

# Antithetic Noise in Diffusion Models

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Scan for project



# Agenda

- Overview
- Antithetic Noise
- Application

# Diffusion Model

- A class of generative models (like GANs, VAEs, autoregressive models)
- Achieve **state-of-the-art** quality in image generation
- **Real world product:**
  - Stable Diffusion
  - MidJourney
  - DALL·E 3 (OpenAI)
  - Imagen (Google DeepMind)

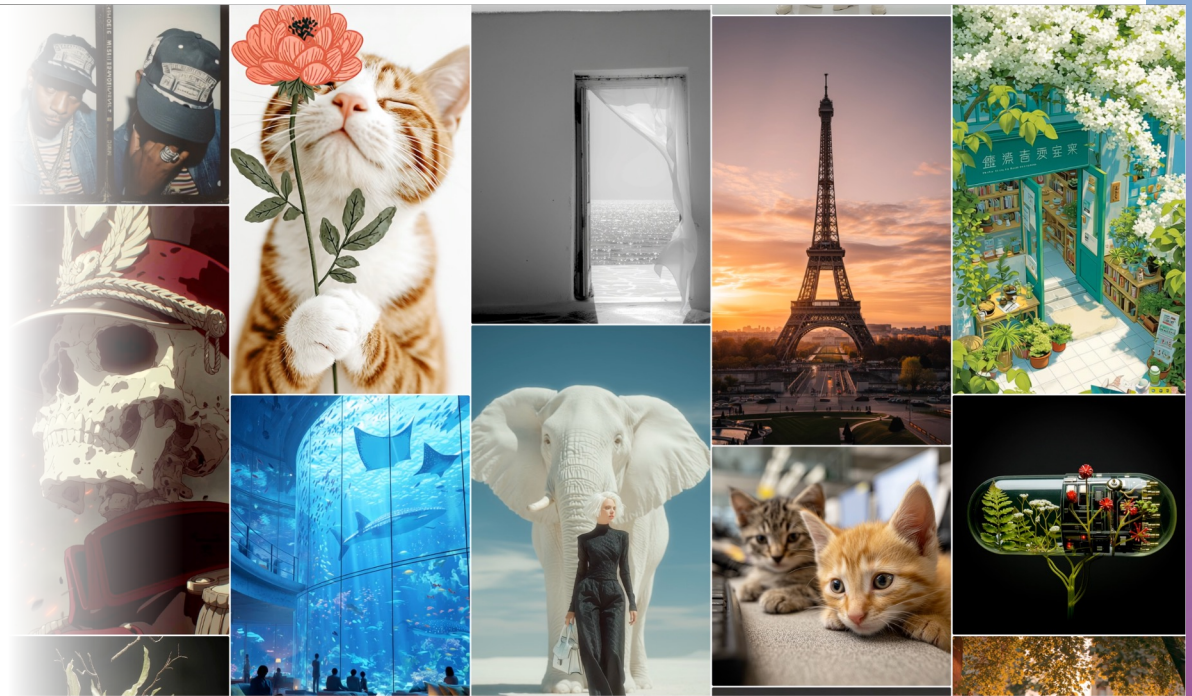


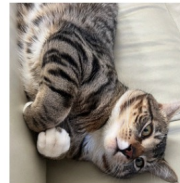
Image from MidJourney gallery

# Diffusion Model

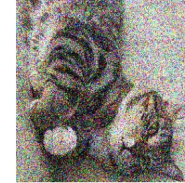
- **Training: image  $\rightarrow$  noise**
  - Corrupted with **additive Gaussian noise**
  - Asymptotically transformed into pure Gaussian noise
- **Goal:** learn the reverse process
- **Generation: noise  $\rightarrow$  image**
  - Model reverses the process with **predicted noise**
  - **Denoises** from pure Gaussian noise to image samples

**Training:**

t = 0

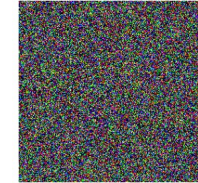


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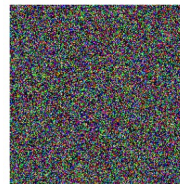
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t = 1000



**Generation:**

t = 1000

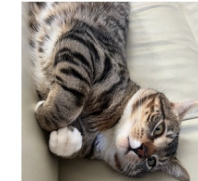


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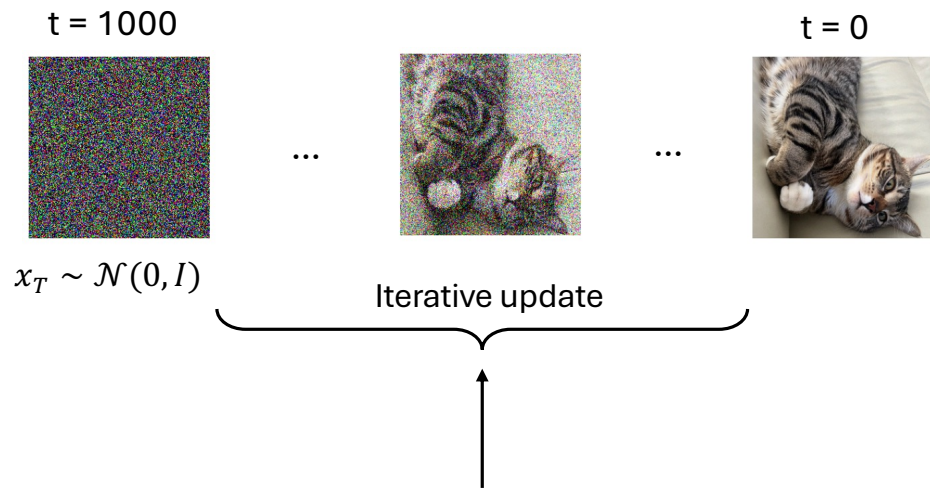
t = 0



$$x_T \sim \mathcal{N}(0, I)$$

# Diffusion Model

- **Network weights**
  - Architectures: U-Net, Transformers, etc.
- **Denosing schedule**
  - Defines how noise is removed step by step
  - DDPM – **stochastic** reverse process
  - DDIM – **deterministic, fast** reverse process
- **Initial Gaussian noise**
  - $x_T \sim \mathcal{N}(0, I)$



One step DDIM update formula

$$\mathbf{x}_{t-1} = \sqrt{\alpha_{t-1}} \left( \frac{\mathbf{x}_t - \sqrt{1 - \alpha_t} \epsilon_{\theta}^{(t)}(\mathbf{x}_t)}{\sqrt{\alpha_t}} \right) + \sqrt{1 - \alpha_{t-1}} \epsilon_{\theta}^{(t)}(\mathbf{x}_t)$$

$\alpha_t$  coefficients determined by the noise schedule

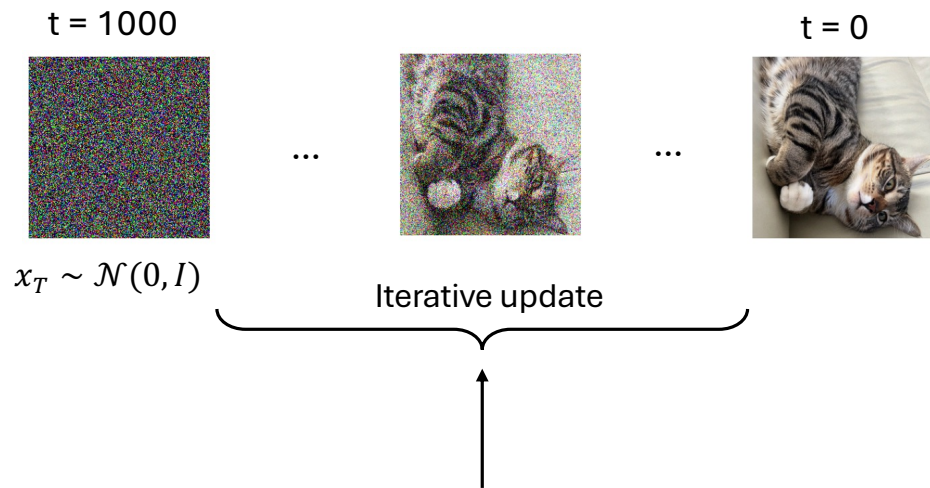
$\epsilon_{\theta}^{(t)}(\mathbf{x})$  noise predicted by the denoising network with weight  $\theta$

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**Question:**

**How would initial noise change outcome?**



$$\mathbf{x}_{t-1} = \sqrt{\alpha_{t-1}} \left( \frac{\mathbf{x}_t - \sqrt{1 - \alpha_t} \epsilon_{\theta}^{(t)}(\mathbf{x}_t)}{\sqrt{\alpha_t}} \right) + \sqrt{1 - \alpha_{t-1}} \epsilon_{\theta}^{(t)}(\mathbf{x}_t)$$

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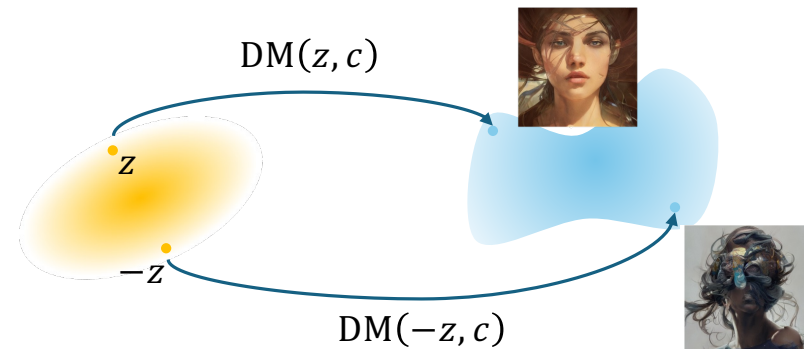
$\epsilon_{\theta}^{(t)}(\mathbf{x})$  noise predicted by the denoising network with weight  $\theta$

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# Antithetic Initial Noise

- **Random/Random (RR):** Two independent Gaussian noise vectors  $z_1$  and  $z_2$  are sampled.
- **Positive/Negative (PN):** A single Gaussian noise vector  $z$  is paired with its negation  $-z$ .

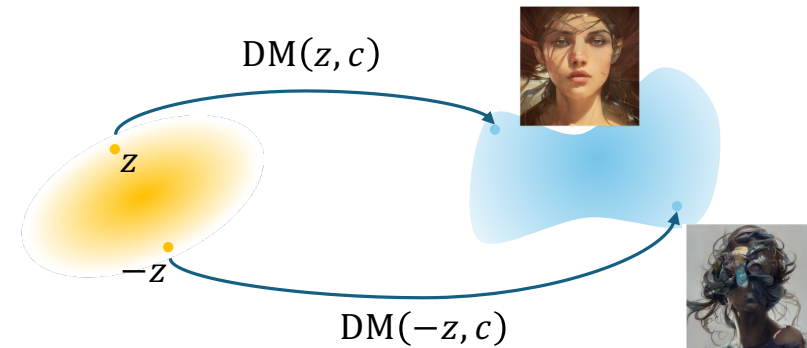


$$z \sim \mathcal{N}(0, I)$$

$c$ , condition if available (text prompt, class label)

# Antithetic Initial Noise

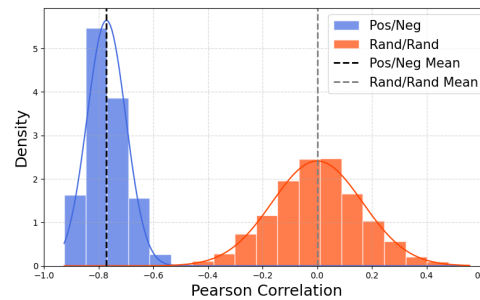
- **Well-studied technique**
  - Classic variance reduction method in Monte Carlo simulation
  - Natural to adapt
- **Preserve quality**
  - Since  $z \sim \mathcal{N}(0,1) \Rightarrow -z \sim \mathcal{N}(0,1)$
  - Generated samples  $DM(z, c)$  and  $DM(-z, c)$  share the same distribution  $\rightarrow$  **no loss in quality**
- **Maximal separation in noise space**
  - $z$  and  $-z$  lie on opposite poles  $\rightarrow$  **largest possible perturbation**



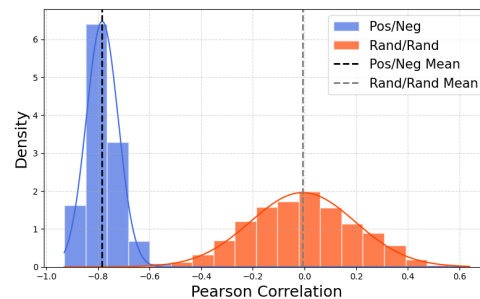
# Discovery

- **Apply Antithetic Noise:**
  - Datasets: CIFAR10, CelebA, Church
  - Architecture : U-Net (SD1.5) and transformer (DiT)

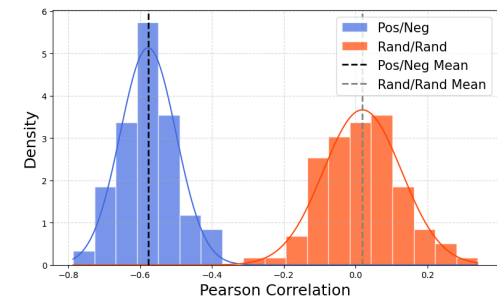
- **Pixel-wise Correlation**
  - Standard Pearson correlation
  - Centralized Pearson correlation
    - Correlation with mean subtracted
    - To correct image structure-level bias



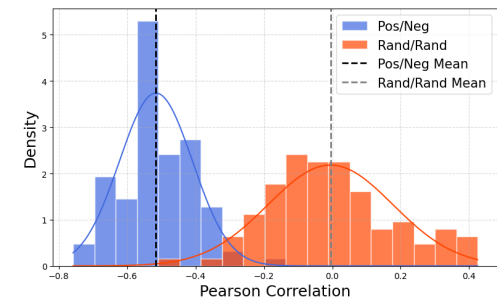
LSUN-Church



CelebA-HQ



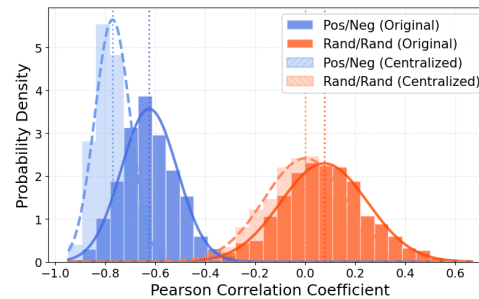
Stable Diffusion



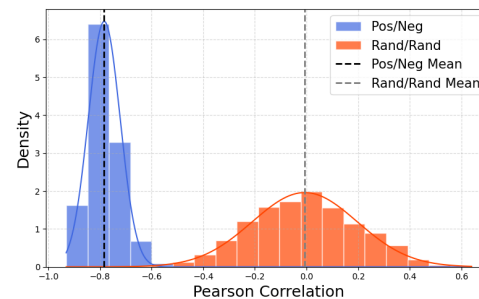
DiT

# Discovery

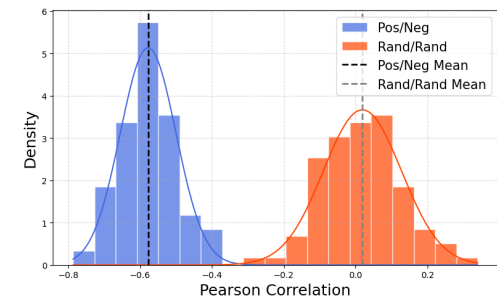
- **Main Finding:**
  - Pairing noise  $z$  with its negation  $-z$  produces **strongly negatively correlated** samples
- **Universal Phenomenon:**
  - Works across all unconditional, conditioned latent-diffusion, and diffusion-posterior samplers
  - Works on generative models beyond diffusion model: VAE, normalizing flow



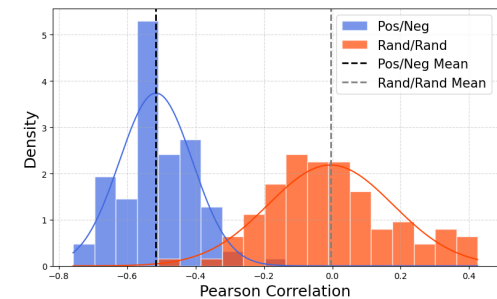
LSUN-Church



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Stable Diffusion



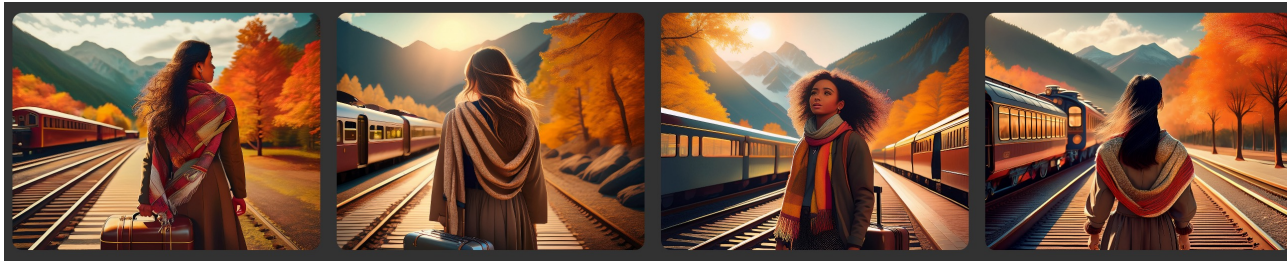
DiT

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# Diversity Enhancement

A young woman standing on a vintage train platform with a suitcase, golden hour light casting long shadows, wind blowing her scarf. Old-school train in the distance, surrounded by mountains and fall trees - nostalgic



Art illustration of a moonlit jungle clearing where luminous giant mushrooms tower over a lone wanderer in a crimson cloak, fireflies tracing golden spirals through misty air; illustrated as a rich digital painting with painterly brush-work soft rim-lighting, and deep teal-violet shadows for a dreamy, storybook fantasy mood



# Diversity Enhancement

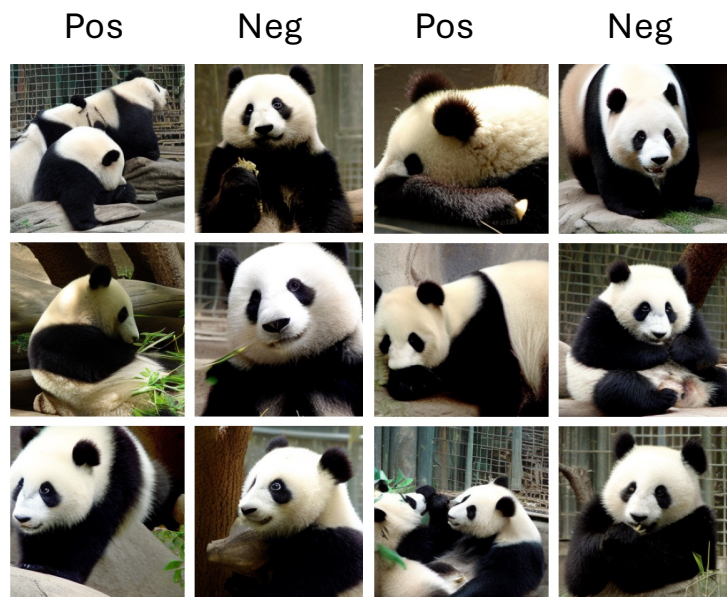
- **Problem:** Standard sampling produces similar images
- **Solution:** Antithetic pairs explore opposite regions of data manifold
- **Benefits:**
  - No quality degradation
  - Zero computational overhead
- **Metric:** higher diversity
  - SSIM (lower better)
  - LPIPS (higher better)

Metric	Unconditional Diffusion			Conditional Diffusion	
	CIFAR-10	Church	CelebA-HQ	SD1.5	DiT
SSIM (%)	88.78	45.69	36.78	28.32	23.99
LPIPS (%)	6.69	3.54	15.14	5.78	10.62

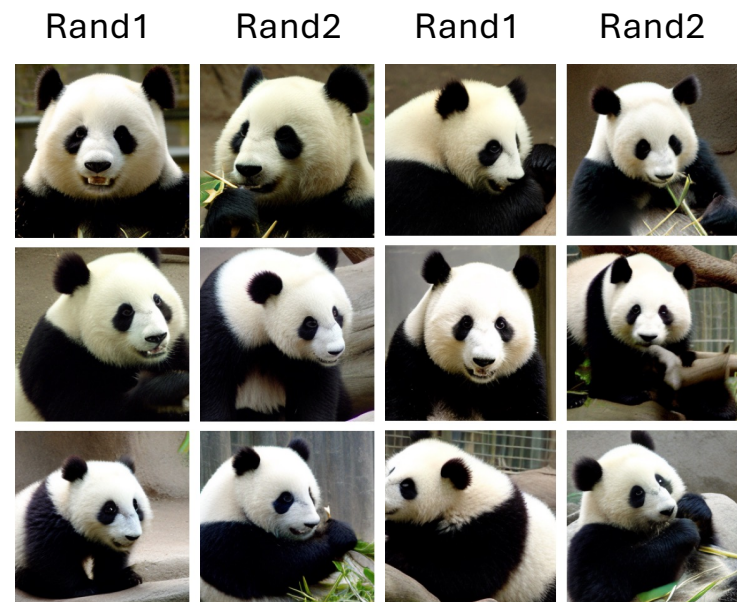
Table 1: Average percentage improvement of PN over RR pairs.

# Diversity Visualization

PN Pairs



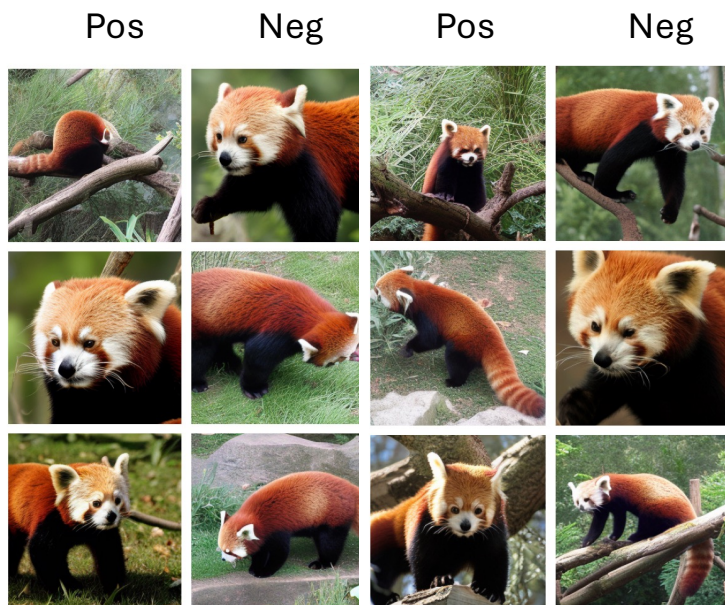
RR Pairs



✓ More diverse !

# Diversity Visualization

PN Pairs



RR Pairs



✓ More diverse !

# Diversity Visualization

PN Pairs



RR Pairs

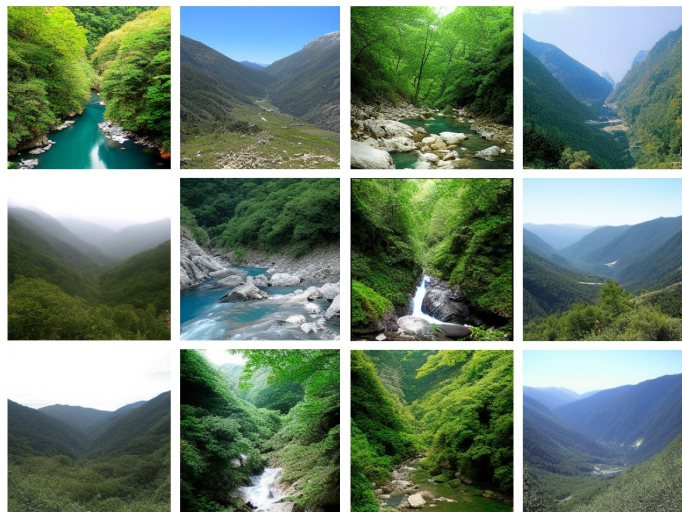


✓ More diverse !

# Diversity Visualization

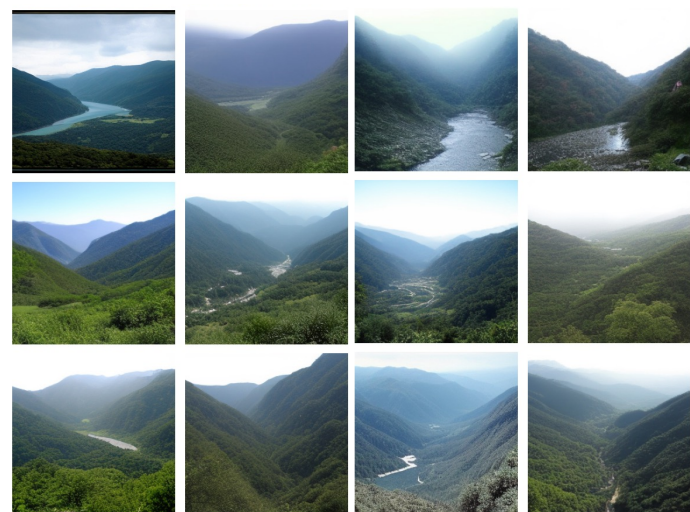
PN Pairs

Pos Neg Pos Neg



RR Pairs

Rand1 Rand2 Rand1 Rand2



✓ More diverse !

# Pixel Statistics Uncertainty

- **Idea:** Use negative correlated samples to reduce variance.
- **Metric:**
  - Confidence interval
  - Relative efficiency w.r.t. MC  $\left(\frac{CI_{MC}}{CI}\right)^2$
- **Estimators:**
  - **Standard Monte Carlo:**
    - draw  $N$  independ  $z_1, \dots, z_N \sim \mathcal{N}(0, I)$
  - **Antithetic Monte Carlo (AMC):**
    - Generate  $2K$  pairs negatively correlated sample

Dataset		Brightness	Pixel Mean	Contrast
Stable Diff.	MC	5.45	5.59	2.18
	AMC	<b>1.26 (18.83)</b>	<b>1.41 (15.84)</b>	<b>0.89 (6.25)</b>
CIFAR-10	MC	2.00	2.04	1.08
	AMC	<b>0.35 (32.66)</b>	<b>0.39 (27.12)</b>	0.23 (22.05)
Church	MC	1.67	1.67	1.02
	AMC	<b>0.14 (136.37)</b>	<b>0.16 (106.79)</b>	<b>0.20 (27.16)</b>
DiT	MC	7.52	7.74	3.32
	AMC	<b>3.13 (5.79)</b>	<b>3.12 (6.16)</b>	<b>1.72 (3.74)</b>

Table 2: CI lengths and efficiency  $\left(\frac{CI_{MC}}{CI}\right)^2$

# Summary

- ✓ **Discovery:** Universal negative correlation from antithetic initial noise
- ✓ **Theory:** Symmetry conjecture explaining the phenomenon
- ✓ **Applications:** Enhanced diversity + uncertainty quantification, and more...
- ✓ **Extensions:** Quasi-Monte Carlo integration
- ✓ **Impact:** Training-free, model-agnostic, zero overhead



# Thank you!