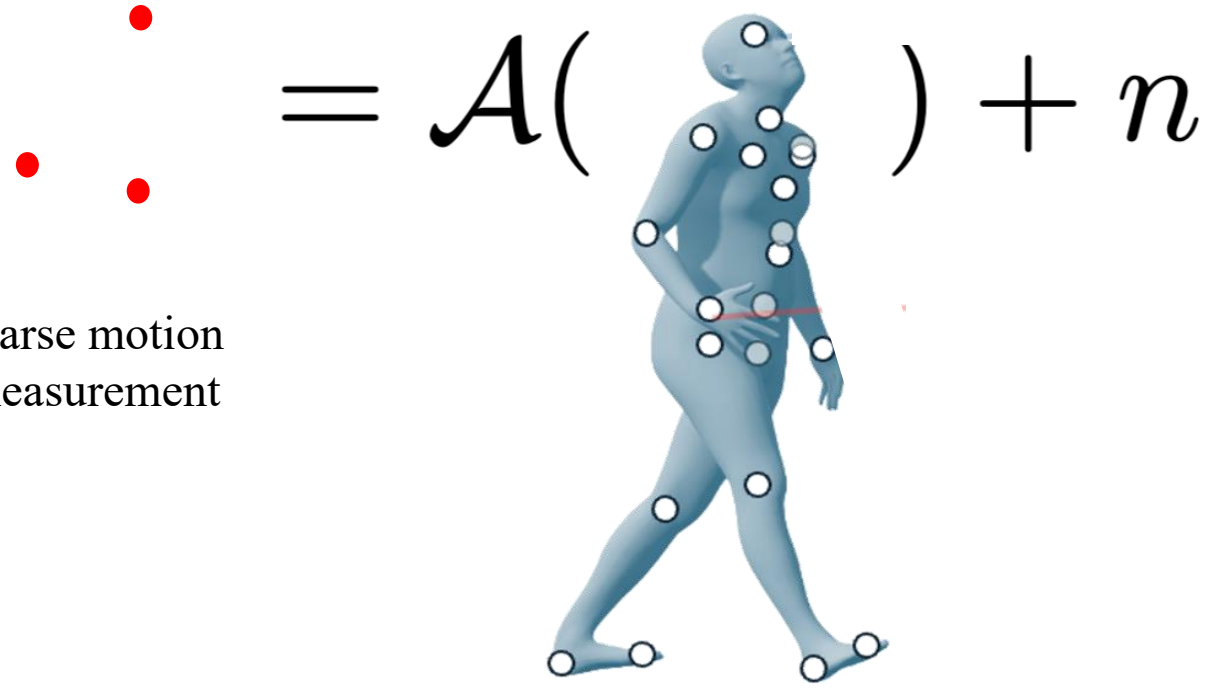
Formulate as an inverse problem



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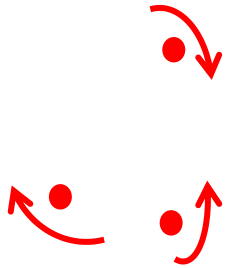
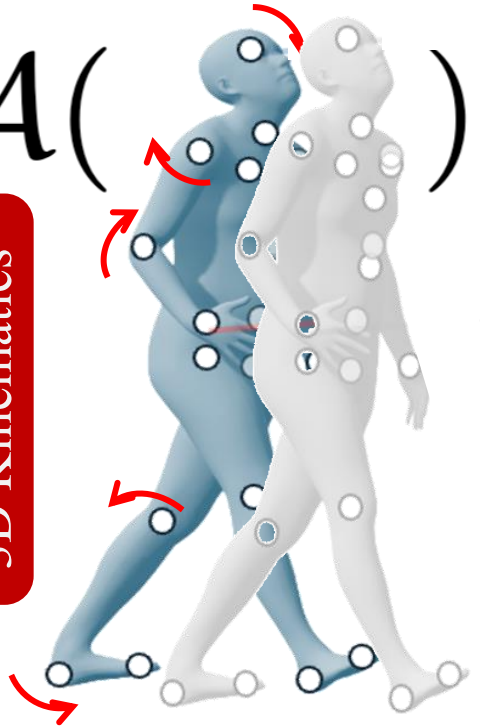


Sparse motion measurement

$$= \mathcal{A}(\quad) + n$$

3D Kinematics



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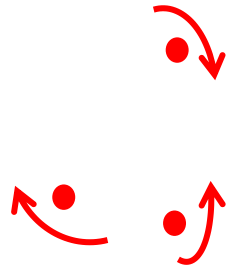

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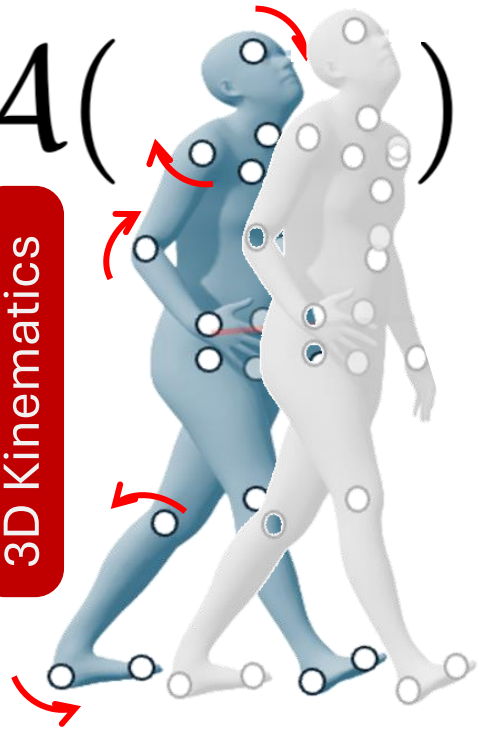

Full body 3D pose

We will solve this inverse problem using generative priors (from diffusion models)

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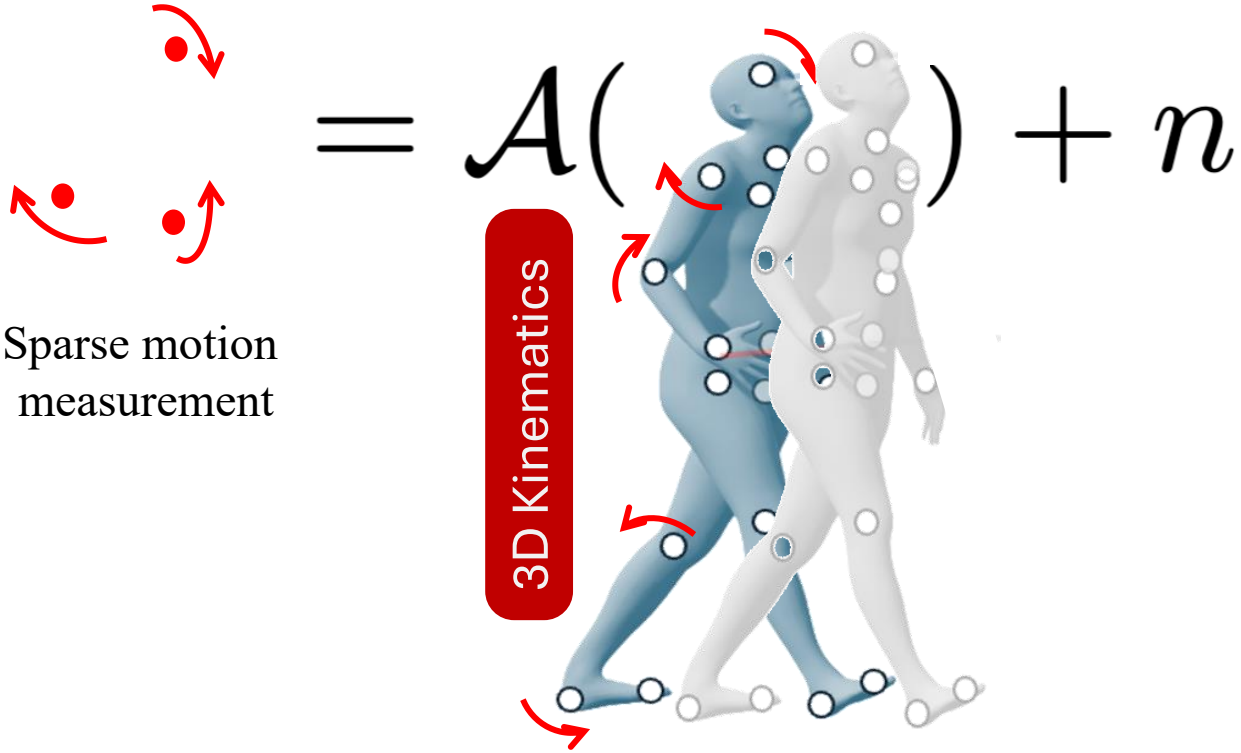

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Generalizable to any body shape (with no user specific training)

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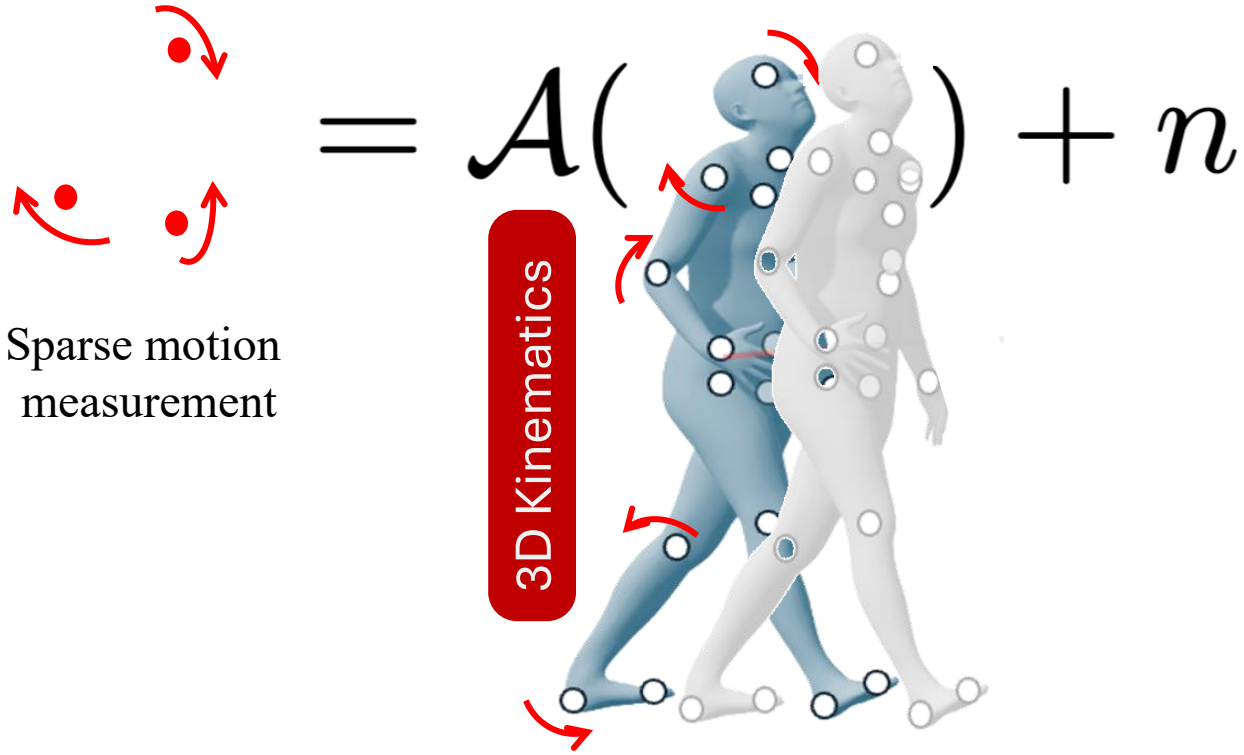
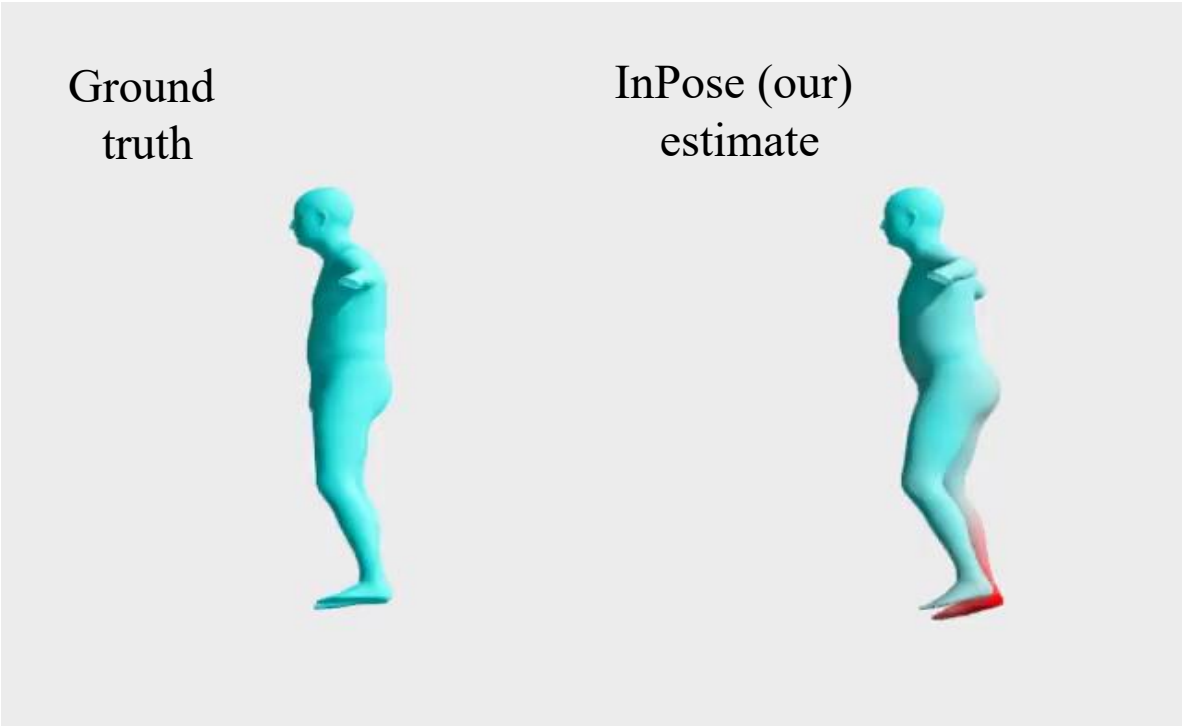


… and show results as follows:



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Roadmap

- Problem formulation
  - Kinematic tree
  - Scale dependent vs. scale free pose
  - 3-point pose estimation
- Learning representations
  - Rotation representation
  - 6D representation
- Background on Diffusion
  - Score function
  - Guided diffusion and Posterior sampling
  - Pseudo-inverse based posterior sampling (ΠGDM)
- InPose: Our work
  - Limitations of past work
  - Key intuition: learning scale free parameters
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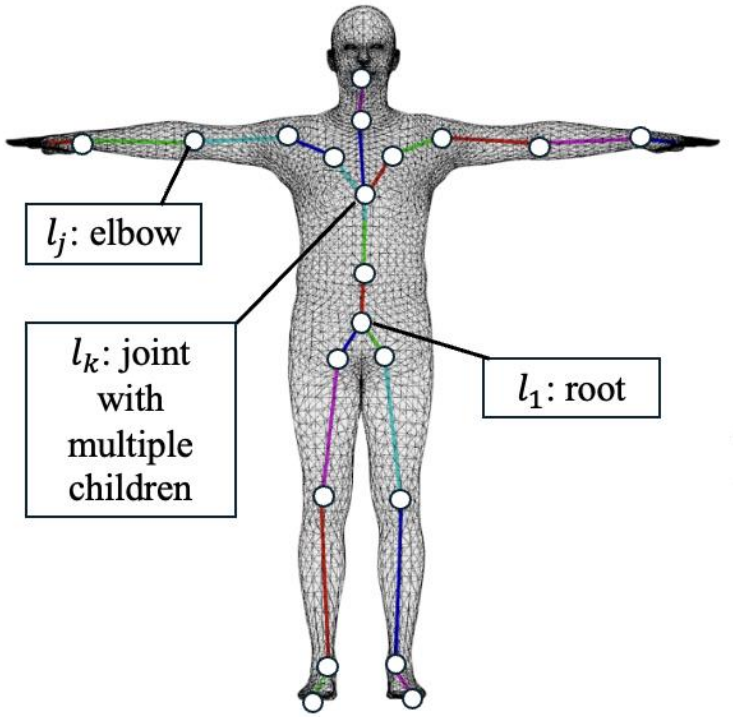
Pose Estimation

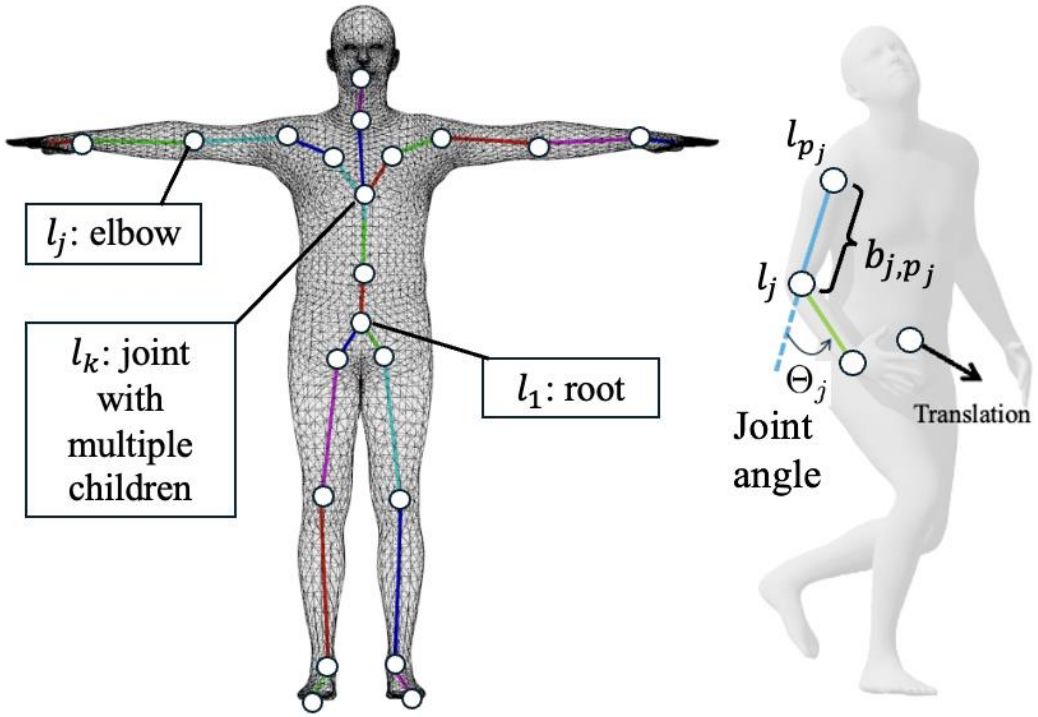
- Pose refers to the state of all joints l_j, $j \in M$ typically, 22 in number.



Pose Estimation

- Pose refers to the state of all joints l_j, $j \in M$ typically, 22 in number.
- Joints have angle $\Theta_j \in SO(3)$ and bone $b_{j,p_j} \in \mathbb{R}^3$ attached to parent p_j.
- Joint states are thus coordinate transforms:

$$ {}^{p_j}T_j = \begin{bmatrix} \Theta_j & b_{j,p_j} \\ 0^{1\times 3} & 1 \end{bmatrix} $$



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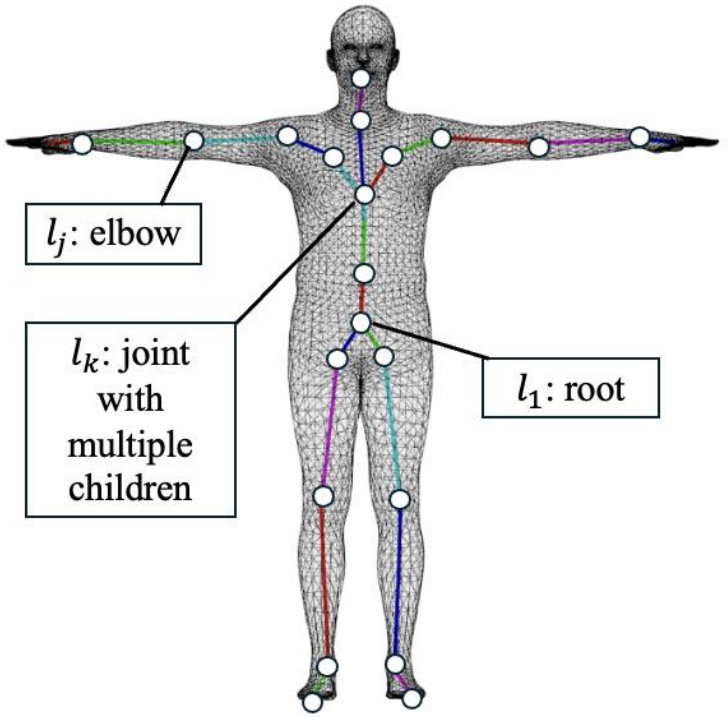
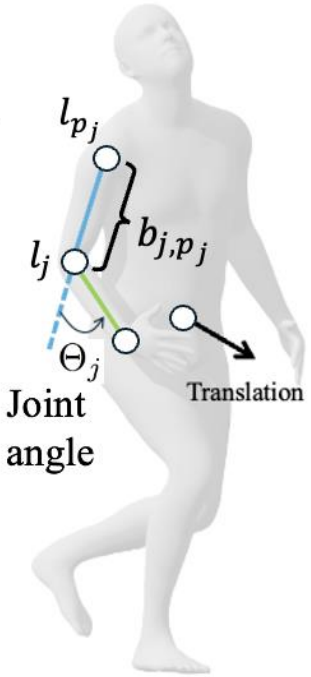
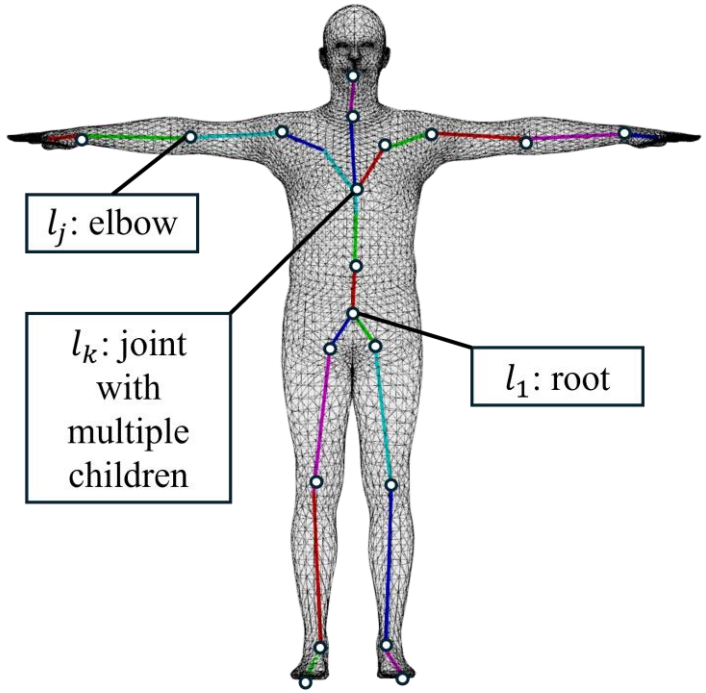




People with different body shapes have different bone lengths for b_{j,p_j}

Kinematic Tree

- Joints define a directed acyclic graph - joints as vertices, parent-joint pairs as edges, with a single parent for each joint.



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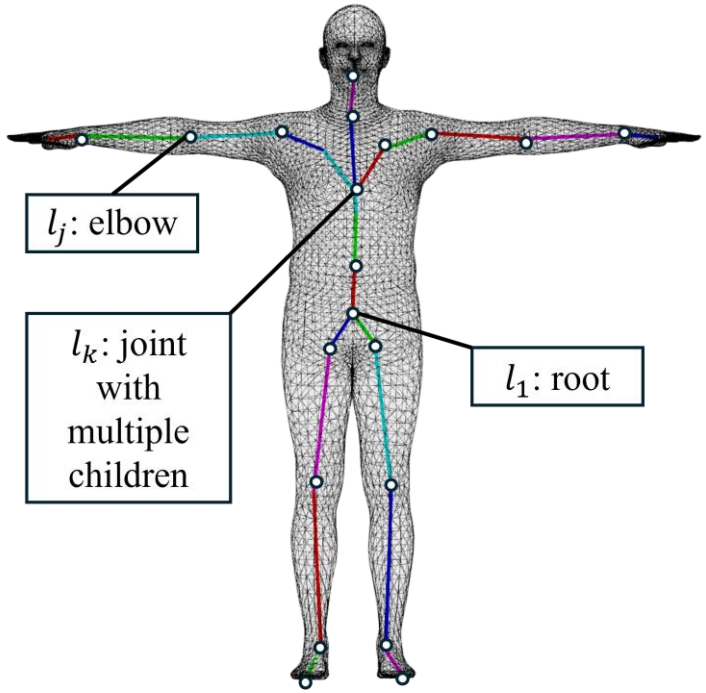
- Thus, we can define a kinematic tree with the pelvis at the root (with state ${}^{G}T_1$).



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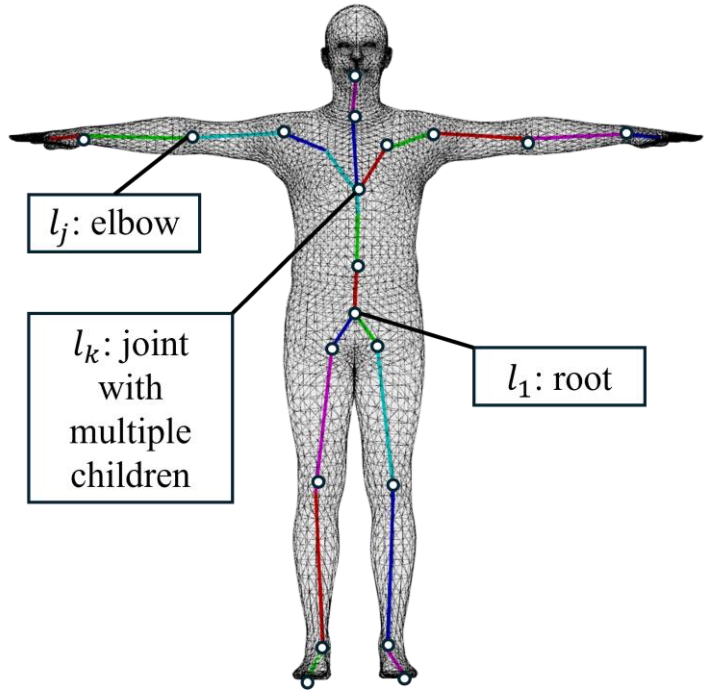
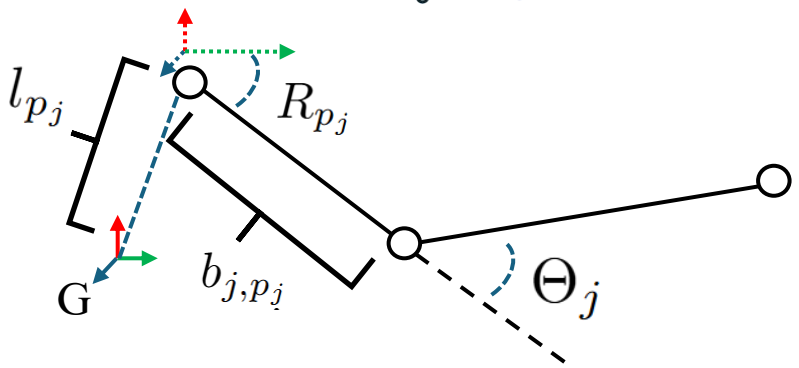
- Thus, we can define a kinematic tree with the pelvis at the root (with state ${}^{G}T_1$).

- Useful to recursively calculate joint state in global frame:

$$ {}^{G}T_j = {}^{G}T_{p_j} \cdot {}^{p_j}T_j, \qquad {}^{G}T_j = \begin{bmatrix} R_j & l_j \\ 0^{1\times 3} & 1 \end{bmatrix} $$

- We term $l_j \in \mathbb{R}^3$ joint location and $R_j \in SO(3)$ the global joint rotation.



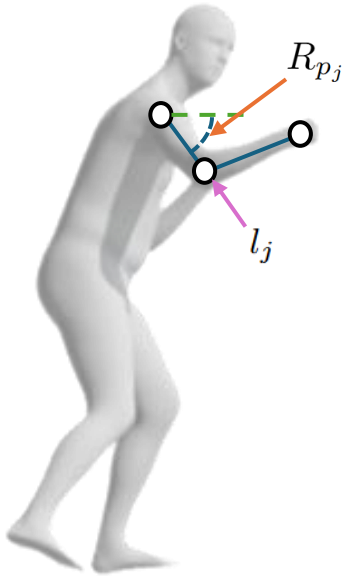


Scale-dependent vs Scale-free Pose

- Using the tree, we can recursively track joint locations and global joint rotations:

$$R_j = R_{p_j} \cdot \Theta_j \qquad (1a)$$

$$l_j = l_{p_j} + R_{p_j} \cdot b_{j,p_j} \qquad (1b)$$



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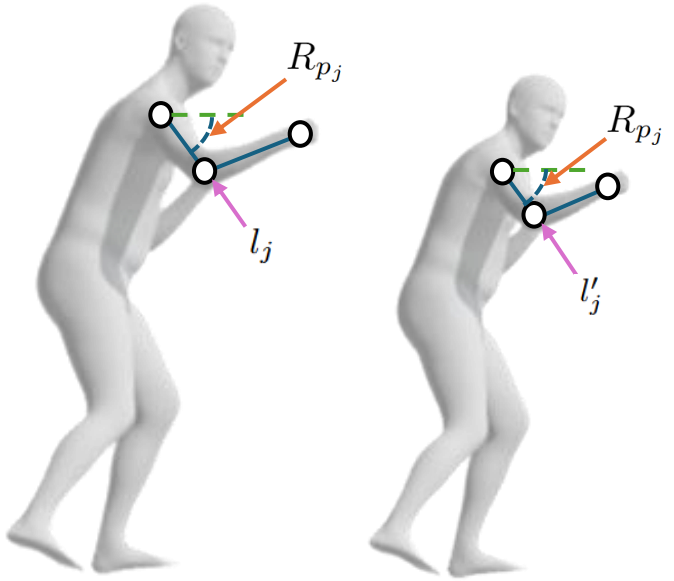
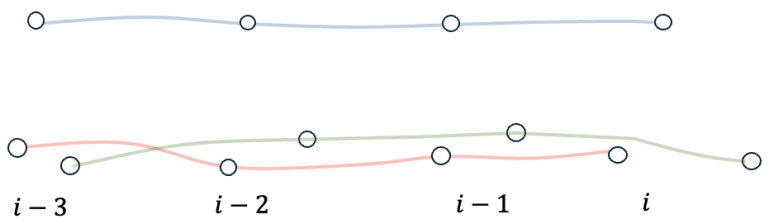
- Users in the same "Pose" have same joint angles regardless of body shape (or bones b_{j,p_j}).
- We thus call joint angles $\{R_j\}$ Scale-free pose and joint locations $\{l_j\}$ Scale-dependent pose.



3-Point Pose Estimation

- AR/VR – Estimate Full Body Pose from underdetermined sensing.
- Observations - $y_m(i) = [l_m(i), R_m(i)]$, where

  m := {head, left wrist, right wrist} joints.



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- Observations - $y_m(i) = [l_m(i), R_m(i)]$, where

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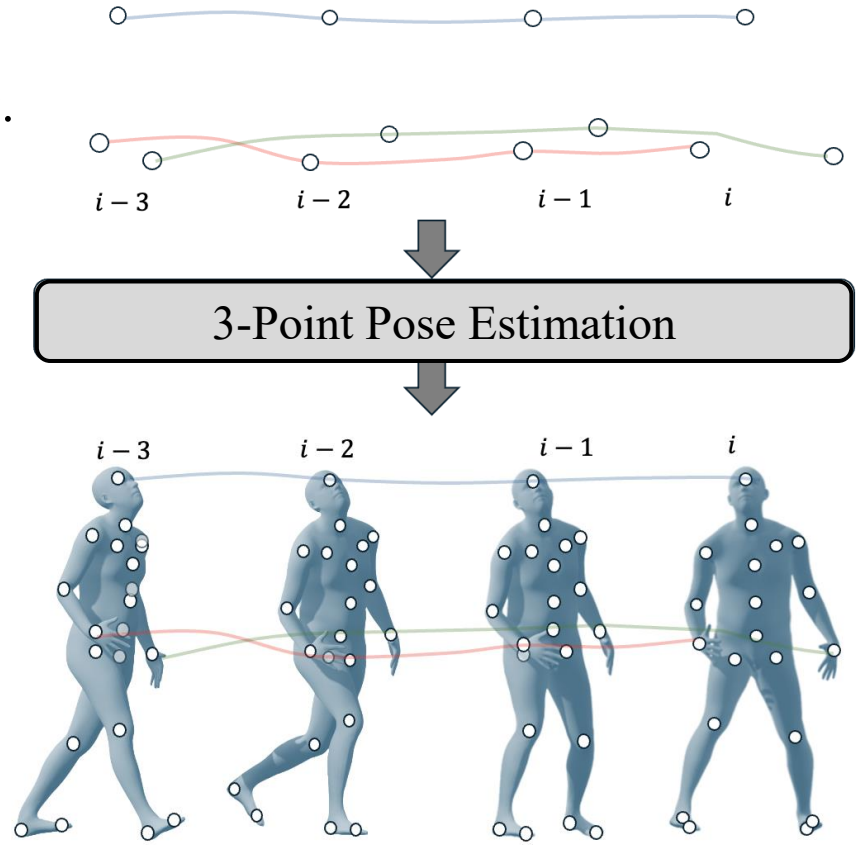
- Estimate $\{l_M(i), R_M(i)\}$ from $y_m(i)$

- Measurements are assumed to be contaminated by Gaussian Noise: $l_m(i) = l_m^+(i) + \sigma_l v(i)$



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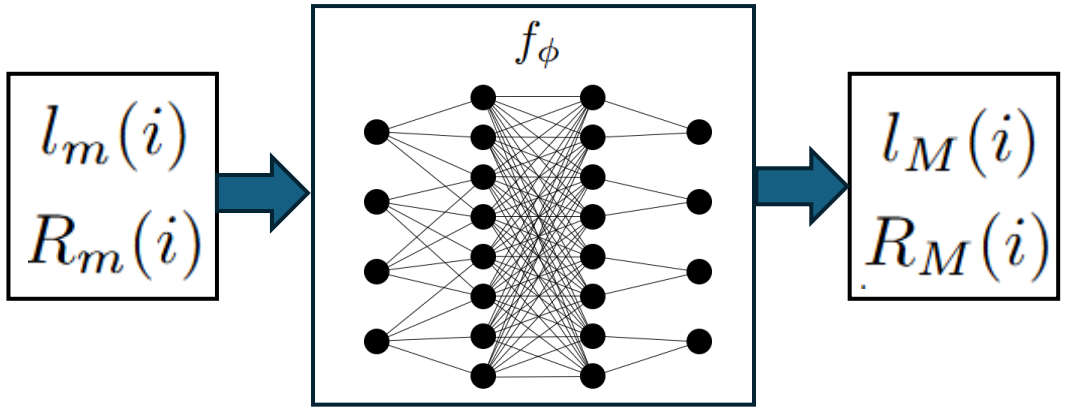
Deep Learning for 3-Point Pose Estimation

- Learn a Neural Network f_ϕ to predict pose:

$$\{l_M(i), R_M(i)\} = f_\phi(l_m(i), R_m(i))$$



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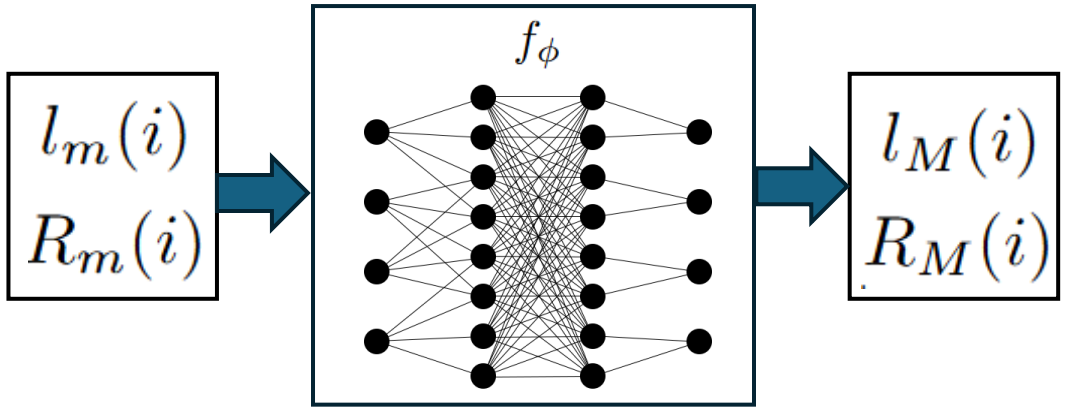
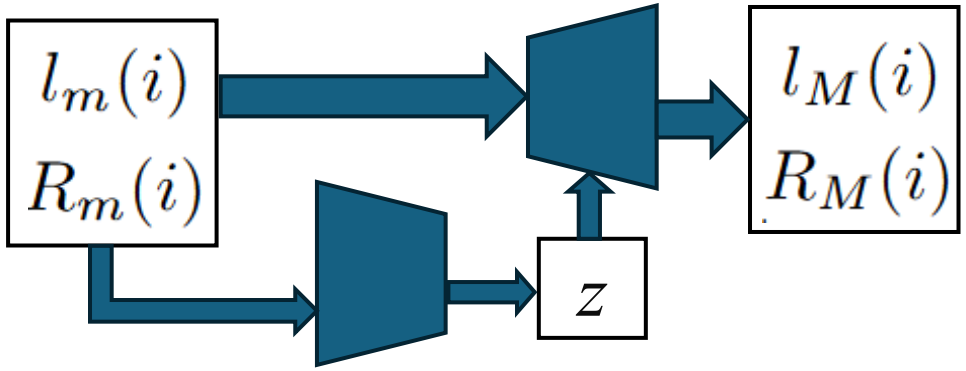


- Use a Conditional VAE to learn the Posterior:

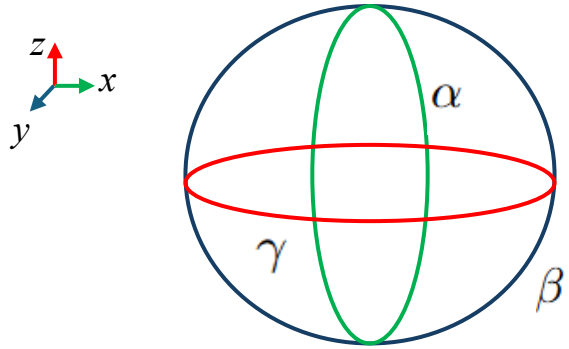
$$\underset{l_M(i), R_M(i)}{\arg\max}\ p(l_M(i), R_M(i) | l_m(i), R_m(i))$$



- Guided Diffusion, etc.

Rotation Representation for Deep Learning

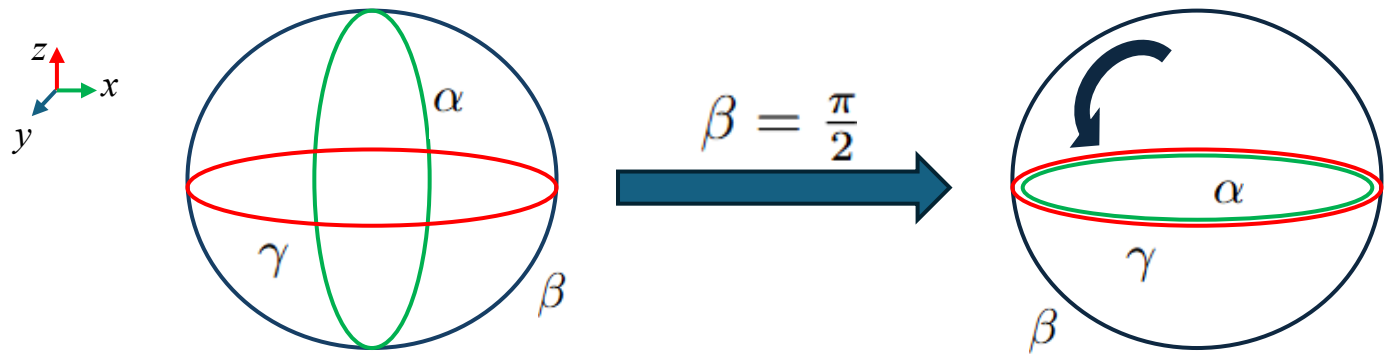
- Options To represent Rotations – Euler, Quaternion, Rotation Matrix
- Issues with various representations:
  - Euler angles



Rotation Representation for Deep Learning

- Options To represent Rotations – Euler, Quaternion, Rotation Matrix
- Issues with various representations:
  - Euler angles – Gimbal lock
  - E.g. at $\alpha, \beta = \frac{\pi}{2}, \gamma$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sin(\alpha+\gamma) & \cos(\alpha+\gamma) & 0 \\ -\cos(\alpha+\gamma) & \sin(\alpha+\gamma) & 0 \end{bmatrix}$$



Rotation Representation for Deep Learning

- Issues with various representations:
  - Quaternion –

$$q = [w \quad ix \quad jy \quad kz]^\top$$

$$q(R) = \begin{bmatrix} \frac{1}{2}\sqrt{1 + R_{11} + R_{22} + R_{33}} \\ i\ \text{sgn}(R_{32} - R_{23})\left|\frac{1}{2}\sqrt{1 + R_{31} + R_{22} + R_{33}}\right| \\ j\ \text{sgn}(R_{13} - R_{31})\left|\frac{1}{2}\sqrt{1 - R_{31} + R_{22} - R_{33}}\right| \\ k\ \text{sgn}(R_{21} - R_{12})\left|\frac{1}{2}\sqrt{1 - R_{31} - R_{22} + R_{33}}\right| \end{bmatrix}$$

Rotation Representation for Deep Learning

- Issues with various representations:
  - Quaternion – Double Cover

    (Discontinuous representation)

$$q = [w \quad ix \quad jy \quad kz]^\top$$

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  - E.g. Rotation by $\gamma = \pi$

$$R(\gamma) = \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$q(R(\pi-)) = [0 \quad 0 \quad 0 \quad 1]^\top$$

$$q(R(\pi+)) = [0 \quad 0 \quad 0 \quad -1]^\top$$

6D Rotation Representation

- Issues with various representations:
  - Rotation matrices – a predicted matrix

    $R = [c^1\ c^2\ c^3]$ may not necessarily have orthonormal

    columns, e.g. $c^1 \times c^2 \neq c^3$

- Columns need to be postprocessed.

6D Rotation Representation

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- Directly use the first 2 columns (6D)[1]:

$$r(R) = [c^{1\top}, c^{2\top}]^\top \in \mathbb{R}^6$$

[1] Yi et al. "On the continuity of rotation representations in neural networks." (CVPR), June 2019

6D Rotation Representation

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- Columns need to be postprocessed.
- Directly use the first 2 columns (6D)[1]:

$$r(R) = [c^{1\top}, c^{2\top}]^\top \in \mathbb{R}^6$$

- Then, use the non-linear function $\mathcal{D}()$ to recover $R_j(i)$
- Most SoTA works use this representation

$$R_j(i) = \mathcal{D}(r_j(i))$$

$$c^1 = \left[r_j^{1:3}(t)\right]^\top,$$

$$c^2 = \left[r_j^{4:6}(t)\right]^\top - \text{proj}_{\bar{c}^1}\left(\left[r_j^{4:6}(t)\right]^\top\right),$$

$$\bar{c}^1 = \frac{c^1}{||c^1||} \quad \bar{c}^2 = \frac{c^2}{||c^2||} \quad \bar{c}^3 = \bar{c}^1 \times \bar{c}^2$$

$$R_j(t) = \left[\bar{c}^1 \quad \bar{c}^2 \quad \bar{c}^3\right]$$

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Diffusion Models

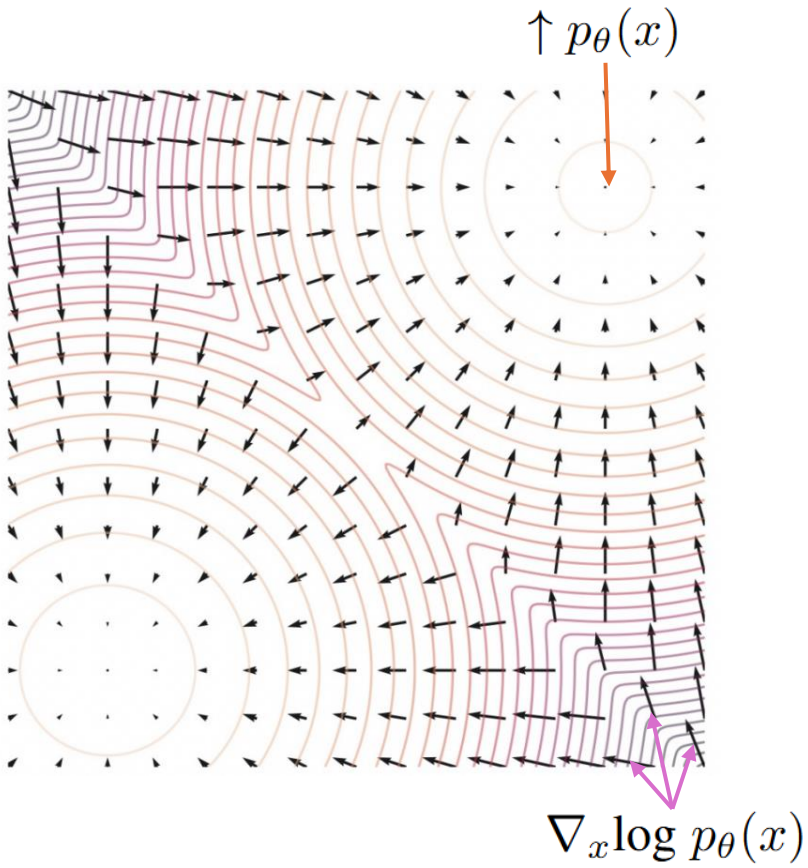
- Let's say we would like to learn a prior $p_\theta(x)$ for some dataset, with $x \in \mathbb{R}^D$
- Can be used to sample from this distribution

Diffusion Models

- Let's say we would like to learn a prior $p_\theta(x)$ for some dataset, with $x \in \mathbb{R}^D$

- We could learn the gradient field $\nabla_x \log p_\theta(x)$
- Points in the direction of higher probability.

We could start at some random point and travel along the gradient field till we reach a sample with high probability



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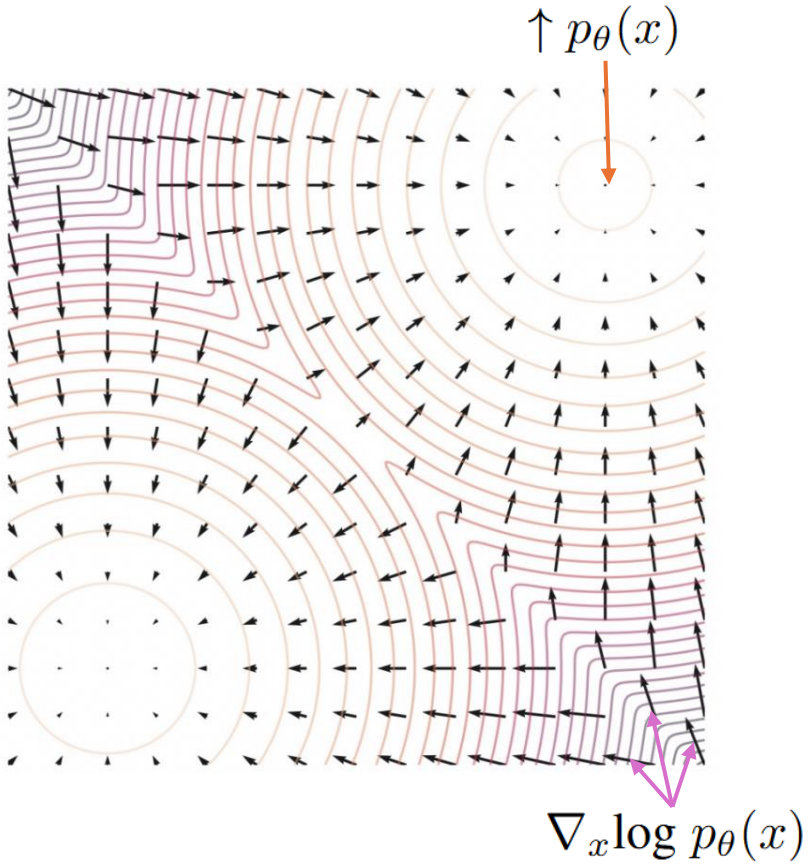
- e.g. Stochastic Gradient Langevin Dynamics:
  - Sample from some easy to sample prior $x_T \sim \pi(x)$
  - Iterate with $z_t \sim \mathcal{N}(0, \mathrm{I}^D)$:

$$x_{t-1} = x_t + \epsilon \nabla_x \log p_\theta(x) + \sqrt{2\epsilon} z_t$$

  - As $\epsilon \to 0,\ T \to \infty$ we find $x_0 \sim p_\theta(x)$



How do we learn the gradient field $\nabla_x \log p_\theta(x)$, which is called the Score function

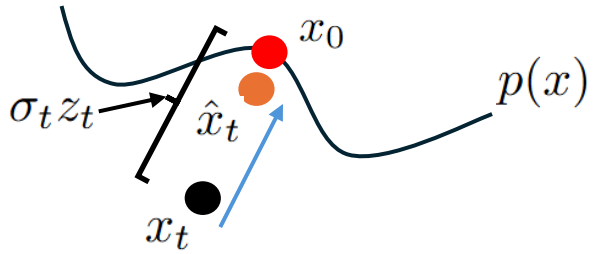
Score Function

- Let's say we noise a sample x_0 with $z_t \sim \mathcal{N}(0, \mathrm{I}^D)$

$$x_t = x_0 + \sigma_t z_t$$

- The MMSE estimate of x_0 is $\hat{x}_t = \mathbb{E}[x_0|x_t]$

- The MMSE can be thought of as a denoiser.

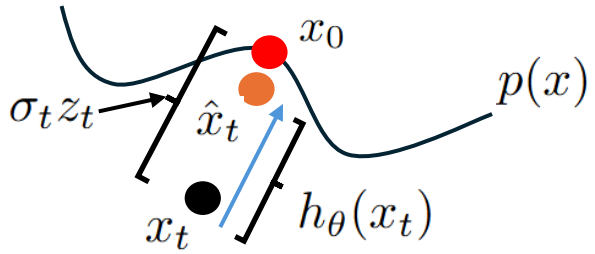


We have a rich body of literature studying denoisers

Score Function

- Let's say we noise a sample x_0 with $z_t \sim \mathcal{N}(0, \mathrm{I}^D)$

$$x_t = x_0 + \sigma_t z_t$$



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- The MMSE can be thought of as a denoiser.
- We have a rich body of literature studying denoisers.
- Let's learn a Denoiser $h_\theta(x_t)$, using a technique called Denoising score matching(DSM):

$$L_{\text{DSM}}(\Theta) = \mathbb{E}_{x_0, x_t}[||h_\theta(x_t) - x_0||^2]$$

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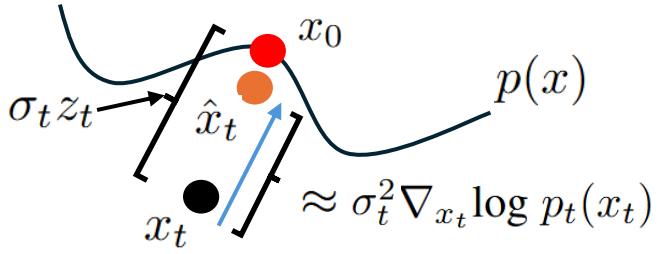


- There exists an important statistical result, called Tweedie's formula that states: $\nabla_{x_t} \log p_t(x_t) = \dfrac{\mathbb{E}[x_0|x_t] - x_t}{\sigma_t^2}$

Use $h_\theta(x_t)$ to approximate the score function:

$$\nabla_{x_t} \log p_t(x_t) \approx \frac{h_\theta(x_t) - x_t}{\sigma_t^2}$$

Algorithms for Diffusion

- Class of Generative Modeling algorithms that exploit the score function.

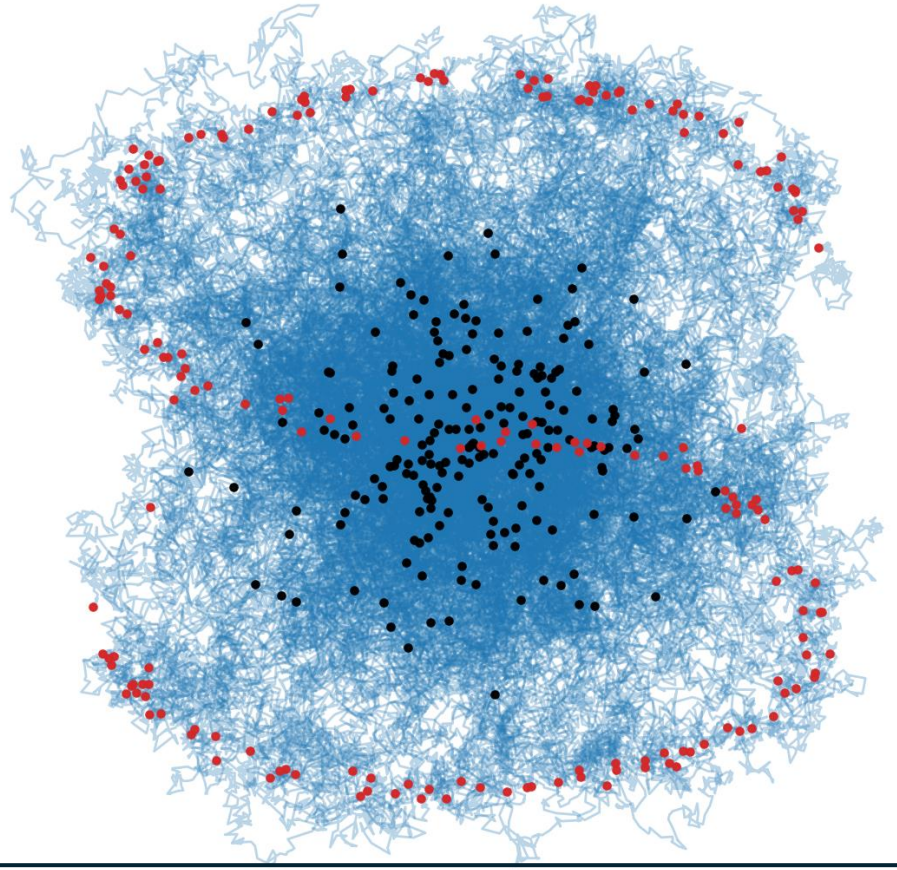
- Some important Diffusion algorithms:
  - Stochastic Gradient Langevin Dynamics.
  - Denoising Diffusion Probabilistic Models (DDPM).

DDPM (Stochastic)

$\sim p_\theta(x)$

$\sim \pi(x)$

Sample Path



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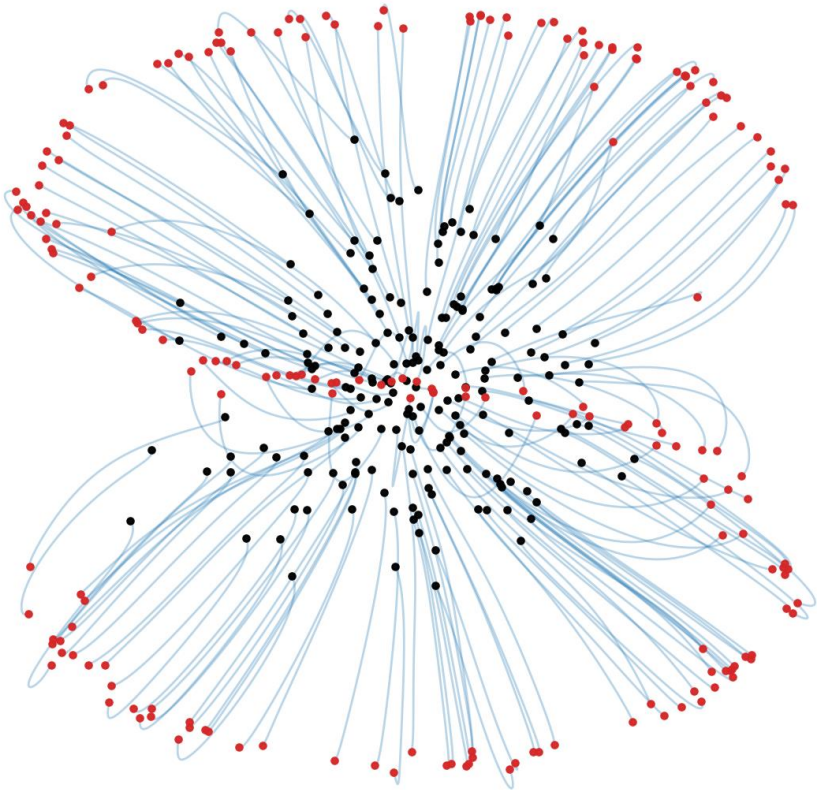

Algorithms for Diffusion

- Class of Generative Modeling algorithms that exploit the score function.

- Some important Diffusion algorithms:
  - Stochastic Gradient Langevin Dynamics.
  - Denoising Diffusion Probabilistic Models (DDPM).
  - Denoising Diffusion Implicit Models (DDIM).
  - Second order ODE solvers such as the Heun solver.

DDPM (Stochastic)

$\sim p_\theta(x)$
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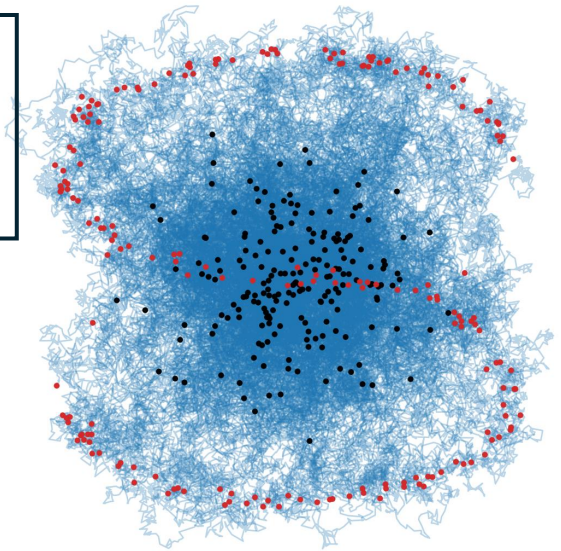
Sample Path



DDIM (Deterministic)

$\sim p_\theta(x)$
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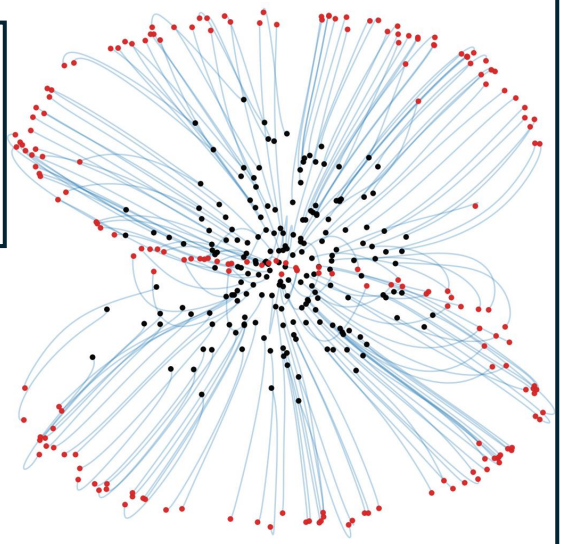
Sample Path



Guided Diffusion

- Let's say we are given a linear measurement $y = Ax_0 + n$, where $n \sim \sigma_y \mathcal{N}(0, \mathrm{I})$.
- We want samples $x_0 \sim p_\theta(x)$ that also belong to a Data distribution.
- In other words - sample from the posterior $p(x|y)$.

Can we use the Diffusion framework itself?

- Option 1: Train a conditional denoiser for Diffusion
  - Called Classifier-free Guidance (CFG):

Posterior Score

- Let's say we are given a linear measurement $y = Ax_0 + n$

- Option 2: Use the posterior score in the Diffusion framework:

$$\nabla_{x_t} \log p_t(x_t|y)$$

- Posterior Score – Substitute the prior score with $\nabla_{x_t} \log p_t(x_t|y)$

Can we derive $\nabla_{x_t} \log p_t(x_t|y)$ from the prior score without any training?

Posterior Score

- Option 2: Use the posterior score in the Diffusion framework: $\nabla_{x_t} \log p_t(x_t|y)$

- Using Bayes Rule, we can split the posterior score:

$$\nabla_{x_t} \log p_t(x_t|y) = \underbrace{\nabla_{x_t} \log p_t(x_t)}_{\text{Prior Score}} + \underbrace{\nabla_{x_t} \log p_t(y|x_t)}_{\text{Likelihood Score}}$$

- To estimate the Likelihood Score, we have:

$$p_t(y|x_t) = \int_{x_0} p(y|x_0) p(x_0|x_t) dx_0$$

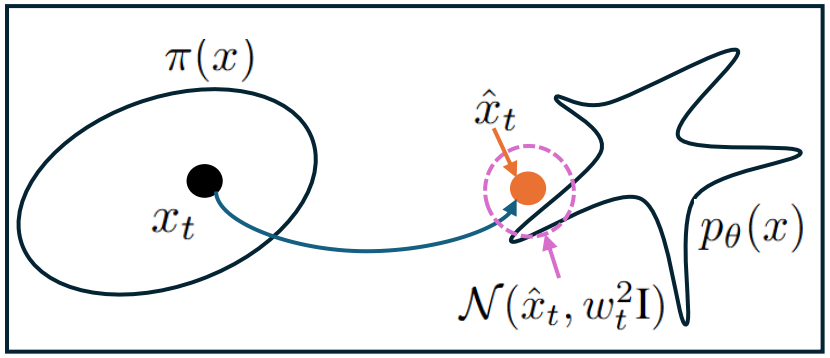
- We know $p(y|x_o) \sim \sigma_y \mathcal{N}(Ax_0, \mathrm{I})$.

How do we approximate $p(x_0|x_t)$?

Pseudoinverse-Guided Diffusion Models

- We use the Pseudoinverse-Guided Diffusion Model (ΠGDM) approximation[2].
- Use an isotropic Gaussian Approximation for $p(x_0|x_t) \sim \mathcal{N}(\hat{x}_t, w_t^2\mathbf{I})$

  where w_t^2 is a data dependent parameter.



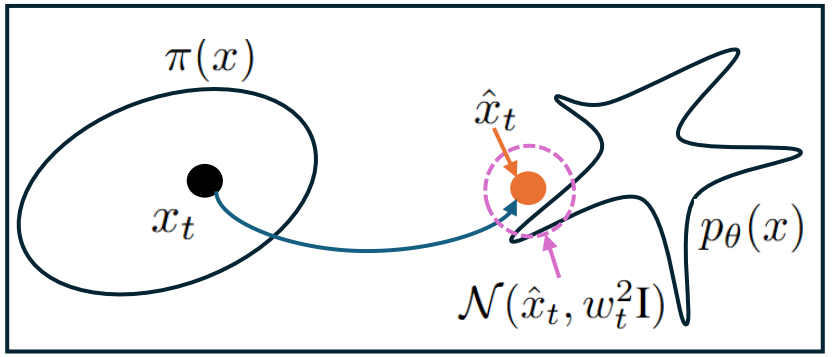
[2] Song et al. "Pseudoinverse-guided diffusion models for inverse problems." (ICLR), 2023

Pseudoinverse-Guided Diffusion Models

- We use the Pseudoinverse-Guided Diffusion Model (ΠGDM) approximation[2].
- Use an isotropic Gaussian Approximation for $p(x_0|x_t) \sim \mathcal{N}(\hat{x}_t, w_t^2\mathrm{I})$

  where w_t^2 is a data dependent parameter.
- Next, using the Gaussian Convolution theorem,

$$p_t(y|x_t) = \int_{x_0} p(y|x_0)p(x_0|x_t)dx_0$$
$$\sim \mathcal{N}(A\hat{x}_t, w_t^2 AA^\top + \sigma_y^2\mathrm{I})$$



- Finally, we can derive the likelihood score as:

$$\nabla_{x_t} \log p_t(y|x_t) \approx \underbrace{(y - A\hat{x}_t)(w_t^2 AA^\top + \sigma_y^2\mathrm{I})^{-1}A}_{\text{vector}} \underbrace{\frac{\partial \hat{x}_t}{\partial x_t}}_{\text{Jacobian}}$$

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Roadmap

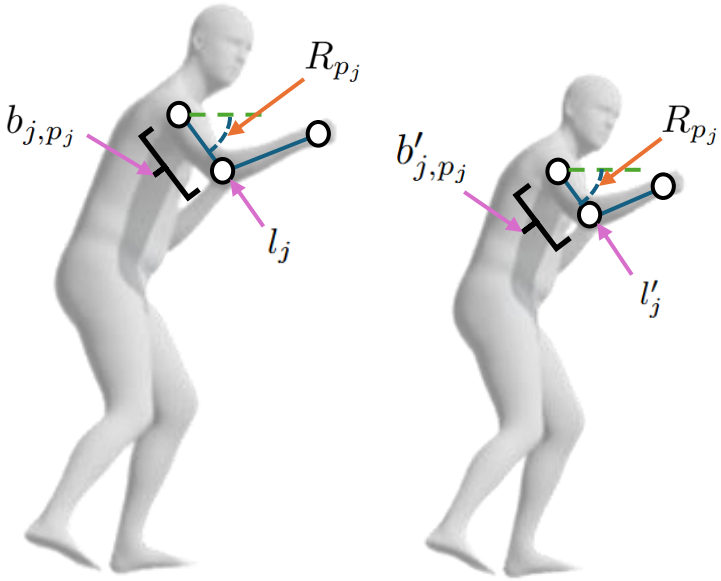
- Problem formulation
  - Kinematic tree
  - Scale dependent vs. scale free pose
  - 3-point pose estimation
- Learning representations
  - Rotation representation
  - 6D representation
- Background on Diffusion
  - Score function
  - Guided diffusion and Posterior sampling
  - Pseudo-inverse based posterior sampling (ΠGDM)
- InPose: Our work
  - Limitations of past work
  - Key intuition: learning scale free parameters
  - Diffusion priors and ΠGDM linear model
  - Theorem
  - InPose pipeline
  - Results

Limitations of Deep Learning Approaches

- E.g. Neural Network f_ϕ to predict pose:

$$\{l_M(i), r_M(i)\} = f_\phi(l_m(i), r_m(i))$$

- Issue – $\{l_j\}$ are a function of bones $\{b_{j,p_j}\}$
- Neural Network trained on one user's data can't generalize to a new user.



Can we reuse the same neural network without retraining?

Limitations of Deep Learning Approaches

- E.g. Neural Network f_ϕ to predict pose:

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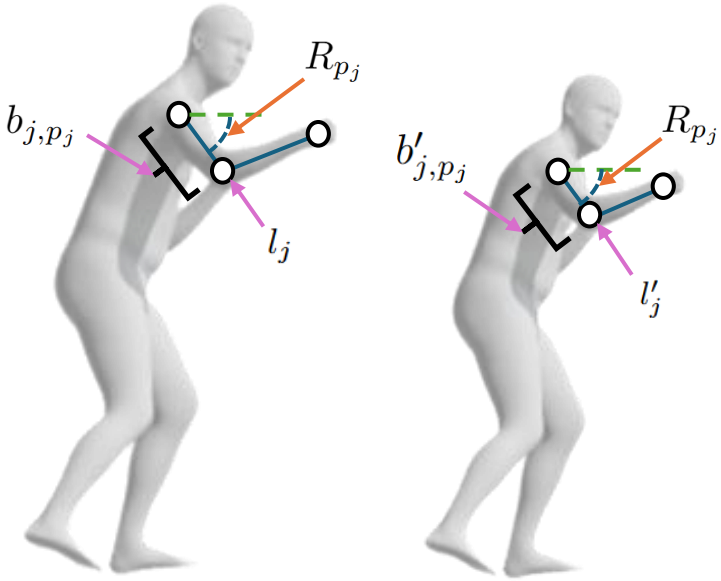
- Issue – $\{l_j\}$ are a function of bones $\{b_{j,p_j}\}$
- Neural Network trained on one user's data can't generalize to a new user.
- Use Guided Diffusion to overcome these issues.



Opportunity 1 – Use scale-free $r_m(i)$ for CFG

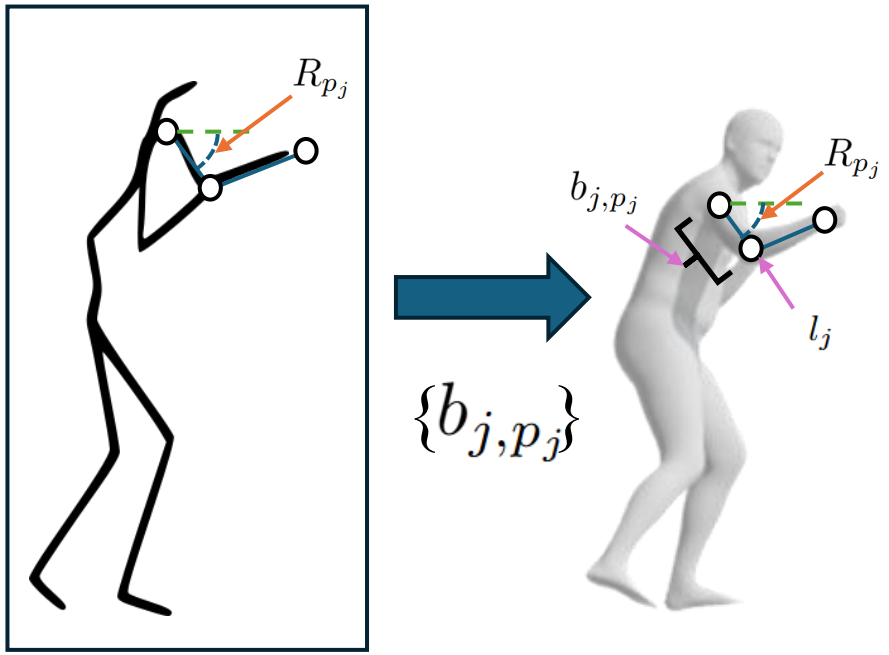
Opportunity 2 – Use ΠGDM to incorporate $l_m(i)$

Diffusion for 3-Point Pose Estimation

- Opportunity 3 – Predict the Scale-free Pose $\{R_j\}$

  .

- Use new user's $\{b_{j,p_j}\}$ to compute Scale-dependent Pose $\{l_j\}$.



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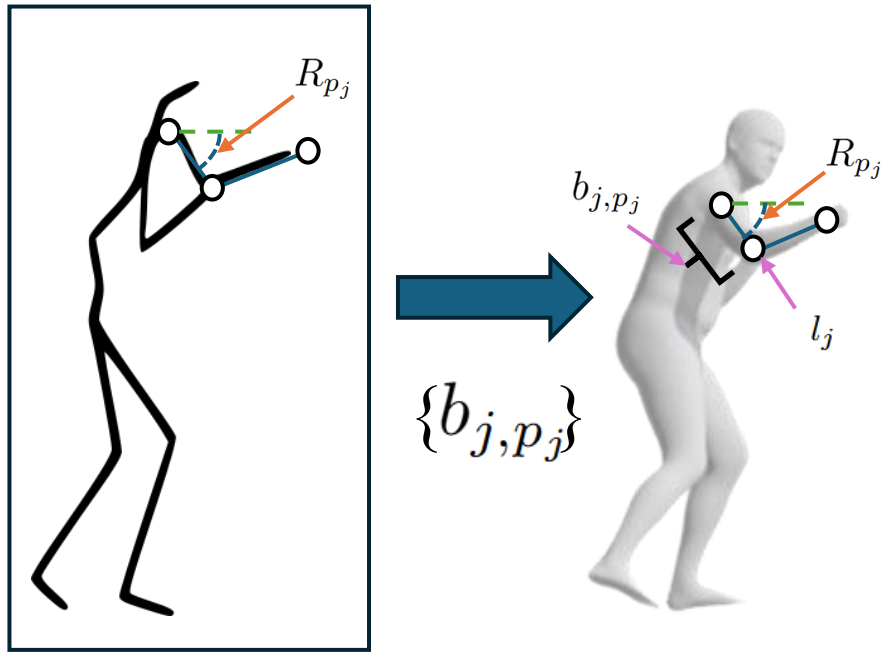

- We thus use the following score function:

$$\nabla_{r_M^t} \log p_t(r_M^t | l_m, r_m) = \underbrace{\nabla_{r_M^t} \log p_t(r_M^t | r_m)}_{\text{CFG Score}} + \underbrace{\nabla_{r_M^t} \log p_t(l_m | r_M^t, r_m)}_{\Pi\text{GDM Score}}$$

ΠGDM linear model

- Recall ΠGDM requires a linear measurement model $y = Ax_0 + n$ with $n \sim \sigma_y \mathcal{N}(0, \mathrm{I})$
- If the rotations $\{R_j\}$ were available to us, we can create the measurement model using Equation (1b) as follows:

$$[R_1 ... R_{p_j}] \cdot [b_{2,1}^\top ... b_{j,p_j}^\top]^\top = l_j$$

$$l_j = l_{p_j} + R_{p_j} \cdot b_{j,p_j} \qquad \text{(1b)}$$

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- Setting $C = [R_1 ... R_{p_j}]$ and $\kappa = [b_{2,1}^\top ... b_{j,p_j}^\top]^\top$, we get:

$$(\mathrm{I}_3 \otimes \kappa^\top) \cdot \mathrm{vec}(C) = l_j$$

where $\otimes$ is the Kronecker product.

- Our linear measurement function is:

$$\mathcal{A} := \mathrm{I}_3 \otimes \kappa^\top$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} b_{2,1}^\top ... b_{j,p_j}^\top \end{bmatrix}}_{\mathcal{A}} \bullet \underbrace{\begin{bmatrix} \mathrm{vec}([R_1 & \cdots & R_{p_j}]) \end{bmatrix}}_{x_0} = \underbrace{\begin{bmatrix} l_j \end{bmatrix}}_{y}$$

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- Our linear measurement function is thus $\mathcal{A} := \mathrm{I}_3 \otimes \kappa^\top$
- But like most SoTA deep-learning models, we represent rotations as $r(R) \in \mathbb{R}^6$
- We need to recover the rotation matrix using the non-linear $\mathcal{D}(r)$

Can ΠGDM be used even after applying $\mathcal{D}()$?

Incorporating the Non-linearity

- Turns out, we can, if we apply the column normalization on the predicted r_M first.
- Let's term the CFG score model $\epsilon_\theta(r_M^t, t, r_m)$, where the *t* parameter is incorporated to inform the denoiser of the current noise level.
- We then have the following Theorem:

Theorem 1. *We are given a well-trained score model ϵ_θ, that learns the score function $\epsilon_\theta(r_M^t, t, r_m)$, and denoises $\hat{r}_M^t$. If the model ensures that $||\hat{r}_j^{t,1:3}|| = ||\hat{r}_j^{t,4:6}|| = 1$, $\langle \hat{r}_j^{t,1:3}, \hat{r}_j^{t,4:6} \rangle = 0$, $\forall j \in M$ then $p_t(\mathcal{D}(r_M^0)|r_M^t) \approx \mathcal{N}(\mathcal{D}(\hat{r}_M^t), w_t^2 \Sigma_{\hat{r}_M^t})$ where $\Sigma_{\hat{r}_M^t}$ is a positive definite matrix.*

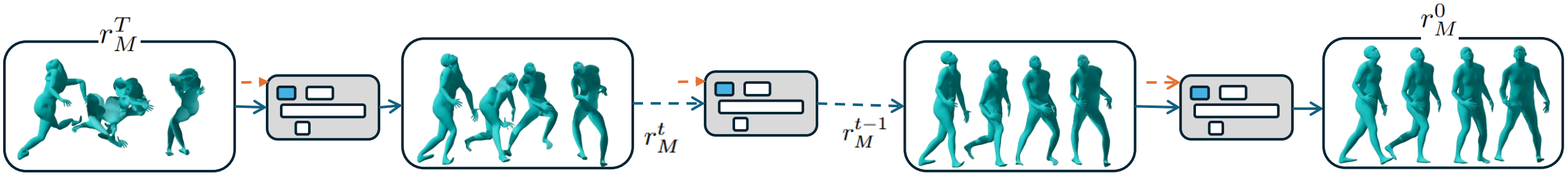
- In other words, we can use a modified Gaussian approximation over the isotropic Gaussian from the original ΠGDM algorithm.

InPose

- We can thus calculate the modified Likelihood score as follows:

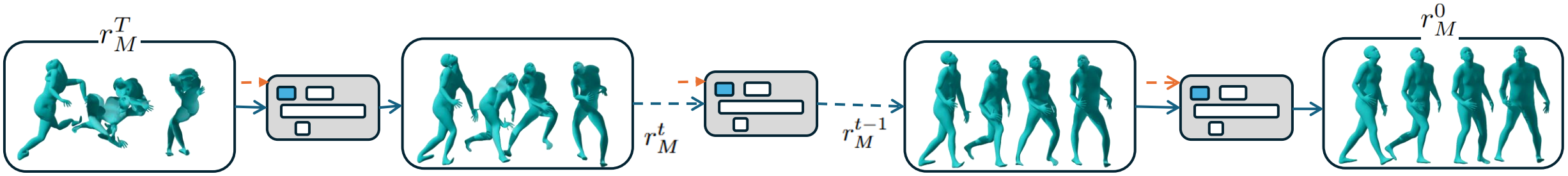
$$\nabla_{r_M^t} \log p_t(l_m | r_M^t, r_m) = \underbrace{((l_m - \mathcal{A} \cdot \mathcal{D}(\hat{r}_M^t))^\top (w_t^2 \mathcal{A} \Sigma_{\hat{r}_M^t} \mathcal{A}^\top + \sigma_l^2 \mathrm{I})^{-1} \mathcal{A}}_{\text{vector}} \underbrace{\frac{\partial \mathcal{D}(\hat{r}_M^t)}{\partial r_M^t})^\top}_{\text{Jacobian}}$$

InPose Pipeline

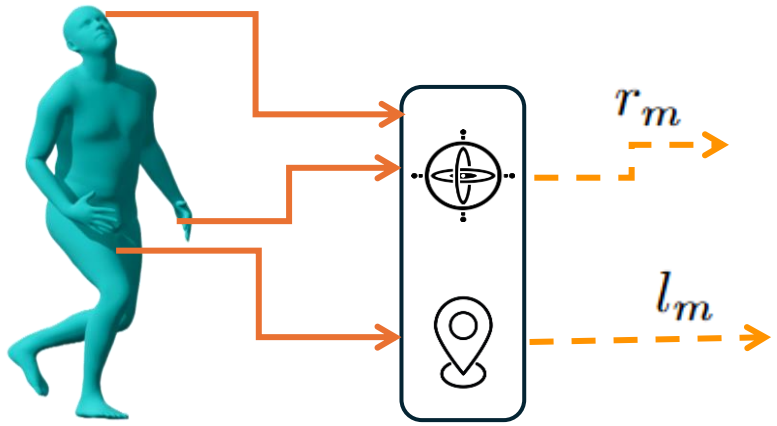


We use DDPM during training and DDIM during inference.

InPose Pipeline

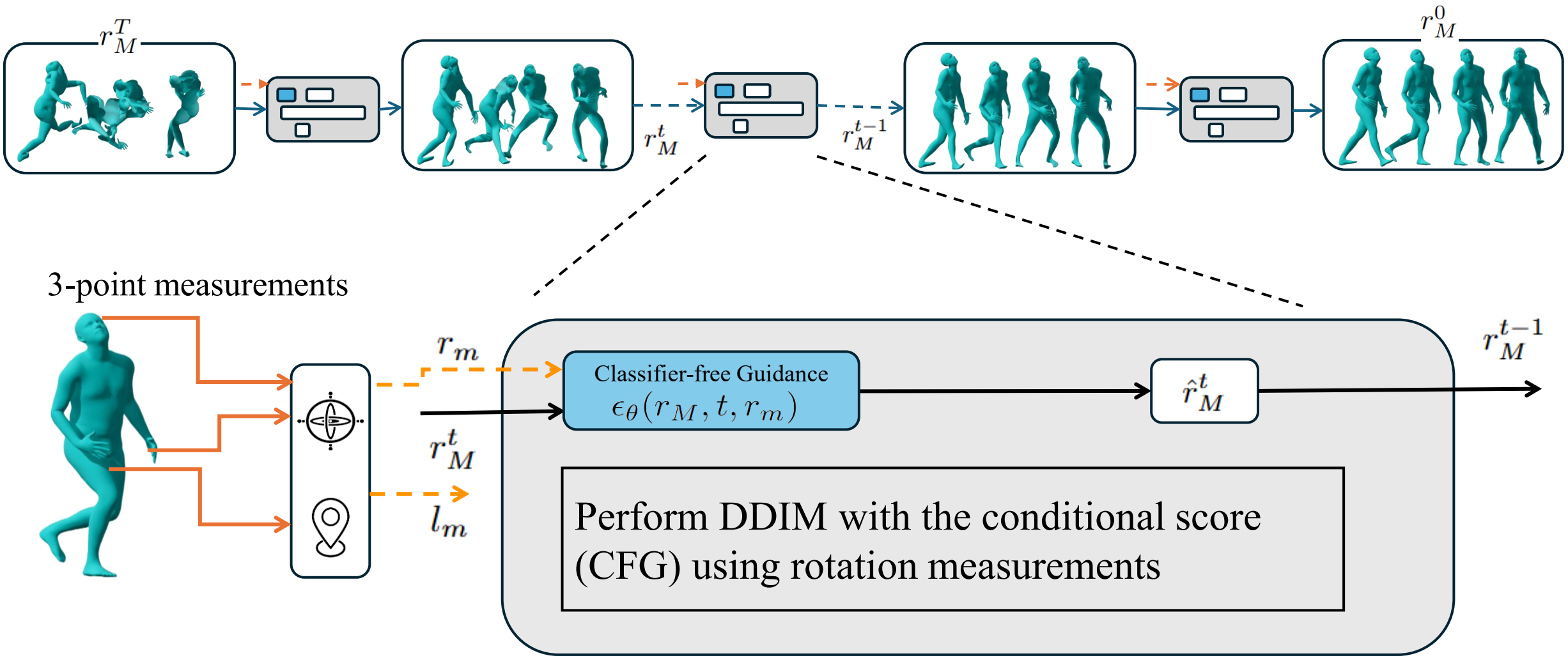


3-point measurements

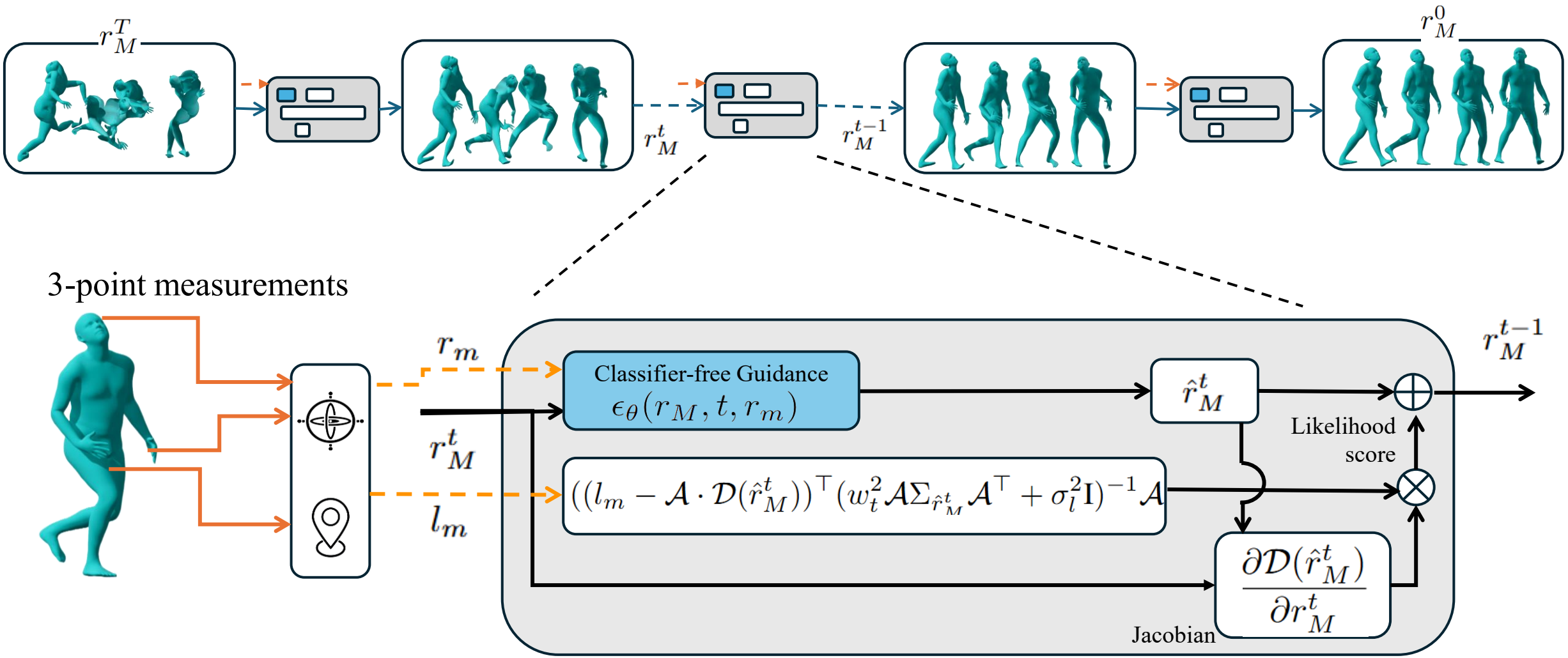


Capture location and rotation measurements from the head and wrists

InPose Pipeline

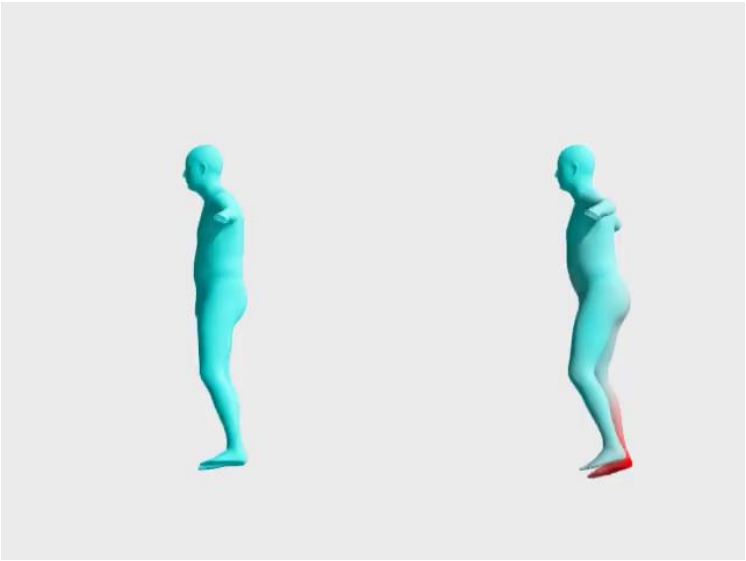


InPose Pipeline

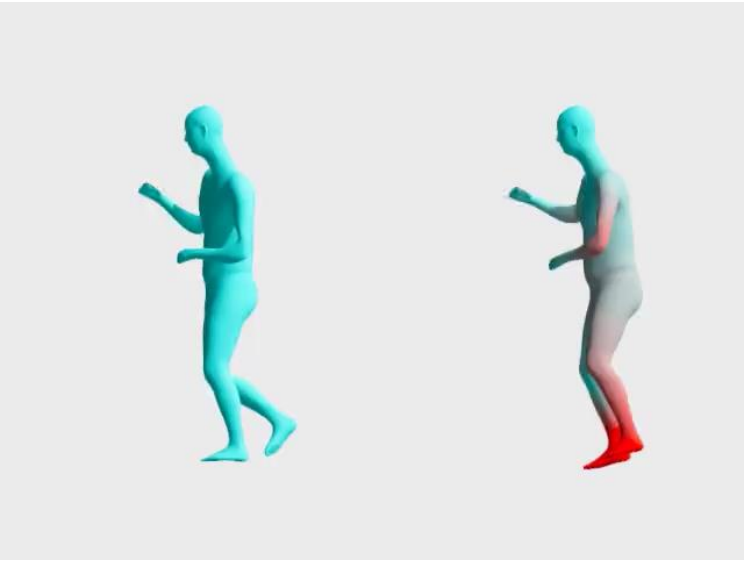


Results

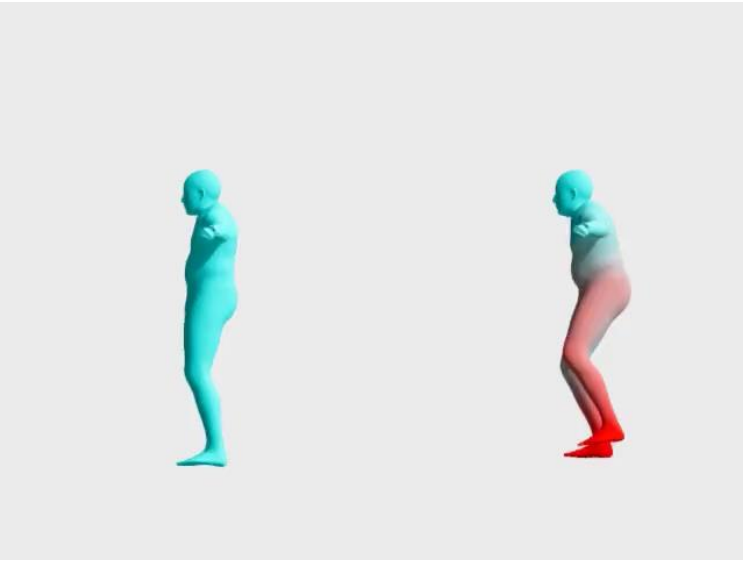
Default body shape



Tall body shape



Short body shape



InPose limitations

- Lower body movement – We don't have lower body measurements:
    - Option 1: infer the lower body movement from the current CFG score (prior-based).
    - Option 2: use the head joint location l_{head} for CFG.
- Differential Inputs – We can't directly use the joint positions in our linear system because we don't have access to the root location beforehand.
    - Use differential inputs such as $l_{\text{head}} - l_{\text{wrist}}$.
    - Likelihood is thus proportional to:

$$\nabla_{r_M^t} \log p_t(l_m | r_M^t, r_m) \propto ((l_{head} - l_{wrists}) - \mathcal{A}\hat{r}_M^t)$$